

Πρόταση: $\forall X \cup Y = Y \text{ τότε } X \subseteq Y.$

Απόδειξη: $(x \in X) \Rightarrow (x \in Y) \equiv \neg(x \in X) \vee (x \in Y)$

$$\equiv \neg(x \in X) \vee (x \in X \cup Y)$$
$$\equiv \neg(x \in X) \vee ((x \in X) \vee (x \in Y))$$
$$\equiv (\neg(x \in X) \vee (x \in X)) \vee (x \in Y)$$
$$\equiv \underline{1} \vee (x \in Y) \equiv \underline{1}$$

'Αρα $(x \in X) \Rightarrow (x \in Y) \equiv \underline{1}$

'Αρα $X \subseteq Y.$

Άσκηση 2B: $A \cup B = A \cap B$ αν και μόνο αν $A = B$

Απόδειξη: Αν $A = B$ τότε $A \cap B = A \cap A = A$
 $A \cup B = A \cup A = A$ } $\implies A \cup B = A \cap B$

Έστω $A \cup B = A \cap B$

$A \cup B = A \cap B$ } $\implies A \cup B \subseteq A$ } $\implies A \cup B = A$ ^{Πρόταση} } $B \subseteq A$
CWS $\rightarrow A \cap B \subseteq A$ } $\implies A \subseteq A \cup B$ } $\implies A = B$

$A \cup B = A \cap B$ } $\implies A \cup B \subseteq B$ } $\implies A \cup B = B$ ^{Πρόταση} } $A \subseteq B$
CWS $\rightarrow A \cap B \subseteq B$ } $\implies B \subseteq A \cup B$ } $\implies A = B$

Alternative solution 2B

$$\forall A \cup B = A \cap B \quad \text{zbtz} \quad A = B$$

$$\begin{aligned} x \in A &\equiv (x \in A) \wedge \perp \equiv (x \in A) \wedge ((x \in B) \vee \neg(x \in B)) \\ &\equiv ((x \in A) \wedge (x \in B)) \vee ((x \in A) \wedge \neg(x \in B)) \\ &\equiv (x \in A \cap B) \vee (x \in A \setminus B) \end{aligned}$$

$$\textcircled{2} A \cap B = A \cup B$$

$$\textcircled{2} \equiv (x \in A \cup B) \vee (x \in A \setminus B)$$

$$\textcircled{2} A \setminus B \subseteq A \subseteq A \cup B \quad \equiv x \in (A \cup B) \cup (A \setminus B)$$

$$\forall a (A \cup B) \cup (A \setminus B) = A \cup B \quad \textcircled{2} \equiv x \in A \cup B$$

$$\forall a \quad \boxed{x \in A \equiv x \in A \cup B} \textcircled{1} \quad \text{O} \mu \text{o} \text{i} \alpha \quad \boxed{x \in B \equiv x \in A \cup B} \textcircled{2}$$

1, 2

$$\rightsquigarrow x \in A \equiv x \in B \quad \forall a \quad A = B$$

Άσκηση 2Γ : $A \setminus B = B \setminus A$ αν και μόνο αν $A = B$.

Απόδειξη : Αν $A = B$ τότε $A \setminus B = A \setminus A = B \setminus A$.

Απόδειξη : Αν $A \setminus B = B \setminus A$

τότε $A \stackrel{\text{⊗}}{=} (A \setminus B) \cup (A \cap B) \stackrel{\text{⊗}}{=} (B \setminus A) \cup (A \cap B) \stackrel{\text{⊗}}{=} B$.

Άρα $A = B$

$$\text{⊗ } X = (X \setminus Y) \cup (X \cap Y)$$

$$x \in X \equiv (x \in X) \wedge \perp$$

$$\equiv (x \in X) \wedge (\neg(x \in Y) \vee (x \in Y))$$

$$\equiv (x \in X \wedge \neg(x \in Y)) \vee (x \in X \wedge (x \in Y))$$

$$\equiv (x \in X \setminus Y) \vee (x \in X \cap Y)$$

$$\equiv x \in (X \setminus Y) \cup (X \cap Y)$$

Άσκηση 2Δ: $A \Delta B = B \setminus A$ αν και μόνο αν $A \subseteq B$.

Απόδειξη: Αν $A \subseteq B$ τότε $x \in A \setminus B = (x \in A) \wedge \neg(x \in B) \equiv \emptyset$

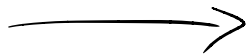
$x \in A$	$x \in B$	$\neg(x \in B)$	$(x \in A) \wedge \neg(x \in B)$
1	1	0	0
0	0	1	0
1	0	1	1
0	1	0	0

$A \subseteq B \rightarrow$ (row 3 is crossed out)

$$\text{Άρα } A \setminus B = \emptyset$$

$$\text{Άρα } A \Delta B = (A \setminus B) \cup (B \setminus A) = \emptyset \cup (B \setminus A) = B \setminus A.$$

$$\text{Άρα } A \Delta B = B \setminus A$$



Απόδειξη (αντιστροφή)

$$A \cup A \Delta B = B \setminus A \quad \text{τότε} \quad (A \setminus B) \cup (B \setminus A) = B \setminus A \xrightarrow{\text{Πρόταση}} \underline{A \setminus B \subseteq B \setminus A}$$

$$X = A \setminus B$$

$$Y = B \setminus A$$

$$\underline{A \setminus B \subseteq B \setminus A} \xrightarrow{\quad} (A \setminus B) \cup B \subseteq (B \setminus A) \cup B \xrightarrow{\quad} A \cup B \subseteq B \quad \left. \begin{array}{l} \\ \text{CW 15 } B \subseteq A \cup B \end{array} \right\} A \cup B = B$$

\downarrow Πρόταση

$$A \subseteq B$$