Calculus Honors - M7H

Implicit Differentiation and Second Derivative Test - Homework 1

1. Find y' from each of the following expressions:

   (i) $x^2 + y^2 = 25$

   (ii) $x^3 + y^3 = 6xy$

   (iii) $x^2 + xy + y^2 = 7$

   (iv) $x^2y + y^2 = 10$

   (v) $\sin(xy) = x + y$

   (vi) $e^{xy} = x^2 + y$

   (vii) $\ln(x^2 + y) = xy$

   (viii) $x^3 + 2y^3 = 6x^2y$

2. For each curve, find the equation of the tangent line at the given point.

   (a) $x^2 + y^2 = 13$ at the point $(2, 3)$

   (b) $x^2 + xy + y^2 = 21$ at the point $(3, 3)$

   (c) $x^3 + y^3 = 16$ at the point where $x = 2$

   (d) $e^{xy} = e^2$ at the point where $x = 1$

3. For each function:

   (a) find the critical points

   (b) use the second derivative test to classify each critical point as a local maximum, local minimum, or neither.

   (i) $f(x) = x^3 - 6x^2 + 9x + 1$

   (ii) $g(x) = x^4 - 8x^2 + 3$

   (iii) $h(x) = x^3 - 3x^2 - 9x + 2$

   (iv) $m(x) = x + \dfrac{1}{x}$

   (v) $q(x) = xe^{-x}$

   (vi) $b(x) = \ln(x^2 + 1) - x$

4. Find the values of the parameters a and b so that the function $f(x) = x^3 + ax^2 + bx$ has critical points at $x = 1$ and $x = 3$, with a local minimum at $x = 1$ and a local maximum at $x = 3$.