

## Άσκηση 2:

$$a) X \subseteq X \cup Y \text{ και } Y \subseteq X \cup Y$$

$$b) X \cap Y \subseteq X \text{ και } X \cap Y \subseteq Y$$

$$d) X \cup \emptyset = X \text{ και } X \cap \emptyset = \emptyset$$

$$δ) X \cup Y = Y \text{ αν και μόνο αν } X \subseteq Y$$

$$ε) X \cap Y = Y \text{ αν και μόνο αν } Y \subseteq X$$

$$\gamma) \boxed{X \cap \emptyset = \emptyset}$$

$$\begin{aligned}
 x \in X \cap \emptyset &\equiv (x \in X) \wedge (x \in \emptyset) \\
 &\equiv (x \in X) \wedge 0 \\
 &\equiv 0 \\
 &\equiv (x \in \emptyset)
 \end{aligned}$$

$$\boxed{X \cup \emptyset = X}$$

$$x \in X \cup \emptyset \equiv (x \in X) \vee (x \in \emptyset)$$

$$\begin{aligned}
 P \vee 0 &\equiv P \\
 &\stackrel{a)}{\equiv} (x \in X) \vee 0 \\
 &\equiv (x \in X)
 \end{aligned}$$

$$a) \underline{X \subseteq X \cup Y}$$

$$(x \in X) \Rightarrow (x \in X \cup Y) \equiv (x \in X) \Rightarrow ((x \in X) \vee (x \in Y))$$

$$\begin{aligned}
 P \rightarrow Q &\equiv (\neg P) \vee Q \\
 &\stackrel{a)}{\equiv} \neg(x \in X) \vee ((x \in X) \vee (x \in Y)) \\
 &\equiv (\neg(x \in X) \vee (x \in X)) \vee (x \in Y)
 \end{aligned}$$

$$\begin{aligned}
 (\neg P) \vee P &\equiv \mathbb{1} \\
 &\stackrel{b)}{\equiv} \mathbb{1} \vee (x \in Y)
 \end{aligned}$$

$$\begin{aligned}
 \mathbb{1} \vee P &\equiv \mathbb{1} \\
 &\stackrel{c)}{\equiv} \mathbb{1}
 \end{aligned}$$

$$b) \underline{X \cap Y \subseteq X}$$

$$\begin{aligned}
 x \in (X \cap Y) \Rightarrow (x \in X) &\equiv ((x \in X) \wedge (x \in Y)) \Rightarrow (x \in X) \\
 &\equiv \neg((x \in X) \wedge (x \in Y)) \vee (x \in X)
 \end{aligned}$$

$$\equiv (\neg(x \in X) \vee \neg(x \in Y)) \vee (x \in X)$$

$$\begin{aligned}
 (P \vee \neg A) \vee P \\
 \equiv P \vee (\neg A \vee P)
 \end{aligned}$$

$$\stackrel{a)}{\equiv} (x \in X) \vee (\neg(x \in X) \vee \neg(x \in Y))$$

$$\equiv ((x \in X) \vee \neg(x \in X)) \vee \neg(x \in Y)$$

$$P \vee (\neg P) \equiv \mathbb{1}$$

$$\stackrel{b)}{\equiv} \mathbb{1} \vee \neg(x \in Y)$$

$$\equiv \mathbb{1}$$

δ)  $X \cup Y = Y$  αν και μόνο αν  $X \subseteq Y$

Αν  $X \cup Y = Y$ , τότε  $X \subseteq Y$

$$P(x) : (x \in X) \Rightarrow (x \in Y)$$

$$\text{υπόθεση } \otimes \equiv (x \in X) \Rightarrow (x \in X \cup Y)$$

$$\equiv (x \in X) \Rightarrow ((x \in X) \vee (x \in Y))$$

$$P \Rightarrow a \quad \text{πν} \vee a \quad \otimes \equiv \neg(x \in X) \vee ((x \in X) \vee (x \in Y))$$

$$P \vee (a \vee R) \quad \otimes \equiv (\neg(x \in X) \vee (x \in X)) \vee (x \in Y)$$

$$\equiv \mathbb{1} \vee (x \in Y)$$

$$\equiv \mathbb{1}$$

Αν  $X \subseteq Y$ , τότε  $X \cup Y = Y$

$$x \in X \cup Y \equiv (x \in X) \vee (x \in Y)$$

$$\otimes \equiv (x \in Y)$$

⊗ γιατί  $((x \in X) \vee (x \in Y)) \Leftrightarrow (x \in Y)$   
είναι ταυτολογία.

$x \in X$	$x \in Y$	$(x \in X) \vee (x \in Y)$	$(x \in X) \vee (x \in Y) \Leftrightarrow (x \in Y)$
1	1	1	1
0	1	1	1
1	0	1	0
0	0	0	1

$X \subseteq Y$

### Άσκηση 3

$$α) X \setminus X = \emptyset$$

$$β) X \setminus \emptyset = X$$

$$γ) \emptyset \setminus X = \emptyset$$

$$δ) (X \cup Y) \setminus Z = X \setminus (Y \cup Z)$$

$$ε) (X \setminus Y) \setminus Z = (X \setminus Z) \setminus Y$$

$$ζ) X \Delta X = \emptyset$$

$$η) X \Delta Y = Y \Delta X$$

$$θ) X \Delta \emptyset = X$$

$$ι) X \Delta Y = (X \cup Y) \setminus (X \cap Y)$$

$$\begin{aligned} α) x \in X \setminus X &\equiv (x \in X) \wedge (x \notin X) \\ &\equiv (x \in X) \wedge \neg(x \in X) \\ &\equiv \emptyset \\ &\equiv (x \in \emptyset) \end{aligned}$$

$$\begin{aligned} β) x \in X \setminus \emptyset &\equiv (x \in X) \wedge (x \notin \emptyset) \\ &\equiv (x \in X) \wedge \mathbb{1} \\ P \wedge \mathbb{1} &\equiv P \\ &\equiv (x \in X) \end{aligned}$$

$$\begin{aligned} γ) x \in \emptyset \setminus X &\equiv (x \in \emptyset) \wedge (x \notin X) \\ &\equiv \emptyset \wedge (x \notin X) \\ &\equiv \emptyset \\ &\equiv (x \in \emptyset) \end{aligned}$$

$$\underline{5) (X \setminus Y) \setminus Z = X \setminus (Y \cup Z)}$$

$$\begin{aligned} x \in (X \setminus Y) \setminus Z &\equiv (x \in (X \setminus Y)) \wedge (x \notin Z) \\ &\equiv ((x \in X) \wedge (x \notin Y)) \wedge (x \notin Z) \\ &\equiv (x \in X) \wedge ((x \notin Y) \wedge (x \notin Z)) \\ &\equiv (x \in X) \wedge (\neg(x \in Y) \wedge \neg(x \in Z)) \\ &\stackrel{\textcircled{5}}{\equiv} (x \in X) \wedge \neg((x \in Y) \vee (x \in Z)) \\ &\equiv (x \in X) \wedge \neg(x \in Y \cup Z) \\ &\equiv x \in X \setminus (Y \cup Z) \end{aligned}$$

$(\neg A) \wedge (\neg B)$   
 $\equiv$   
 $\neg(A \vee B)$

$$\underline{6) (X \setminus Y) \setminus Z = (X \setminus Z) \setminus Y}$$

$$\begin{aligned} (X \setminus Y) \setminus Z &\stackrel{\textcircled{35}}{=} X \setminus (Y \cup Z) \\ &\stackrel{\textcircled{18}}{=} X \setminus (Z \cup Y) \\ &\stackrel{\textcircled{35}}{=} (X \setminus Z) \setminus Y \end{aligned}$$

$$\underline{7) X \Delta X = \emptyset}$$

$$\begin{aligned} X \Delta X &= (X \setminus X) \cup (X \setminus X) \\ &\stackrel{\textcircled{3a}}{=} \emptyset \cup \emptyset \\ &= \emptyset \end{aligned}$$

$$\underline{7) X \Delta Y = Y \Delta X}$$

$$X \Delta Y = (X \setminus Y) \cup (Y \setminus X) \stackrel{\textcircled{18}}{=} (Y \setminus X) \cup (X \setminus Y) = Y \Delta X$$

$$\eta) \quad \underline{X \Delta Y = (X \cup Y) \setminus (X \cap Y)}$$

$$x \in X \Delta Y \equiv x \in (X \setminus Y) \cup (Y \setminus X)$$

$$\equiv (x \in (X \setminus Y)) \vee (x \in (Y \setminus X))$$

$$\equiv ((x \in X) \wedge (x \notin Y)) \vee ((x \in Y) \wedge (x \notin X))$$

$$\begin{array}{l} \text{P} \vee (\text{Q} \wedge \text{R}) \\ \dots \\ (\text{P} \vee \text{Q}) \wedge (\text{P} \vee \text{R}) \end{array}$$

$$\equiv \left[ ((x \in X) \wedge (x \notin Y)) \vee (x \in Y) \right] \wedge \left[ ((x \in X) \wedge (x \notin Y)) \vee (x \notin X) \right]$$

$$\begin{array}{l} (\text{P} \vee \text{Q}) \wedge \text{R} \\ \dots \\ (\text{P} \wedge \text{R}) \vee (\text{Q} \wedge \text{R}) \end{array}$$

$$\equiv \left[ ((x \in X) \vee (x \in Y)) \wedge ((x \notin Y) \vee (x \in Y)) \right] \wedge \left[ ((x \in X) \vee (x \notin X)) \wedge ((x \notin Y) \vee (x \notin X)) \right]$$

$$\begin{array}{l} \text{P} \vee (\text{P}) \\ \dots \\ \mathbb{1} \end{array}$$

$$\equiv \left[ ((x \in X) \vee (x \in Y)) \wedge \mathbb{1} \right] \wedge \left[ \mathbb{1} \wedge ((x \notin Y) \vee (x \notin X)) \right]$$

$$\begin{array}{l} \text{P} \wedge \mathbb{1} \\ \dots \\ \mathbb{1} \end{array}$$

$$\equiv ((x \in X) \vee (x \in Y)) \wedge ((x \notin Y) \vee (x \notin X))$$

$$\begin{array}{l} (\text{P}) \vee (\neg \text{P}) \\ \dots \\ \neg(\text{P} \wedge \neg \text{P}) \end{array} \equiv \text{De Morgan} \equiv ((x \in X) \vee (x \in Y)) \wedge (\neg((x \in X) \wedge (x \in Y))) \equiv x \in (X \cup Y) \setminus (X \cap Y)$$