Calculus Honors - M7H

Extrema - Homework 1

1. Find the local extrema (if there are any) of the following functions (do not forget to find the domains). Show your work.

   (i) $f(x) = 1 + \sqrt{4 - x^2}$

   (ii) $g(x) = x^{\frac{2}{3}}$

   (iii) $h(x) = x^2 + \dfrac{2}{x}$

   (iv) $t(x) = x^2 e^{3x}$

   (v) $p(x) = x^2 + 32\sqrt{x}$

   (vi) $q(x) = \dfrac{x^2}{x-2}$

   (vii) $m(x) = x^2 \ln x$

   (viii) $d(x) = e^x - e^{-x}$

2. Find the extrema of the following functions and classify them as local or global. Show your work.

   (i) $f(x) = mx + b$, where $m, b \in \mathbb{R}, m > 0$, $x \in [-5, 9]$

   (ii) $g(x) = -\dfrac{1}{x^2},\ x \in \left[\dfrac{1}{2}, 3\right]$

   (iii) $h(x) = 2x^3 + 24x^2 - 120x + 6,\ x \in [-10, 3]$

   (iv) $F(x) = \dfrac{1}{x} + \ln x,\ x \in [1, 4]$

   (v) $G(x) = x^2 + 32\sqrt{x},\ x \in [1, 5]$

   (vi) $H(x) = \dfrac{x-2}{x^2-2}, x \in [-1, 1]$

   (vii) $k(x) = (x-1)^2(x+2),\ x \in [-3, 2]$

   (viii) $r(x) = 2x \ln x + x^2 - 4x + 3,\ x \in [1, e]$

3. Find $a, b \in \mathbb{R}$ such that the function $f(x) = ax^3 + bx^2 - 3x + 1$ has relative extrema at $x = -1$ and $x = 1$. For these values of a and b, specify what each extremum is.

4. Consider a rectangle with side lengths x and y and perimeter 10 m.

   (i) Write the area of the rectangle as a function $A(x)$ and find the domain of $A(x)$

   (ii) Prove that from all the rectangles with perimeter 10 m, the square has the maximum area

   (iii) Find the maximum area

5. Consider the function $f(x) = e^x - x - 1$

   (i) Find the domain of f

   (ii) Study the monotonicity of f and find its local extrema

   (iii) Prove that $e^x > 1 + x$ for every $x > 0$