

Calculus Honors - M7H

Continuity - Homework 1

1. Consider the function

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$$

Determine whether f is continuous in \mathbb{R} .

2. Let

$$h(x) = \begin{cases} \sqrt{x+1}, & x < 3 \\ 2x - 5, & x \geq 3 \end{cases}$$

Determine whether h is continuous in \mathbb{R} .

3. Determine the value of c that makes $f(x)$ continuous at $x = 2$:

$$f(x) = \begin{cases} 3x - 4, & x < 2 \\ cx + 1, & x \geq 2 \end{cases}$$

4. Let

$$f(x) = \begin{cases} \frac{\sin(2x)}{x} & x \neq 0 \\ k & x = 0 \end{cases}$$

Find the value of k such that $f(x)$ is continuous.

5. Prove that the equation $x^3 - 2x + 8 = 2x + 7$ has a solution in the interval $(0, 2)$.

6. Let $a, b \in \mathbb{R}$ with $a > 0$. Find a and b so that $f(x)$ is continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{\sqrt{x+a^2} - a}{x}, & x < 0 \\ b, & x = 0, \\ x + 2, & x > 0 \end{cases}$$

7. Consider the polynomial $g(x) = x^{2n+1} + x^{2n} + x^{2n-1} + \dots + x + 1$.

- (i) Find $\lim_{x \rightarrow +\infty} g(x)$
- (ii) Find $\lim_{x \rightarrow -\infty} g(x)$
- (iii) Prove that $g(x)$ has at least one real root