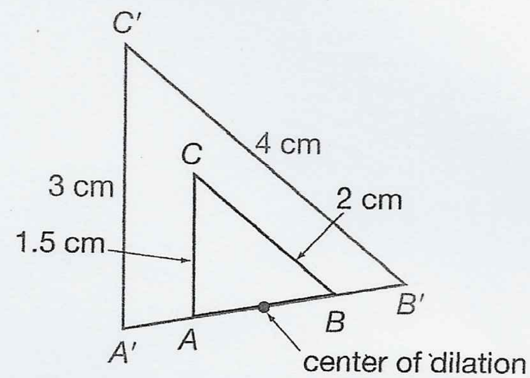
LESSON 10

Dilations and Similarity

Defining Dilation and Similarity

UNDERSTAND A **dilation** is a transformation that moves the points of a line, line segment, or figure either toward or away from a point called the **center of dilation.** The center of dilation can be any point inside the figure, on the figure, or outside the figure.

In the diagram on the right, blue triangle ABC was dilated to produce green triangle $A'B'C'$. The figures have the same shape, but $\triangle A'B'C'$ is twice the size of $\triangle ABC$.

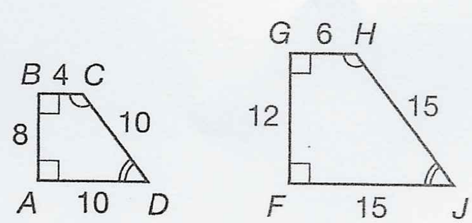


If the center of dilation lies on a line or line segment, the dilated image of the line will be collinear with its preimage. So $\overline{A'B'}$ is collinear with its corresponding side, $\overline{AB}$. If a line or line segment does not pass through the center of dilation, the dilated image will be parallel to the preimage. So, $\overline{B'C'} \parallel \overline{BC}$ and $\overline{A'C'} \parallel \overline{AC}$. (The symbol $\parallel$ means "is parallel to.")

When a dilation is applied to a line segment or closed figure, it changes the size of the image according to a **scale factor**, k. If $k > 1$, the figure is enlarged. If $0 < k < 1$, the figure is reduced. In the figure above, $\triangle ABC$ was dilated by a scale factor of 2, so it was enlarged. Each side of $\triangle A'B'C'$ is twice as long as its corresponding side on $\triangle ABC$.

UNDERSTAND Dilating a figure produces a figure that is the same shape as the original figure, but a different size. Like rigid motions, dilations preserve angle measures. Unlike rigid motions, dilations do not preserve the lengths of line segments. Instead, they produce a figure with sides that are proportional to the sides of the preimage. So, the original figure and its dilated image are **similar** figures.

Trapezoid $ABCD$ was dilated by a scale factor of $\frac{3}{2}$ to form trapezoid $FGHJ$ on the right. The angle marks show that corresponding angles are congruent. Corresponding side lengths are proportional. So, $ABCD \sim FGHJ$. (The symbol $\sim$ means "is similar to.")



$\angle F \cong \angle A \quad \angle G \cong \angle B \quad \angle H \cong \angle C \quad \angle J \cong \angle D$

$\frac{FG}{AB} = \frac{12}{8} = \frac{3}{2} \quad \frac{GH}{BC} = \frac{6}{4} = \frac{3}{2} \quad \frac{HJ}{CD} = \frac{15}{10} = \frac{3}{2} \quad \frac{FJ}{AD} = \frac{15}{10} = \frac{3}{2}$

A **regular polygon** is a polygon in which all sides have the same length and all angles have the same measure. Any two regular polygons of the same type—having the same number of sides—are similar to each other.