

$$1) \quad x, y, z \in \mathbb{Z}$$

$$\text{Av } x|y \text{ και } y|z, \text{ τότε } x|\boxed{3y^2} + \boxed{5yz} - \boxed{8z} - \boxed{7x}$$

Λύση: $x|y \xRightarrow{(6)} x|3y^2$

$$x|y \xRightarrow{(6)} x|5yz$$

(Μεταβατική) $x|y \text{ και } y|z \xRightarrow{(10)} x|z \xRightarrow{(6)} x|8z$

$$(1) x|x \xRightarrow{(6)} x|7x$$

$$\left. \begin{array}{l} x|3y^2 \\ x|5yz \end{array} \right\} \xRightarrow{(9)} x|3y^2 + 5yz$$

$$\left. \begin{array}{l} x|8z \\ x|7x \end{array} \right\} \xRightarrow{(9)} x|8z + 7x \xRightarrow{(6)} x|-(8z + 7x)$$

"
 $-8z - 7x$

$$\left. \begin{array}{l} x|3y^2 + 5yz \\ x|-(8z + 7x) \end{array} \right\} \xRightarrow{(9)} x| \frac{3y^2 + 5yz + (-8z - 7x)}{11}$$

$3y^2 + 5yz - 8z - 7x$

$$2) \ a, b \in \mathbb{Z}$$

$$a \mid 2b-1 \text{ and } a \mid 7b+10 \Rightarrow a = ?$$

$$\begin{array}{lcl} a \mid 2b-1 & \xrightarrow{\textcircled{6}} & a \mid 7 \cdot (2b-1) = 14b-7 \\ & & a \mid 14b-7 \\ a \mid 7b+10 & \xrightarrow{\textcircled{6}} & a \mid 2 \cdot (7b+10) = 14b+20 \\ & & a \mid 14b+20 \end{array} \quad \left. \vphantom{\begin{array}{l} a \mid 2b-1 \\ a \mid 7b+10 \end{array}} \right\} \xrightarrow{\textcircled{9}} \begin{array}{r} a \mid (14b-7) - (14b+20) \\ \quad \quad \quad \parallel \\ \quad \quad \quad 27 \\ a \mid 27 \end{array}$$

$$a \mid 27 \Rightarrow a = \pm 1, \pm 3, \pm 9, \pm 27$$

$$3) d, a \in \mathbb{Z}$$

$$d \mid (2a+1) \text{ and } d \mid (3a-1) \Rightarrow d = ?$$

Answer

$$\begin{array}{l} d \mid 2a+1 \Rightarrow d \mid 3 \cdot (2a+1) = 6a+3 \\ d \mid 3a-1 \Rightarrow d \mid 2 \cdot (3a-1) = 6a-2 \end{array} \quad \left. \vphantom{\begin{array}{l} d \mid 2a+1 \\ d \mid 3a-1 \end{array}} \right\} \Rightarrow d \mid (6a+3) - (6a-2)$$

$$d \mid (6a+3) - (6a-2) = d \mid 6a+3-6a+2 = 5$$

$$\Rightarrow d \mid 5 \Rightarrow d = \pm 1, \pm 5$$

Άσκηση: $m, a \in \mathbb{Z}$, $m > 1$

Αν $m \mid a$, τότε $m \nmid a+1$.

p : $m \mid a$ (m διαιρεί a), $m > 1 \rightarrow$ Υπόθεση

q : $m \nmid a+1$ (m δεν διαιρεί $a+1$) \rightarrow Συμπέρασμα

$\neg q$: $m \mid a+1$

$p \Rightarrow q$

(είναι άτοπο απαγωγή)

Υποθέσω $p \wedge \neg q$ αληθές

Έστω $m \mid a$ και $m \mid a+1$

\Downarrow
 $\exists k_1 \in \mathbb{Z}: a = k_1 \cdot m$

\Downarrow
 $\exists k_2 \in \mathbb{Z}: a+1 = k_2 \cdot m$

$a = k_1 \cdot m$
 $a+1 = k_2 \cdot m$ } $\Rightarrow 1 = (a+1) - a = k_2 \cdot m - k_1 \cdot m = (k_2 - k_1) \cdot m$

Αρα $1 = (k_2 - k_1) \cdot m$
 $k_2 - k_1 \in \mathbb{Z}$ } $\Rightarrow m \mid 1 \Rightarrow m = \pm 1$

ΑΤΟΠΟ

γιατί $m > 1$

Αρα, $p \Rightarrow q$, δηλαδή $m \nmid a+1$

Άσκηση: $m, a \in \mathbb{Z}$

Αν $m \mid a+1$, τότε $m \mid a^2+2a+1$

$$a^2+2a+1 = (a+1)^2$$

Λύση: $m \mid a+1 \Rightarrow \exists k \in \mathbb{Z} : a+1 = k \cdot m$

$$\stackrel{\cdot (a+1)}{\Rightarrow} (a+1) \cdot (a+1) = (a+1) \cdot k \cdot m$$

$$\Rightarrow (a+1)^2 = \underbrace{(a+1) \cdot k}_{\in \mathbb{Z}} \cdot m$$

$$\Rightarrow m \mid (a+1)^2$$