

Review Assignment 01

Surface Integrals

PROBLEM 0.1.

- (1) Find the volume of the solid bounded above and below by the sphere $x^2 + y^2 + z^2 = 9$ and inside the cylinder $x^2 + y^2 = 4$.
- (2) Find the volume of the solid bounded above and below by the sphere $\rho = 4$ and below by the cone $\phi = \pi/6$.
- (3) Find the volume of the solid bounded above and below by the sphere $\rho = 1$ and below by the cone $\phi = \pi/4$.

PROBLEM 0.2.

- (1) Find the Fundamental Vector product of the upper half unit sphere. Then compute the surface area of this surface.
- (2) Find the Fundamental Vector product of the surface which is the portion of the cone $\phi = \pi/4$ between the planes $z = 2$ and $z = 5$. Then compute the surface area of this surface.

PROBLEM 0.3.

- (1) Compute the surface integrals $\int_{\mathcal{R}} z \, dS$, where \mathcal{R} is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the horizontal planes $z = 2$ and $z = 5$.
- (2) Compute the surface integrals $\int_{\mathcal{R}} z^2 \, dS$, where \mathcal{R} is the portion of the cone $\phi = \pi/6$ between the horizontal planes $z = 2$ and $z = 5$.
- (3) Compute the surface integrals $\int_{\mathcal{R}} z^3 \, dS$, where \mathcal{R} is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the horizontal planes $z = 2$ and $z = 5$.

PROBLEM 0.4.

- (1) Use spherical coordinate change of variable to evaluate

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx.$$

- (2) Use cylindrical coordinate change of variable to evaluate

$$\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} x^2 \, dz \, dy \, dx.$$

- (3) Find the volume of the solid bounded above and below by the sphere $\rho = 4$ and below by the cone $\phi = \pi/3$.

PROBLEM 0.5.

- (1) Find the equation of the plane that contains the line $x = 2 - 5t, y = 1 + 3t, z = 7 + 2t$ and is perpendicular to the plane $3x - 6y + 4z = 0$.
- (2) Find the equation of the plane that contains the line $x = 1 - 5t, y = 2 + 3t, z = -1 + 2t$ and is perpendicular to the plane $x - y + z = 0$.
- (3) Find the equation of the plane that contains the lines $x = 1 - 5t, y = 2 + 3t, z = -1 + 2t$ and $x = 1 - 5t, y = 2 + 3t, z = -1 + 2t$.