

Γράφημα $f(x) = \sin x$, $x \in \mathbb{R}$

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$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x \leq \sin^2 x + \cos^2 x = 1$$

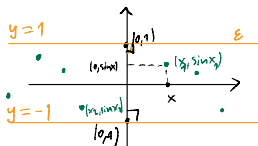
$$\Rightarrow (0 \leq) \sin^2 x \leq 1$$

$$\Rightarrow \sqrt{\sin^2 x} \leq \sqrt{1}$$

$$\Rightarrow |\sin x| \leq 1$$

$$\Rightarrow -1 \leq \sin x \leq 1$$

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$(x, \sin x)$ $x \in \mathbb{R}$

find $\sin x$, $0 \leq x \leq 2\pi$

$$f(x) = 0 \Leftrightarrow \sin x = 0$$

$$\Rightarrow \sin x = 0 \Leftrightarrow x = 0$$

$$\Rightarrow x = \begin{cases} 2k\pi + 0 \\ 2k\pi + \pi \end{cases}, k \in \mathbb{Z}$$

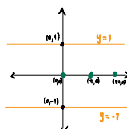
$$\Rightarrow x = \begin{cases} 2k\pi \\ 2k\pi + \pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow x = k\pi, k \in \mathbb{Z}$$

$$0 \leq x \leq 2\pi$$

$$0 \leq k\pi \leq 2\pi \Rightarrow k = 0, 1, 2 \Rightarrow x = \begin{cases} 0 \\ \pi \\ 2\pi \end{cases}$$

$$y = \sin x \Rightarrow (0, 0) \in \sin x \Rightarrow (0, 0) \in \sin x$$



$$f(x) = \sin x, 0 \leq x \leq 2\pi$$

$$f(x) = 1 \Leftrightarrow \sin x = 1$$

$$\Leftrightarrow \sin x = \sin \frac{\pi}{2}$$

$$\Rightarrow x = \begin{cases} 2k\pi + \frac{\pi}{2} \\ 2k\pi + \frac{5\pi}{2} \end{cases} = 2k\pi + \frac{\pi}{2}, k \in \mathbb{Z}$$

$$0 \leq 2k\pi + \frac{\pi}{2} \leq 2\pi$$

$$0 \leq (2k + \frac{1}{2}) \cdot \pi \leq 2\pi$$

$$\Rightarrow 0 \leq 2k + \frac{1}{2} \leq 2$$

$$\Rightarrow k = 0, 1$$

$$\Rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}$$

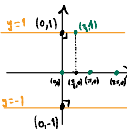
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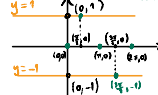
$$\Rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}$$

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$$(\frac{\pi}{2}, 1) = (\frac{\pi}{2}, \sin \frac{\pi}{2}) \in \sin x$$



$$(\frac{5\pi}{2}, -1) \in \sin x$$



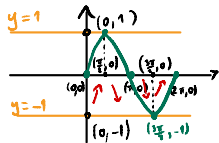
$$f(x) = \sin x, 0 \leq x \leq 2\pi$$

$$[0, \frac{\pi}{2}]: \nearrow \text{increasing}$$

$$[\frac{\pi}{2}, \pi]: \searrow \text{decreasing}$$

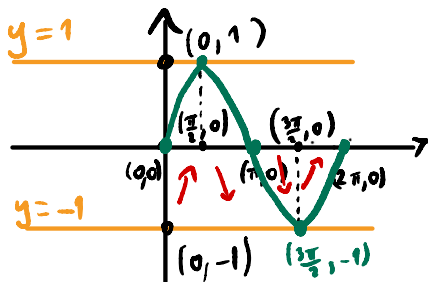
$$[\pi, \frac{3\pi}{2}]: \searrow \text{decreasing}$$

$$[\frac{3\pi}{2}, 2\pi]: \nearrow \text{increasing}$$



$$f(x) = \sin x, \quad 0 \leq x \leq 2\pi$$

Γραφική:



$$f(x) = \sin x, \quad 2\pi \leq x \leq 4\pi$$

⊙ $f(x) = \sin x$ $\sin x$

περίοδος 2π \Rightarrow επανάληψη

$$f(x+2\pi) = f(x), \text{ για κάθε } x \in \mathbb{R}$$

Ανάλυση:

$$\begin{aligned} f(x+2\pi) &= \sin(x+2\pi) \\ &= \sin\left(\underbrace{(x+\pi)+\pi}_{\substack{\text{sin}(y+\pi) \\ = -\sin y}}\right) \\ &= -\sin(x+\pi) \\ &= -(-\sin x) \\ &= \sin x \\ &= f(x) \end{aligned}$$

$$\sin(2\pi) \stackrel{\text{⊙}}{=} \sin 0 = 0$$

$$\sin\left(\frac{5\pi}{2}\right) \stackrel{\text{⊙}}{=} \sin \frac{\pi}{2} = 1$$

$$\sin(3\pi) \stackrel{\text{⊙}}{=} \sin \pi = 0$$

$$\sin\left(\frac{7\pi}{2}\right) \stackrel{\text{⊙}}{=} \sin\left(\frac{3\pi}{2}\right) = -1$$

$$\sin(4\pi) \stackrel{\text{⊙}}{=} \sin(2\pi) = 0$$

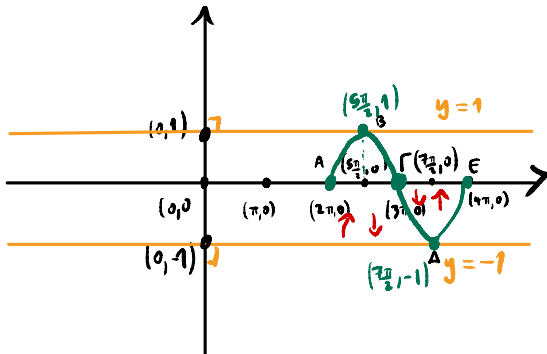
$$A = (2\pi, 0) \in \Gamma_{\sin x}$$

$$B = \left(\frac{5\pi}{2}, 1\right) \in \Gamma_{\sin x}$$

$$\Gamma = (3\pi, 0) \in \Gamma_{\sin x}$$

$$\Delta = \left(\frac{7\pi}{2}, -1\right) \in \Gamma_{\sin x}$$

$$E = (4\pi, 0) \in \Gamma_{\sin x}$$



Γράφημα $f(x) = \sin x$, $x \geq 0$

