AP Precalculus - M3Y & M3Z
Rational Functions - Homework 1

1. Divide the following polynomials. Show your work and write the polynomial long division, as well as the degree of the remainder (or that the remainder is zero):

   (i) $4x^2 - 10x + 6$ by $4x + 2$

   (ii) $2x^5 - 3x^4 - x^2 + x + 4$ by $x^2 + 1$

   (iii) $x^5 + 3x + 2$ by $x^3 + 2x + 1$

   (iv) $-3x^5 + 4x^3 + 3x^2 + 12x - 10$ by $x^2 + 2x - 1$

   (v) $x^4 + 1$ by $x - 1$

   (vi) $x^5 + 1$ by $x^6 - 1$

2. Let $f(x) = x^3 - 2x^2 - x + 2$. In this question, you will solve the equation $f(x) = 0$ without using a calculator.

   (i) Verify that 1 is a root of the polynomial $f(x)$ (that is, show that $f(1) = 0$)

   (ii) By the theorem we proved in class, if 1 is a root of $f(x)$, then $x - 1$ divides $f(x)$.
   Divide $f(x)$ by $x - 1$ and write the polynomial long division (you should get a remainder of 0).

   (iii) Your polynomial division from part (ii) should look like $f(x) = (x - 1)q(x)$, for some polynomial $q(x)$ that you found in part (ii), and $\deg q(x) = 2$.
   Solve the equation $q(x) = 0$, that is, find the roots of $q(x)$ ($q(x) = 0$ should be a quadratic equation).
   The solutions to the initial equation $f(x) = 0$ are $x = 1$ and the two solutions that you will get from the equation $q(x) = 0$.

3. For each one of the following rational functions find the domain and determine whether the function has horizontal and/or vertical asymptotes. If it does, find the equation of each asymptote.

   (i) $\dfrac{2x^3 + 4x^2 - 6x + 1}{x^3 + x^2 + x}$

   (ii) $\dfrac{-2x^3 + 5x + 7}{3x^2 - 9x - 12}$

   (iii) $\dfrac{2}{5x - 6}$

   (iv) $\dfrac{x^3 + 5x^2 + 6x + 1}{x^3 - 2x^2 - x + 2}$

   (v) $\dfrac{-x}{3(x - 6)(x - 5)(x - 4)(x - 3)}$