

THE FLORIDA INTERNATIONAL UNIVERSITY

Fall Semester 2024: Examination 1–

Multivariable Calculus Dr. George Kafkoulis

MAC 2313 Dual Enrollment FIU & Archimedean Upper Conservatory

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Note: By accepting the NDA statement, you agree not to disclose the content of this Exam.

Writing Time: 100 minutes

Permitted Materials: Nothing

Student Name & Signature

in acceptance of the above

NON-DISCLOSURE AGREEMENT : _____

| | Score |
|---|-------|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

Answer **all** questions in this section using the answer booklet(s) provided. Answers are expected to be succinct but complete. Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided **ONLY**. Partial credit will be given. You have **100 minutes** to complete this exam.

Student Name: _____

The Exam Problems

Problem 1

- (1) State and prove the Cauchy-Schwarz-Bunyakovsky inequality (C-S-B).
(2) Suppose that $\vec{a}, \vec{b} \in V_3$. State the Lagrange Identity. Use the Lagrange Identity and (C-S-B) to prove that

$$|\vec{a}| |\vec{b}| \iff \vec{a} \times \vec{b} = \vec{0}_3$$

- (3) Define the cross-product $\vec{a} \times \vec{b}$ of two vectors in V_3 .
(4) Compute $(3\hat{i} - 5\hat{j} + 2\hat{k}) \times (-\hat{i} - \hat{j} + \hat{k})$.

Problem 2

- (1) Define the concept of “ F is multilinear alternating form in V_n ”. State the theorem that gives the formula of each F is multilinear alternating form in V_n . What is the definition of the vector determinant $\text{Det}_n(\vec{a}_1, \dots, \vec{a}_n)$.
(2) Write all permutations of S_3 as products of cycles and products of transpositions.
(3) Compute the determinant providing the triple scalar-product

$$[(3\hat{i} - 5\hat{j} + 2\hat{k}) \times (-\hat{i} - \hat{j} + \hat{k})] \cdot (2\hat{i} - 4\hat{j} + 5\hat{k})$$

Problem 3

- (1) Define “ X is linearly independent”. Define the linear-span “ $L(X)$ ” of a set X .
(2) Suppose that $\vec{a}, \vec{b} \in V_3$ are two non-parallel vectors. Prove that $\{\vec{a}, \vec{b}, \vec{a} \times \vec{b}\}$ is a basis of V_3 .
(3) (Extra Credit: 5 points) Prove that if X is linearly independent and $b \notin L(X)$, then $X \cup \{b\}$ is linearly independent.

Problem 4

- (1) Define “the line $L(\vec{x}_0; \vec{a})$ ” and “the plane $L(\vec{x}_0; \vec{a}, \vec{b})$ ”. Compute the line passing through $\hat{i} + \hat{j} + \hat{k}$ and it is parallel to the line $L(\langle 7, 8, 9 \rangle; \langle 1, 2, 3 \rangle)$.
(2) Define “ (X, E) is an equivalence relationship.” Define “ $[y]_E$ is the equivalence class of y in the equivalence relationship (X, E) ”.
(3) Define “ $\vec{x} \in V_n$ ”. What is the vector $\langle 3, 4, 5, 6 \rangle$.
(4) (Extra Credit: 10 points) Suppose that $X \subset V_n$ is linearly independent and $Y = \{ \frac{\vec{x}}{\|\vec{x}\|_n} \mid \vec{x} \in V_n \}$. Prove that Y is linearly independent.

Do not write your solutions here.

Student Name: _____

The Solutions

Problem 1

- (1) State and prove the Cauchy-Schwarz-Bunyakovsky inequality (C-S-B).
(2) Suppose that $\vec{a}, \vec{b} \in V_3$. State the Lagrange Identity. Use the Lagrange Identity and (C-S-B) to prove that

$$\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = \vec{0}_3$$

- (3) Define the cross-product $\vec{a} \times \vec{b}$ of two vectors in V_3 .
(4) Compute $(3\hat{i} - 5\hat{j} + 2\hat{k}) \times (-\hat{i} - \hat{j} + \hat{k})$.

Write your solutions here. Use the back page, if needed

Student Name: _____

Problem 2

- (1) Define the concept of “ F is multilinear alternating form in V_n . State the theorem that gives the formula of each F is multilinear alternating form in V_n . What is the definition of the vector determinant $\text{Det}_n(\vec{a}_1, \dots, \vec{a}_n)$.
- (2) Write all permutations of S_3 as products of cycles and products of transpositions.
- (3) Compute the the determinant providing the triple scalar-product

$$[(3\hat{i} - 5\hat{j} + 2\hat{k}) \times (-\hat{i} - \hat{j} + \hat{k})] \cdot (2\hat{i} - 4\hat{j} + 5\hat{k})$$

Write your solutions here. Use the back page, if needed

Student Name: _____

Problem 3

- (1) Define “ X is linearly independent”. Define the linear-span “ $L(X)$ ” of a set X .
- (2) Suppose that $\vec{a}, \vec{b} \in V_3$ are two non-parallel vectors. Prove that $\{\vec{a}, \vec{b}, \vec{a} \times \vec{b}\}$ is a basis of V_3 .
- (3) (Extra Credit: 5 points) Prove that if X is linearly independent and $b \notin L(X)$, then $X \cup \{b\}$ is linearly independent.

Write your solutions here. Use the back page, if needed

Student Name: _____

Problem 4

(1) Define “the line $L(\vec{x}_0; \vec{a})$ ” and “the plane $L(\vec{x}_0; \vec{a}, \vec{b})$ ”. Compute the line passing through $\hat{i} + \hat{j} + \hat{k}$ and it is parallel to the line $L(\langle 7, 8, 9 \rangle; \langle 1, 2, 3 \rangle)$.

(2) Define “ (X, E) is an equivalence relationship.” Define “ $[y]_E$ is the equivalence class of y in the equivalence relationship (X, E) ”.

(3) Define “ $\vec{x} \in V_n$ ”. What is the vector $\langle 3, 4, 5, 6 \rangle$.

(4) (Extra Credit: 10 points)

Suppose that $X \subset V_n$ is linearly independent and $Y = \{ \frac{\vec{x}}{\|\vec{x}\|_n} \mid \vec{x} \in V_n \}$. Prove that Y is linearly independent.

Write your solutions here. Use the back page, if needed

Student Name: _____

Problem 5 (*Extra Credit: 10 points*)

Write your solutions here. Use the back page, if needed

Student Name: _____

Write your solutions here. Use the back page, if needed

————— *End of Examination* —————