Name____ Date__________

1.5 Reteach

Recall that an absolute value function is a function that contains an absolute value expression.

Key Idea

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form.** The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

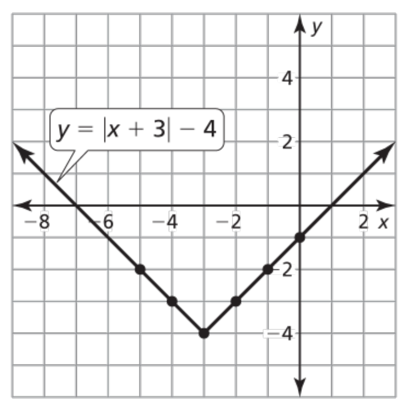
EXAMPLE Describing Characteristics

Graph $f(x) = |x + 3| - 4$. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

SOLUTION

Step 1 Make a table of values.

x	–5	–4	–3	–2	–1	0
$f(x)$	–2	–3	–4	–3	–2	–1



Step 2 Plot the ordered pairs.

Step 3 Draw a V-shaped graph though the plotted points.

Positive and Negative: The x-intercepts are –7 and 1. The function is positive when $x < -7$, negative when $-7 < x < 1$, and positive when $x > 1$.

Increasing and Decreasing: The vertex is $(-3, -4)$. The function is decreasing when $x < -3$ and increasing when $x > -3$.

End Behavior: The graph shows that the function values increase as x approaches both positive and negative infinity. So, $y \to +\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

Name ____ Date __________

1.5 Reteach (continued)

EXAMPLE Using Characteristics to Graph Absolute Value Functions

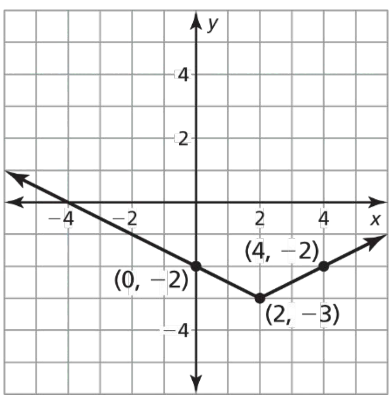
Graph each absolute value function f with the given characteristics.

a. f has a range of $(-3, \infty)$, a graph that is symmetric about the line $x = 2$, and a y-intercept of -2.

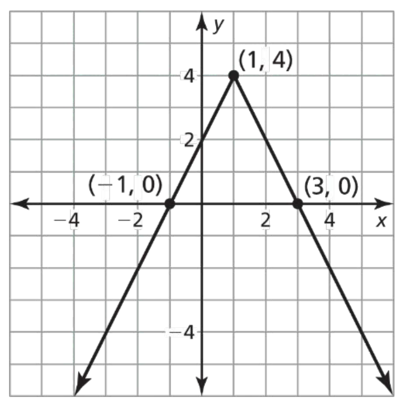
b. f is positive over the interval $(-1, 3)$, negative over the intervals $(-\infty, -1)$ and $(3, \infty)$ a graph that is symmetric about the line $x = 1$, and the maximum value is 4.

SOLUTION

a. Because the graph is symmetric about $x = 2$, the x-value of the vertex is 2. Because the range is $(-3, \infty)$, the y-value of the vertex is -3. Plot the vertex $(2, -3)$. Because the y-intercept is -2, plot the point $(0, -2)$ and its reflection in the line of symmetry, $(4, -2)$. Then draw the graph.



b. Because f is positive over the interval $(-1, 3)$, negative over the intervals $(-\infty, -1)$ and $(3, \infty)$, you know that the x-intercepts are -1 and 3. Because the x-intercepts are -1 and 3, the vertex has an x-coordinate halfway in between -1 and 3, or 1. The maximum value is 4, so the vertex is $(1, 4)$. So, plot the points $(-1, 0)$, $(3, 0)$, and $(1, 4)$. Then draw the graph.



Do these questions by yourself before looking at the answer key

In Exercises 1–2, graph *f*. Determine when the function is positive, negative, increasing, or decreasing. Then describe the end behavior of the function.

1. $f(x) = |x - 4| + 1$

2. $f(x) = -|x + 3| + 3$

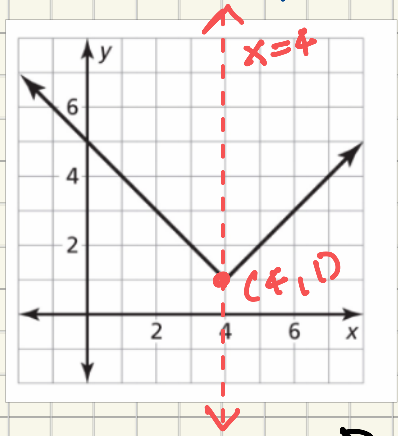
In Exercises 3–4, graph the absolute value function *f* with the given characteristics.

3. f has a range of $(-\infty, 2)$ and a graph that is symmetric about the line $x = -3$ and has a y-intercept of -4.

4. f is positive over the intervals $(-\infty, 0)$ and $(6, \infty)$, negative over the interval $(0, 6)$, and the minimum value is -3.

Answers

① $f(x) = |x-4| + 1 \rightarrow h=4,\ k=1$



Vertex: (4,1)
Line of symm: x = 4

x	2	3	4	5	6
f(x)	3	2	1	2	3

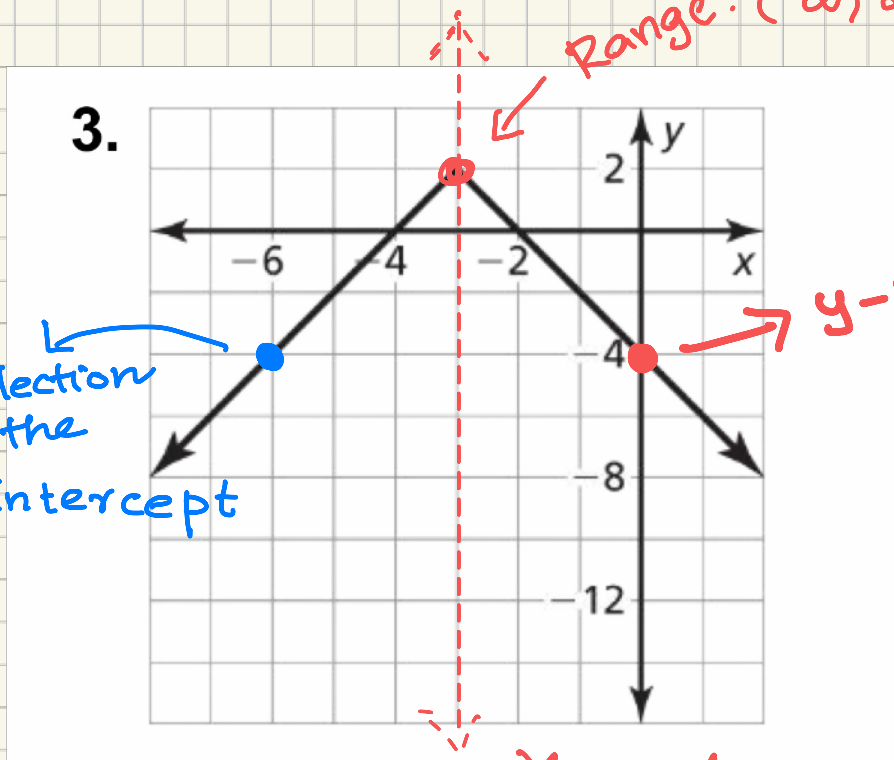
D: $(-\infty, \infty)$
R: $[1, \infty)$

Increasing: $(4, \infty)$ or $x > 4$
Decreasing: $(-\infty, 4)$ or $x < 4$

Positive: $(-\infty, \infty)$
Negative: None
End Behavior: as $x \rightarrow +\infty$, $f(x) \rightarrow +\infty$
as $x \rightarrow -\infty$, $f(x) \rightarrow +\infty$

② Do it by Yourself

Range: $(-\infty, 2) \rightarrow$ 2 is the Max



y-intercept

Reflection of the y-intercept

X = −3, line of symmetry (Given)

④ Do it by Yourself

Name____ Date__________

H 1.6 Reteach

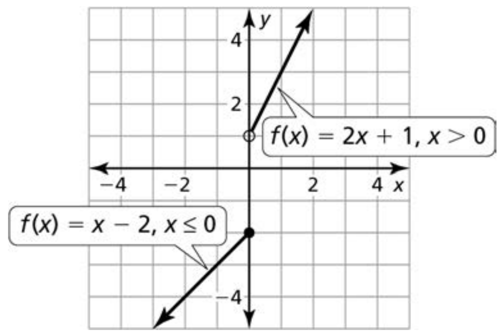
Key Idea

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 2, & \text{if } x \le 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.
- The expression $2x + 1$ represents the value of f when x is greater than 0.



EXAMPLE Evaluating a Piecewise Function

Evaluate $f(x) = \begin{cases} 4x + 3, & \text{if } x < 1 \\ x + 7, & \text{if } x \ge 1 \end{cases}$ when (a) $x = 1$ and (b) when $x = 0$.

SOLUTION

a. Because $x = 1$ and $1 \ge 1$, use the second equation.

$f(x) = x + 7$	Write the second equation.
$f(1) = 1 + 7$	Substitute 1 for *x*.
$f(1) = 8$	Add.

▶ The value of f is 8 when $x = 1$.

b. Because $x = 0$ and $0 < 1$, use the first equation.

$f(x) = 4x + 3$	Write the first equation.
$f(0) = 4(0) + 3$	Substitute 0 for *x*.
$f(0) = 3$	Simplify.

▶ The value of f is 3 when $x = 0$.

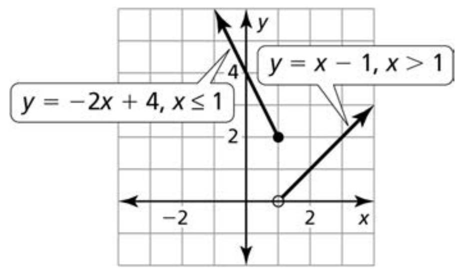
Name ______________________________ Date ________

1.6 Reteach (continued)

EXAMPLE Graphing a Piecewise Function

Graph $y = \begin{cases} -2x + 4, & \text{if } x \le 1 \\ x - 1, & \text{if } x > 1 \end{cases}$. Describe the domain, range, and end behavior of the function.

SOLUTION



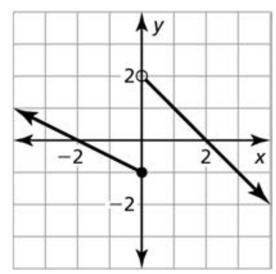
Step 1 Graph $y = -2x + 4$ for $x \le 1$. Because 1 is included in the domain for this equation, use a closed circle at $(1, 2)$.

Step 2 Graph $y = x - 1$ for $x > 1$. Because 1 is not included in the domain for this equation, use an open circle at $(1, 0)$.

▶ The domain is all real numbers. The range is $y > 0$. The graph shows that $y \to +\infty$ as $x \to -\infty$ and $y \to +\infty$ as $x \to +\infty$.

EXAMPLE Writing a Piecewise Function

Write a piecewise function represented by the graph.



SOLUTION

Left Piece When $x \le 0$, the graph is the line represented by $y = -\frac{1}{2}x - 1$.

Right Piece When $x > 0$, the graph is the line represented by $y = -x + 2$.

▶ So, a piecewise function is $f(x) = \begin{cases} -\frac{1}{2}x - 1, & \text{if } x \le 0 \\ -x + 2, & \text{if } x > 0 \end{cases}$.

In Exercises 1 and 2, evaluate the function when $x = -4, -2, -1, \frac{1}{2}$, and 2.

1. $f(x) = \begin{cases} 2x + 3, & \text{if } x < 0 \\ x - 5, & \text{if } x \ge 0 \end{cases}$

2. $f(x) = \begin{cases} -3x - 2, & \text{if } x \le -2 \\ 4x + 1, & \text{if } x > -2 \end{cases}$

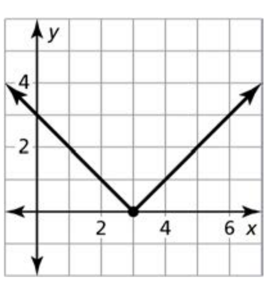
In Exercises 3 and 4, graph the function. Describe the domain, range, and end behavior of the function.

3. $f(x) = \begin{cases} -x, & \text{if } x < 3 \\ x + 4, & \text{if } x \ge 3 \end{cases}$

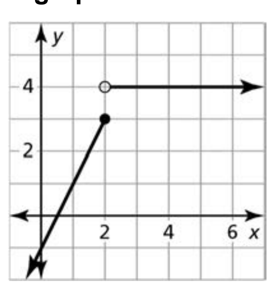
4. $f(x) = \begin{cases} -3x, & \text{if } x \le -1 \\ 3x, & \text{if } x > -1 \end{cases}$

In Exercises 5 and 6, write a piecewise function for the graph.

5.



6.



Franciso Alvarez

In Exercises 1 and 2, evaluate the function when $x = -4, -2, -1, \frac{1}{2}$, and 2.

1. $f(x) = \begin{cases} 2x + 3, & \text{if } x < 0 \\ x - 5, & \text{if } x \geq 0 \end{cases}$

$f(-4) = 2\cdot(-4)+3$
$f(-4) = -8+3$
$f(-4) = -5$

$f(-2) = 2\cdot(-2)+3$
$f(-2) = -4+3$
$f(-2) = -1$

$f(-1) = 2\cdot(-1)+3$
$f(-1) = -2+3$
$f(-1) = 1$

$f(\frac{1}{2}) = \frac{1}{2} - 5$
$f(\frac{1}{2}) = -\frac{9}{2}$

$f(2) = 2-5$
$f(2) = -3$

2. $f(x) = \begin{cases} -3x - 2, & \text{if } x \leq -2 \\ 4x + 1, & \text{if } x > -2 \end{cases}$

$f(-4) = -3(-4)-2$
$f(-4) = 12-2$
$f(-4) = 10$

$f(-2) = -3(-2)-2$
$f(-2) = 6-2$
$f(-2) = 4$

$f(-1) = 4\cdot(-1)+1$
$f(-1) = -4+1$
$f(-1) = -3$

$f(\frac{1}{2}) = 4(\frac{1}{2})+1$
$f(\frac{1}{2}) = 2+1$
$f(\frac{1}{2}) = 3$

$f(2) = 4(2)+1$
$f(2) = 8+1$
$f(2) = 9$

In Exercises 3 and 4, graph the function. Describe the domain, range, and end behavior of the function.

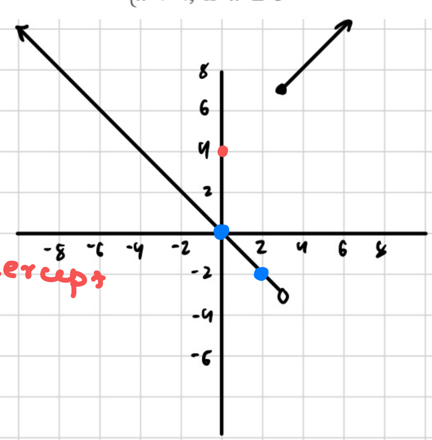
3. $f(x) = \begin{cases} -x, & \text{if } x < 3 \\ x + 4, & \text{if } x \geq 3 \end{cases}$

3

$f(x) = -1x + 0$

use $y = mx + b$

first mark b, y-intercept
then slope $= \frac{\text{rise}}{\text{run}}$



Domain: $(-\infty, \infty)$
Range: $(-3, \infty)$
End Behavior: $x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

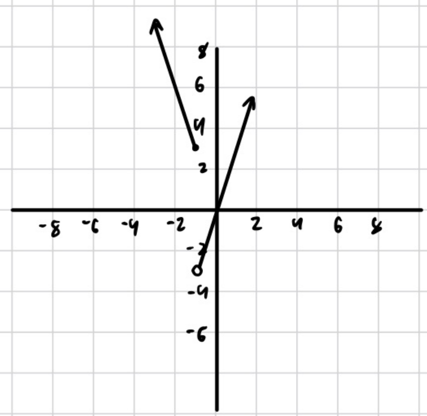
4. $f(x) = \begin{cases} -3x, & \text{if } x \leq -1 \\ 3x, & \text{if } x > -1 \end{cases}$

$m = -\frac{3}{1}$

$y = -3x + \underbrace{0}_{b}$

$y = 3x + \underbrace{0}_{b}$

$m = \frac{3}{1}$



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Range: $(-3, \infty)$
End Behavior: $x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

Franciso Alvarez

In Exercises 5 and 6, write a piecewise function for the graph.

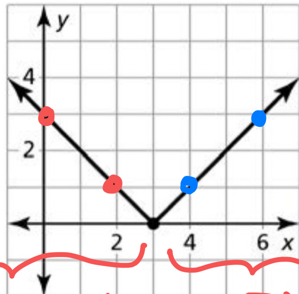
L

$b = 3$

$m = \frac{-2}{2} = -1$

$\Rightarrow y = -1x + 3$

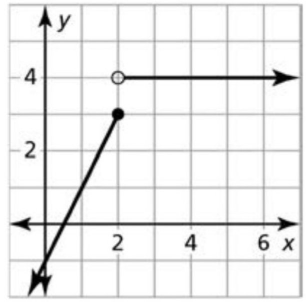
5.



Left piece   Right Piece

$f(x) = \begin{cases} -x + 3, & \text{if } x < 3 \\ x - 3, & \text{if } x \geq 3 \end{cases}$

6.



$f(x) = \begin{cases} 2x - 0.5, & \text{if } x \leq 2 \\ 4, & \text{if } x > 2 \end{cases}$

Right

we do not have b

But we have slope

$m = \frac{-2}{-2} = 1$

$\boxed{m = 1}$

choose any point on the graph, say: (4,1)

$y = mx + b$

$1 = 1(4) + b$

$1 = 4 + b$

$1 - 4 = b$

$\boxed{-3 = b}$

$y = 1x - 3$

or $y = x - 3$

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$f(-2) = -1$

$f(-1) = 2 \cdot (-1) + 3$

$f(-1) = -2 + 3$

$f(-1) = 1$

$f(\frac{1}{2}) = \frac{1}{2} - 5$

$f(\frac{1}{2}) = -\frac{9}{2}$

$f(2) = 2 - 5$

$f(2) = -3$

2. $f(x) = \begin{cases} -3x - 2, & \text{if } x \leq -2 \\ 4x + 1, & \text{if } x > -2 \end{cases}$

$f(-4) = -3(-4) - 2$

$f(-4) = 12 - 2$

$f(-4) = 10$

$f(-2) = -3(-2) - 2$

$f(-2) = 6 - 2$

$f(-2) = 4$

$f(-1) = 4 \cdot (-1) + 1$

$f(-1) = -4 + 1$

$f(-1) = -3$

$f(\frac{1}{2}) = 4(\frac{1}{2}) + 1$

$f(\frac{1}{2}) = 2 + 1$

$f(\frac{1}{2}) = 3$

$f(2) = 4(2) + 1$

$f(2) = 8 + 1$

$f(2) = 9$

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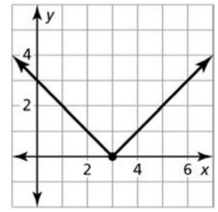
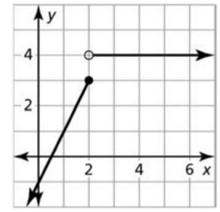


$f(x) = \begin{cases} -x + 3, & \text{if } x < 3 \\ x - 3, & \text{if } x \geq 3 \end{cases}$

6.



$f(x) = \begin{cases} 2x - 0.5, & \text{if } x \leq 2 \\ 4, & \text{if } x > 2 \end{cases}$

In Exercises 3 and 4, graph the function. Describe the domain, range, and end behavior of the function.

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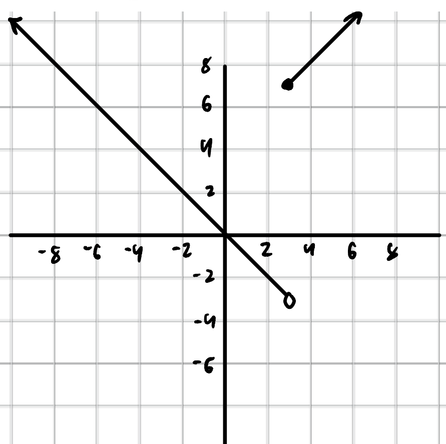


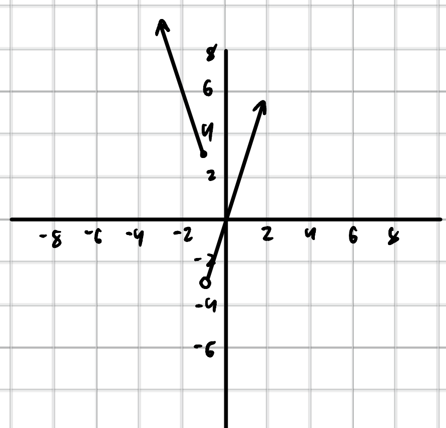
Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

End Behavior: $x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$

4. $f(x) = \begin{cases} -3x, & \text{if } x \leq -1 \\ 3x, & \text{if } x > -1 \end{cases}$



Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

End Behavior: $x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$