AP Precalculus - M3Y & M3Z

Polynomials - Homework 3

1. Expand the following:
   (i) $(x+1)^5$
   (ii) $(2y-1)^6$
   (iii) $(3+a)^4$
   (iv) $(4w-3v)^5$

2. Find the following coefficients:
   (i) The coefficient of x^4y^{23} in the expansion of $(-x+y)^{27}$
   (ii) The coefficient of a^9b^9 in the expansion of $(a-2b)^{18}$
   (iii) The coefficient of $p^{2025}q^{2023}$ in the expansion of $(-p-q)^{4048}$

3. Prove that, if $n \in \mathbb{N}$, then:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^{n+1}$$

4. Solve the following inequalities:
   (i) $6x^2 - 30x + 70 \geq 3x^2 + 3x - 20$
   (ii) $x^3 + 2x^2 - 3x < 0$
   (iii) $x^4 - 13x^2 + 36 > 0$
   (iv) $(x^2 - 15x + 56)(-x^2 - 2x + 24)(x^2 - 5x + 9) \geq 0$
   (v) $(x+8)^2(x+5)(x+7)^3 > 0$
   (vi) $-x^3 + 3x^2 + 25x > 2x^25x$
   (vii) $x^4 + 4x^3 - 12x^2 \leq 0$