

Ασκηση 1:  $\sin x = \cos x$

Λύση Α:  $\sin x = \cos x$

$$\Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right)$$

Αρα,

$$x = \begin{cases} 2k\pi + \frac{\pi}{2} - x \\ 2k\pi + \left(\pi - \left(\frac{\pi}{2} - x\right)\right) \end{cases} \quad k \in \mathbb{Z}$$

$$x = 2k\pi + \frac{\pi}{2} - x \quad \text{ή} \quad x = 2k\pi + \frac{\pi}{2} + x$$

$$\Rightarrow 2x = 2k\pi + \frac{\pi}{2}$$

$$\Leftrightarrow 0 = 2k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z}$$

**ΑΔΥΝΑΤΗ.**

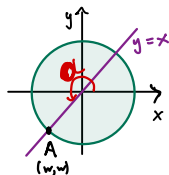
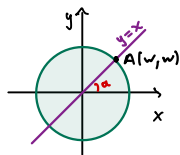
$$\Rightarrow x = k\pi + \frac{\pi}{4}$$

$k \in \mathbb{Z}$

Λύση Β:  $\sin a = \cos a$

Έστω  $A(\cos a, \sin a)$  σημείο του μοναδιαίου κύκλου.

$\cos a = \sin a \Rightarrow A$  είναι σημείο της ευθείας  $y = x$



Η κλίση της  $y = x$  είναι 1.

$$\Rightarrow a = \frac{\pi}{4} \quad \text{ή} \quad a = \pi + \frac{\pi}{4} \left(= \frac{5\pi}{4}\right).$$

$$\Rightarrow a = 2k\pi + \frac{\pi}{4} \quad \text{ή} \quad a = 2k\pi + \pi + \frac{\pi}{4}, k \in \mathbb{Z}$$

$$\Rightarrow a = 2k\pi + \frac{\pi}{4} \quad \text{ή} \quad a = (2k+1) \cdot \pi + \frac{\pi}{4}, k \in \mathbb{Z}$$

$$\Rightarrow a = k \cdot \pi + \frac{\pi}{4}, k \in \mathbb{Z}$$

Άσκηση 2:  $|\sin \alpha| = |\cos \alpha|$

Λύση A:  $|\sin \alpha| = |\cos \alpha| \Leftrightarrow \begin{cases} \sin \alpha = \cos \alpha \\ \sin \alpha = -\cos \alpha \end{cases}$

$\sin \alpha = \cos \alpha$

•  
•  
•

$\alpha = k \cdot \pi + \frac{\pi}{4}$

$k \in \mathbb{Z}$

$\sin \alpha = -\cos \alpha$

$\Leftrightarrow -\sin \alpha = \cos \alpha$

$\Leftrightarrow \sin(-\alpha) = \cos \alpha$

$\Leftrightarrow \sin(-\alpha) = \sin\left(\frac{\pi}{2} - \alpha\right)$

Άρα,

$-\alpha = 2k\pi + \frac{\pi}{2} - \alpha$

$0 = 2k\pi + \frac{\pi}{2}$  **ΑΔΥΝΑΤΗ**

ή

$-\alpha = 2k\pi + \left(\pi - \left(\frac{\pi}{2} - \alpha\right)\right)$

$\Leftrightarrow -\alpha = 2k\pi + \frac{\pi}{2} + \alpha$

$\Leftrightarrow 2\alpha = -2k\pi - \frac{\pi}{2}$

$\Leftrightarrow \alpha = -k\pi - \frac{\pi}{4}, k \in \mathbb{Z}$

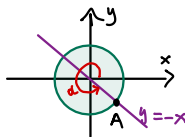
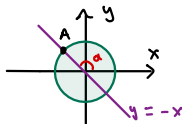
Λύση B:  $\sin \alpha = \cos \alpha \leadsto$  Άσκηση 1

$\alpha = k\pi + \frac{\pi}{4}, k \in \mathbb{Z}$

$\sin \alpha = -\cos \alpha$

Έστω  $A(\cos \alpha, \sin \alpha)$  σημείο του μοναδιαίου κύκλου.

$\sin \alpha = -\cos \alpha \Rightarrow A$  σημείο της ευθείας  $y = -x$ .



Η κλίση της  $y = -x$  είναι  $-1$

$\Rightarrow \alpha = \frac{3\pi}{4}$  ή  $\alpha = \pi + \frac{3\pi}{4}$

$\Rightarrow \alpha = 2k\pi + \frac{3\pi}{4}$  ή  $\alpha = 2k\pi + \pi + \frac{3\pi}{4}$

$\Rightarrow \alpha = 2k\pi + \frac{3\pi}{4}$  ή  $\alpha = (2k+1) \cdot \pi + \frac{3\pi}{4}$

Άρα,  $\alpha = k \cdot \pi + \frac{3\pi}{4}, k \in \mathbb{Z}$

$\Rightarrow \alpha = k \cdot \pi + \frac{4\pi}{4} - \frac{\pi}{4}, k \in \mathbb{Z}$

$\Rightarrow \alpha = k \cdot \pi + \pi - \frac{\pi}{4}, k \in \mathbb{Z}$

$\Rightarrow \alpha = (k+1) \cdot \pi - \frac{\pi}{4}, k \in \mathbb{Z}$

$\Rightarrow \alpha = k\pi - \frac{\pi}{4}, k \in \mathbb{Z}$