

Άσκηση 4: $\sin x = \cos x$

Άσκηση A: $\sin x = \cos x$

$$\Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right)$$

Άποδι,

$$x = \begin{cases} 2k\pi + \frac{\pi}{2} - x \\ 2k\pi + \left(\pi - \left(\frac{\pi}{2} - x\right)\right) \end{cases} \quad k \in \mathbb{Z}$$

$$x = 2k\pi + \frac{\pi}{2} - x \quad \mid \quad x = 2k\pi + \frac{\pi}{2} + x$$

$$\Rightarrow 2x = 2k\pi + \frac{\pi}{2} \quad \Leftrightarrow 0 = 2k\pi + \frac{\pi}{2} \quad k \in \mathbb{Z}$$

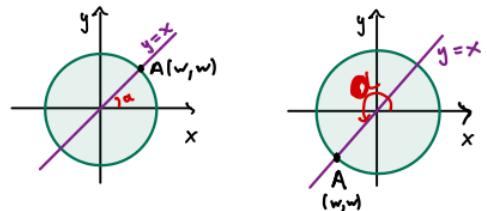
A ΔΥΝΑΤΗ.

$$\Rightarrow x = k\pi + \frac{\pi}{4} \quad k \in \mathbb{Z}$$

Άσκηση B: $\sin a = \cos a$

Έστω $A(\cos a, \sin a)$ σημείο του πολαρικού κώνου.

$$\cos a = \sin a \Rightarrow A \text{ είναι σημείο της ευθείας } y = x$$



H κάθιση της $y=x$ είναι 1.

$$\Rightarrow a = \frac{\pi}{4} \quad \mid \quad a = \pi + \frac{\pi}{4} \left(= \frac{5\pi}{4}\right).$$

$$\Rightarrow a = 2k\pi + \frac{\pi}{4} \quad \mid \quad a = 2k\pi + \pi + \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow a = 2k\pi + \frac{\pi}{4} \quad \mid \quad a = (2k+1) \cdot \pi + \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow a = k \cdot \pi + \frac{\pi}{4} \quad | \quad k \in \mathbb{Z}$$

Aσκήση 2: $|\sin a| = |\cos a|$

Άνωγ Α: $|\sin a| = |\cos a| \Leftrightarrow \begin{cases} \sin a = \cos a \\ \sin a = -\cos a \end{cases}$

$$\sin a = \cos a$$

$$\begin{matrix} 0 \\ 0 \\ \vdots \end{matrix}$$

$$a = k \cdot \pi + \frac{\pi}{4}$$

$$k \in \mathbb{Z}$$

$$\begin{aligned} & \sin a = -\cos a \\ \Leftrightarrow & -\sin a = \cos a \\ \Leftrightarrow & \sin(-a) = \cos a \\ \Leftrightarrow & \sin(-a) = \sin\left(\frac{\pi}{2} - a\right) \\ \text{Άρα,} & \end{aligned}$$

$$-a = 2k\pi + \frac{\pi}{2} - a$$

$$0 = 2k\pi + \frac{\pi}{2} \quad \text{ΑΔΥΝΑΤΗ}$$

$$\begin{matrix} \downarrow \\ n \end{matrix}$$

$$-a = 2k\pi + \left(\pi - \left(\frac{\pi}{2} - a\right)\right)$$

$$\Leftrightarrow -a = 2k\pi + \frac{\pi}{2} + a$$

$$\Leftrightarrow 2a = -2k\pi - \frac{\pi}{2}$$

$$\Leftrightarrow a = -k\pi - \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

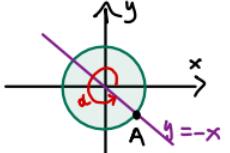
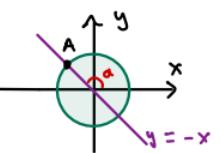
Άνωγ Β: $\sin a = \cos a \rightsquigarrow$ Ασκήση 1

$$a = k\pi + \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$\sin a = -\cos a$$

Έτσι a ($\cos a, \sin a$) αντίστοιχο του πνοαδιάτου κύκλου.

$\sin a = -\cos a \Rightarrow a$ σημείο της επίξιας $y = -x$.



Η κρίση της $y = -x$ είναι -1

$$\Rightarrow a = \frac{3\pi}{4} \quad \text{ή} \quad a = \pi + \frac{3\pi}{4}$$

$$\Rightarrow a = 2k\pi + \frac{3\pi}{4} \quad \text{ή} \quad a = 2k\pi + \pi + \frac{3\pi}{4}$$

$$\Rightarrow a = 2k\pi + \frac{3\pi}{4} \quad \text{ή} \quad a = (2k+1) \cdot \pi + \frac{3\pi}{4}$$

Άρα, $a = k \cdot \pi + \frac{3\pi}{4}, \quad k \in \mathbb{Z}$

$$\Rightarrow a = k \cdot \pi + \frac{4\pi}{4} - \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow a = k \cdot \pi + \pi - \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow a = (k+1) \cdot \pi - \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$\Rightarrow a = k\pi - \frac{\pi}{4}, \quad k \in \mathbb{Z}$$