Classwork 3 – Trigonometry

1) $\sin x = -\frac{\sqrt{3}}{2}$

2) $\sin\left(2x + \frac{\pi}{4}\right) = \frac{1}{2}$

3) $\sin x = -1$

4) $\sin x = -\frac{1}{2}$

5) $\sin x = \frac{\sqrt{2}}{2}$

6) $2\sin(3x) = \sqrt{3}$

7) $(1 - \sin x) \cdot (2\sin x - \sqrt{3}) = 0$

8) $\sin\left(x + \frac{\pi}{3}\right) = -1$

9) $2\sin^2 x + \sin x - 1 = 0$

10) $\sin^2 x + 5 \cdot \cos^2 x = 4$

2) $\sin\left(2x+\frac{\pi}{4}\right) = \frac{1}{2}$

Λύση: $\sin\left(2x+\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{6}\right)$

$$2x+\frac{\pi}{4} = \begin{cases} 2k\pi + \frac{\pi}{6} \\ \\ 2k\pi + \left(\pi - \frac{\pi}{6}\right) \end{cases} \quad k \in \mathbb{Z}$$

$$2x = \begin{cases} 2k\pi + \frac{\pi}{6} - \frac{\pi}{4} \\ \\ 2k\pi + \left(\pi - \frac{\pi}{6}\right) - \frac{\pi}{4} \end{cases} \quad k \in \mathbb{Z}$$

$$x = \begin{cases} \frac{1}{2}\cdot\left(2k\pi + \frac{\pi}{6} - \frac{\pi}{4}\right) \\ \\ \frac{1}{2}\cdot\left(2k\pi + \left(\pi - \frac{\pi}{6}\right) - \frac{\pi}{4}\right) \end{cases} \quad k \in \mathbb{Z}$$

3) $\sin x = -1$.

Λύση: $\sin x = \sin\left(\frac{3\pi}{2}\right)$

$$x = \begin{cases} 2k\pi + \frac{3\pi}{2} \\ \\ 2k\pi + \left(\pi - \frac{3\pi}{2}\right) \end{cases}$$

$k \in \mathbb{Z}$

1) $\sin x = -\frac{\sqrt{3}}{2}$

Λύση: $\frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$-\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3} = \sin\left(-\frac{\pi}{3}\right)$

Άρα, $\sin x = -\frac{\sqrt{3}}{2} \Leftrightarrow \sin x = \sin\left(-\frac{\pi}{3}\right)$

$$x = \begin{cases} 2k\pi + \left(-\frac{\pi}{3}\right), & k \in \mathbb{Z} \\ 2k\pi + \left(\pi - \left(-\frac{\pi}{3}\right)\right), & k \in \mathbb{Z} \end{cases}$$

$$x = \begin{cases} 2k\pi - \frac{\pi}{3}, & k \in \mathbb{Z} \\ 2k\pi + \frac{4\pi}{3}, & k \in \mathbb{Z} \end{cases}$$

Παράδειγμα: $\sin x = 0$

Λύση: $0 = \sin 0$

Άρα, $\sin x = 0$

$\Leftrightarrow \sin x = \sin 0$

Άρα, $$x = \begin{cases} 2k\pi + 0, & k \in \mathbb{Z} \\ 2k\pi + (\pi - 0), & k \in \mathbb{Z} \end{cases}$$

$$x = \begin{cases} 2k\pi, & k \in \mathbb{Z} \\ 2k\pi + \pi, & k \in \mathbb{Z} \end{cases} = \begin{cases} 2k\pi, & k \in \mathbb{Z} \\ (2k+1)\cdot\pi, & k \in \mathbb{Z} \end{cases}$$

2κ = άρτιος
11 − περιττός
$\Rightarrow x = k \cdot \pi, \; k \in \mathbb{Z}$

9) $2\sin^2 x + \sin x - 1 = 0$

Λύση: Θέτω $y = \sin x$

Άρα, $2\sin^2 x + \sin x - 1 = 0$

$\Leftrightarrow 2y^2 + y - 1 = 0$

$\Delta = 1^2 - 4 \cdot 2 \cdot (-1) = 9$

$y_1 = \frac{-1 + \sqrt{9}}{4} = \frac{-1+3}{4} = \frac{1}{2}$

$y_2 = \frac{-1 - \sqrt{9}}{4} = \frac{-4}{4} = -1$

Άρα, $y = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$

Άρα, $\sin x = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$

$\sin x = \frac{1}{2}$	$\sin x = -1$
$\sin x = \sin\frac{\pi}{6}$	$\sin x = \sin\left(\frac{3\pi}{2}\right)$
$x = \begin{cases} 2k\pi + \frac{\pi}{6}, k \in \mathbb{Z} \\ 2k\pi + \left(\pi - \frac{\pi}{6}\right), k \in \mathbb{Z} \end{cases}$	$x = \begin{cases} 2k\pi + \frac{3\pi}{2}, k \in \mathbb{Z} \\ 2k\pi + \left(\pi - \frac{3\pi}{2}\right), k \in \mathbb{Z} \end{cases}$

(10) $\sin^2 x + 5 \cdot \cos^2 x = 4$

Λύση:

$\sin^2 x + 5 \cdot \cos^2 x = 4 \Leftrightarrow$

$\sin^2 x + 5 \cdot (1 - \sin^2 x) = 4 \Leftrightarrow$

$\sin^2 x + 5 - 5\sin^2 x = 4 \Leftrightarrow$

$-4\sin^2 x = -1 \Leftrightarrow$

$\sin^2 x = \frac{1}{4} \Leftrightarrow$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}} \Leftrightarrow$

$|\sin x| = \frac{1}{2}$

$|\sin x| = \frac{1}{2}$

$\sin x = \frac{1}{2} \Leftrightarrow$

$\sin x = \sin\frac{\pi}{6}$

$\Downarrow$

$x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \frac{5\pi}{6} \end{cases}$

$k \in \mathbb{Z}$

$\sin x = -\frac{1}{2}$

$\sin x = \sin\left(-\frac{\pi}{6}\right)$

$\Downarrow$

$x = \begin{cases} 2k\pi - \frac{\pi}{6} \\ 2k\pi + \frac{7\pi}{6} \end{cases}$

$k \in \mathbb{Z}$

7) $2\sin(3x) = \sqrt{3}$

Λύση: $2\sin(3x) = \sqrt{3}$

$\Leftrightarrow \sin(3x) = \frac{\sqrt{3}}{2}$

$\Leftrightarrow \sin(3x) = \sin\frac{\pi}{3}$

Άρα, $3x = \begin{cases} 2k\pi + \frac{\pi}{3} \\ 2k\pi + \left(\pi - \frac{\pi}{3}\right) \end{cases}$ $k \in \mathbb{Z}$

Άρα $x = \begin{cases} \frac{1}{3} \cdot \left(2k\pi + \frac{\pi}{3}\right) \\ \frac{1}{3} \cdot \left(2k\pi + \frac{2\pi}{3}\right) \end{cases}$ $k \in \mathbb{Z}$

8) $(1-\sin x)\cdot(2\sin x - \sqrt{3}) = 0$

Λύση: $(1-\sin x)\cdot(2\sin x - \sqrt{3}) = 0$

$\Leftrightarrow 1-\sin x = 0$ ή $2\sin x - \sqrt{3} = 0$

$\Leftrightarrow 1 = \sin x$ ή $\sin x = \frac{\sqrt{3}}{2}$

$\Leftrightarrow \sin x = \sin\frac{\pi}{2}$ ή $\sin x = \sin\frac{\pi}{3}$

$\Leftrightarrow x = \begin{cases} 2k\pi + \frac{\pi}{2} \\ 2k\pi + \left(\pi - \frac{\pi}{2}\right) \end{cases}$ ή $x = \begin{cases} 2k\pi + \frac{\pi}{3} \\ 2k\pi + \left(\pi - \frac{\pi}{3}\right) \end{cases}$

$\Leftrightarrow x = 2k\pi + \frac{\pi}{2}$ ή $x = \begin{cases} 2k\pi + \frac{\pi}{3} \\ 2k\pi + \frac{2\pi}{3} \end{cases}$

$k \in \mathbb{Z}$