

Γενικές λύσεις τριγωνομετρικών εξισώσεων

A) $\sin x = \sin \theta$

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$$

ακέραιος αριθμοί

$$x(k) = \begin{cases} 2k\pi + \theta \\ 2k\pi + (\pi - \theta) \end{cases}, k \in \mathbb{Z}$$

B) $\cos x = \cos \theta$

$$x(k) = \begin{cases} 2k\pi + \theta \\ 2k\pi - \theta \end{cases}, k \in \mathbb{Z}$$

Γ) $\tan x = \tan \theta$

$$x(k) = \begin{cases} 2k\pi + \theta \\ 2k\pi + (\pi + \theta) \end{cases}$$

Παράδειγμα 1: $\sin x = 1$

Λύση: $1 = \sin \frac{\pi}{2}$

Άρα, $\sin x = 1 \Leftrightarrow \sin x = \sin \frac{\pi}{2}$

Άρα, $x = \begin{cases} 2k\pi + \frac{\pi}{2} \\ 2k\pi + (\pi - \frac{\pi}{2}) \end{cases}, k \in \mathbb{Z}$

Άρα, $x = 2k\pi + \frac{\pi}{2}, k \in \mathbb{Z}$

Παράδειγμα 2: $\cos x = \frac{\sqrt{2}}{2}$

Λύση: $\frac{\sqrt{2}}{2} = \cos \frac{\pi}{4}$. Άρα $\cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \cos x = \cos \left(\frac{\pi}{4}\right)$

Άρα $x = \begin{cases} 2k\pi + \frac{\pi}{4} \\ 2k\pi - \frac{\pi}{4} \end{cases}, k \in \mathbb{Z}$

Παράδειγμα 3: $\tan x = \frac{\sqrt{3}}{3}$

Λύση: $\frac{\sqrt{3}}{3} = \tan \frac{\pi}{6}$. Άρα $\tan x = \frac{\sqrt{3}}{3} \Leftrightarrow \tan x = \tan \left(\frac{\pi}{6}\right)$

Άρα, $x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + (\pi + \frac{\pi}{6}) \end{cases}, k \in \mathbb{Z}$

Homework 7 - Trigonometry

- 1) $\cos x = 0$ 2) $\cos x = -\frac{\sqrt{2}}{2}$
3) $\cos x = -1$ 4) $\tan x = 0$
5) $\tan x = \frac{\sqrt{3}}{3}$ 6) $\tan x = -\frac{\sqrt{3}}{3}$
7) $\cot x = \sqrt{3}$ 8) $\cot x = -\frac{\sqrt{3}}{3}$
9) $(\sqrt{3} + \tan x) \cdot (1 - \tan x) = 0$
10) $(2\cos x + 1) \cdot (\tan^3 x - 3) \cdot \cot x = 0$
11) $3\tan^2 x = 3 + 2\sqrt{3}\tan x$

$$12) \tan x \cdot \cot(2x) = 1$$

$$13) \sin x + \cos\left(\frac{\pi}{4} - x\right) = 0$$

$$14) \tan(2x) - \cot\left(\frac{\pi}{3} + 3x\right) = 0$$

$$15) \tan x \cdot \sin x + 1 = \sin x + \tan x$$

$$16) \frac{1}{\cos^2 x} - 2\tan x = 4$$

Να λύσετε τις παραπάνω
εξισώσεις.