

AP Precalculus - M3Y & M3Z

Polynomials - Homework 2

1. Determine whether the following functions are even, odd, or neither. Show your work.

(i) $f(x) = x^2 + 5$

(iv) $r(x) = 5x - 3$

(ii) $g(x) = x^3 - x^2 + x$

(v) $k(x) = x^{2024} + 7x - 1$

(iii) $h(x) = x^5 + x^3 + x$

(vi) $q(x) = -x^3 - 3x$

2. Prove the following:

(i) If $f(x)$ is an even function and $g(x)$ is an even function, then:

(a) $h(x) = f(x) + g(x)$ is an even function

(b) $k(x) = f(x)g(x)$ is an even function

(c) $s(x) = cf(x)$ is an even function for every $c \in \mathbb{R}$

(d) $t(x) = \frac{f(x)}{g(x)}$ is an even function

(ii) If $f(x)$ is an even function and $g(x)$ is an odd function, then:

(a) $k(x) = f(x)g(x)$ is an odd function

(b) $t(x) = \frac{f(x)}{g(x)}$ is an odd function

(iii) If $f(x)$ is an odd function and $g(x)$ is an odd function, then:

(a) $h(x) = f(x) + g(x)$ is an odd function

(b) $k(x) = f(x)g(x)$ is an even function

(c) $s(x) = cf(x)$ is an odd function for every $c \in \mathbb{R}$

(d) $t(x) = \frac{f(x)}{g(x)}$ is an even function

3. Can a non-zero function $f(x)$ be both odd and even? Explain.

4. Find the end-behavior as x increases without a bound and as x decreases without a bound of the following functions:

(i) $f(x) = x^5 - 4x^2 + 3x - 1$

(iv) $r(x) = \frac{1}{1000}x^2 - 10000x + 5$

(ii) $g(x) = -4x^{16} - 3x^2 + 7x - 9$

(v) $p(x) = 2x - 9$

(iii) $h(x) = -3x^7 + 2x^6 - 2x^5 - x^3 + 2x + 1$

(vi) $q(x) = -9x - 2$

5. Consider the polynomial function $f(x) = 2x^4 - 3x^3 - x^2 - 2$.

- (i) What is the greatest number of local minima and maxima that f can have? Explain.
- (ii) What is the greatest number of inflection points that f can have? Explain.
- (iii) Does f have a local extrema? Explain.
- (iv) Find the end-behavior of f as x increases without a bound and as x decreases without a bound.
- (v) Can f have a global maximum? Explain.