

$$10) \cot(a-b) = \frac{\cot a \cdot \cot b + 1}{\cot b - \cot a}$$

$$\cot(a-b) = \frac{\cos(a-b)}{\sin(a-b)}$$

$$\begin{aligned} &= \frac{\cos a \cdot \cos b + \sin a \cdot \sin b}{\sin a \cdot \cos b - \cos a \cdot \sin b} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{1}{\sin a \cdot \sin b} \cdot (\cos a \cdot \cos b + \sin a \cdot \sin b)}{\frac{1}{\sin a \cdot \sin b} \cdot (\sin a \cdot \cos b - \cos a \cdot \sin b)} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{\cos a \cdot \cos b}{\sin a \cdot \sin b} + \frac{\cancel{\sin a} \cdot \cancel{\sin b}}{\cancel{\sin a} \cdot \cancel{\sin b}}}{\frac{\cancel{\sin a} \cdot \cos b}{\cancel{\sin a} \cdot \sin b} - \frac{\cos a \cdot \cancel{\sin b}}{\sin a \cdot \cancel{\sin b}}} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{\cos a \cdot \cos b}{\sin a \cdot \sin b} + 1}{\frac{\cos b}{\sin b} - \frac{\cos a}{\sin a}} \end{aligned}$$

$$= \frac{\cot a \cdot \cot b + 1}{\cot b - \cot a}$$

$$11) \cot(a+b) = \frac{-\cot a \cdot \cot b + 1}{-\cot b - \cot a}$$

$$\cot(a+b) = \cot(a - (-b))$$

$$\stackrel{(10)}{=} \frac{\cot a \cdot \cot(-b) + 1}{\cot(-b) - \cot a}$$

$$\stackrel{(*)}{=} \frac{\cot a \cdot (-\cot b) + 1}{-\cot b - \cot a}$$

$$= \frac{-\cot a \cdot \cot b + 1}{-\cot b - \cot a}$$

(\*)

$$\cot(-b) = \frac{\cos(-b)}{\sin(-b)}$$

$$= \frac{\cos b}{-\sin b}$$

$$= -\cot b$$

12)

$$\cot(2a) = \frac{-\cot^2 a + 1}{-2 \cot a}$$

$$\cot(2a) = \cot(a + a)$$

$$\begin{aligned} & \textcircled{11} \quad \frac{-\cot a \cdot \cot a + 1}{- \cot a - \cot a} \\ &= \frac{-\cot^2 a + 1}{-2 \cot a} \end{aligned}$$

## Ταυτότητες Αποτέλεσμα νιοφά

$$1) \cos^2 a = \frac{1 + \cos(2a)}{2}$$

Απόδειξη:

$$\otimes \cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$\otimes \cos(2a) = \cos^2 a - \sin^2 a \quad (2a = a+a)$$

$$\begin{aligned}\cos(2a) &= \cos^2 a - \sin^2 a \\ &= \cos^2 a - (1 - \cos^2 a) \\ &= \cos^2 a - 1 + \cos^2 a \\ &= 2\cos^2 a - 1\end{aligned}$$

$$\text{'Αρα } \cos(2a) = 2 \cdot \cos^2 a - 1$$

$$\Leftrightarrow \cos(2a) + 1 = 2 \cdot \cos^2 a$$

$$\Leftrightarrow \cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\text{Εφαρμογή: } \cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\begin{aligned}\cos^2\left(\frac{\pi}{8}\right) &= \frac{1 + \cos\left(2 \cdot \frac{\pi}{8}\right)}{2} = \frac{1 + \cos \frac{\pi}{4}}{2} \\ &= \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}\end{aligned}$$

$$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$2) \quad \sin^2 a = \frac{1 - \cos(2a)}{2}$$

Απόδειξη:

$$\otimes \cos(2a) = \cos^2 a - \sin^2 a$$

$$\begin{aligned} \cos(2a) &= (1 - \sin^2 a) - \sin^2 a \\ &= 1 - \sin^2 a - \sin^2 a \\ &= 1 - 2\sin^2 a \end{aligned}$$

Αρα  $\cos(2a) = 1 - 2\sin^2 a$

$$\Leftrightarrow 2\sin^2 a = 1 - \cos(2a)$$

$$\Leftrightarrow \sin^2 a = \frac{1 - \cos(2a)}{2}$$

$$3) \quad \tan^2 a = \frac{1 - \cos(2a)}{1 + \cos(2a)}$$

Απόδειξη:

$$\begin{aligned} \tan^2 a &= \left( \frac{\sin a}{\cos a} \right)^2 \\ &= \frac{\sin^2 a}{\cos^2 a} \\ &\stackrel{(1)}{=} \frac{1 - \cos(2a)}{\cancel{2}} \\ &\stackrel{(2)}{=} \frac{1 + \cos(2a)}{\cancel{2}} \\ &= \frac{1 - \cos(2a)}{1 + \cos(2a)} \end{aligned}$$

$$\sin^2\left(\frac{\pi}{8}\right) = \frac{1 - \cos\left(2 \cdot \frac{\pi}{8}\right)}{2} = \frac{1 - \cos\left(\frac{\pi}{4}\right)}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4}$$

$$\text{Apa } \sin\left(\frac{\pi}{8}\right) = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\tan\left(\frac{\pi}{8}\right) = \frac{\sin\left(\frac{\pi}{8}\right)}{\cos\left(\frac{\pi}{8}\right)} = \frac{\frac{\sqrt{2 - \sqrt{2}}}{2}}{\frac{\sqrt{2 + \sqrt{2}}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{\sqrt{2 + \sqrt{2}}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}} = \sqrt{\frac{(2 - \sqrt{2}) \cdot (2 - \sqrt{2})}{(2 + \sqrt{2}) \cdot (2 - \sqrt{2})}} = \sqrt{\frac{(2 - \sqrt{2})^2}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{2} - 2}{2} = \sqrt{2} - 1$$