

$$1) \cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$$

Cinemath

$$2) \cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

Ausführung:

$$\begin{aligned} \cos(a+b) &= \cos(a - (-b)) \\ &= \cos a \cdot \cos(-b) + \sin a \cdot \sin(-b) \\ &= \cos a \cdot \cos b + \sin a \cdot (-\sin b) \\ &= \cos a \cdot \cos b - \sin a \cdot \sin b \end{aligned}$$

$$3) \sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$$

Ausführung:

$$\sin(\underbrace{a-b}_{\varphi}) \stackrel{①}{=} \cos\left(\frac{\pi}{2} - \underbrace{(a-b)}_{\varphi}\right)$$

$$= \cos\left(\frac{\pi}{2} - a + b\right)$$

$$= \cos\left(\left(\frac{\pi}{2} - a\right) + b\right)$$

$$\stackrel{②}{=} \cos\left(\frac{\pi}{2} - a\right) \cdot \cos b - \sin\left(\frac{\pi}{2} - a\right) \cdot \sin b$$

$$\stackrel{③, ④}{=} \sin a \cdot \cos b - \cos a \cdot \sin b$$

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$$\textcircled{3} \cos\left(\frac{\pi}{2} - \varphi\right) = \sin \varphi$$

$$\textcircled{4} \sin\left(\frac{\pi}{2} - \varphi\right) = \cos \varphi$$

$$4) \sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$$

$$5) \sin(2a) = 2 \cdot \sin a \cdot \cos a$$

Απόδειξη:

$$\sin(a+b) = \sin(a - (-b))$$

$$\stackrel{(3)}{=} \sin a \cdot \cos(-b) - \cos a \cdot \sin(-b)$$

$$\stackrel{(\otimes \otimes)}{=} \sin a \cdot \cos b - \cos a \cdot (-\sin b)$$

$$= \sin a \cdot \cos b + \cos a \cdot \sin b$$

Απόδειξη:

$$\sin(2a) = \sin(a+a)$$

$$\stackrel{(4)}{=} \sin a \cdot \cos a + \cos a \cdot \sin a$$

$$= 2 \cdot \sin a \cdot \cos a$$

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$$\cos(-\varphi) = \cos \varphi \quad (\otimes)$$

$$\sin(-\varphi) = -\sin \varphi \quad (\otimes \otimes)$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$

Απόδειξη:

$$\tan(a-b) \stackrel{(*)}{=} \frac{\sin(a-b)}{\cos(a-b)}$$

$$\stackrel{(3)}{=} \frac{\sin a \cdot \cos b - \cos a \cdot \sin b}{\cos a \cdot \cos b + \sin a \cdot \sin b}$$

$$\stackrel{(1)}{=} \frac{\sin a \cdot \cos b - \cos a \cdot \sin b}{\cos a \cdot \cos b + \sin a \cdot \sin b}$$

$$= \frac{\frac{1}{\cos a \cdot \cos b} (\sin a \cdot \cos b - \cos a \cdot \sin b)}{\frac{1}{\cos a \cdot \cos b} (\cos a \cdot \cos b + \sin a \cdot \sin b)}$$

$$= \frac{\frac{\sin a \cdot \cos b}{\cos a \cdot \cos b} - \frac{\cos a \cdot \sin b}{\cos a \cdot \cos b}}{\frac{\cos a \cdot \cos b}{\cos a \cdot \cos b} + \frac{\sin a \cdot \sin b}{\cos a \cdot \cos b}}$$

$$= \frac{\frac{\sin a}{\cos a} - \frac{\sin b}{\cos b}}{1 + \frac{\sin a}{\cos a} \cdot \frac{\sin b}{\cos b}}$$

$$\stackrel{(*)}{=} \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$

$$\frac{\sin}{\cos} \stackrel{(*)}{=} \tan$$

7)

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

Απόδειξη:

$$\tan(a+b) = \tan(a - (-b))$$

$$\stackrel{\textcircled{b}}{=} \frac{\tan a - \tan(-b)}{1 + \tan a \cdot \tan(-b)}$$

$$\stackrel{\textcircled{x}}{=} \frac{\tan a - (-\tan b)}{1 + \tan a \cdot (-\tan b)} = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

$\textcircled{x}$   $\tan(-\varphi) = -\tan \varphi$

$$8) \cos(2a) = \cos^2 a - \sin^2 a$$

Απόδειξη:

$$\begin{aligned} \cos(2a) &= \cos(a+a) \\ &\stackrel{(2)}{=} \cos a \cdot \cos a - \sin a \cdot \sin a \\ &= \cos^2 a - \sin^2 a \end{aligned}$$

$$9) \tan(2a) = \frac{2 \tan a}{1 - \tan^2 a}$$

Απόδειξη:

$$\tan(2a) = \frac{\sin(2a)}{\cos(2a)}$$

$$\stackrel{(5)}{=} \frac{2 \sin a \cdot \cos a}{\cos^2 a - \sin^2 a}$$

$$\stackrel{(8)}{=} \frac{2 \sin a \cdot \cos a}{\cos^2 a - \sin^2 a}$$

$$\begin{aligned} &= \frac{\frac{1}{\cos^2 a} \cdot (2 \sin a \cdot \cos a)}{\frac{1}{\cos^2 a} \cdot (\cos^2 a - \sin^2 a)} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{2 \sin a \cdot \cos a}{\cos^2 a}}{\frac{\cos^2 a - \sin^2 a}{\cos^2 a}} = \frac{2 \cdot \frac{\sin a}{\cos a}}{\frac{\cos^2 a}{\cos^2 a} - \frac{\sin^2 a}{\cos^2 a}} = \frac{2 \tan a}{1 - \tan^2 a} \end{aligned}$$