

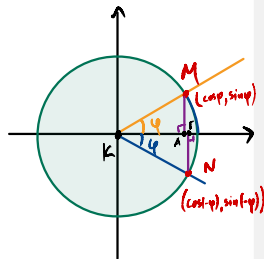
Από το M φέρω

$MA \perp x'x$ και

από το N φέρω $NG \perp x'x$

Συγκρίνω τα τρίγωνα

$\triangle KAM$ και $\triangle KGN$.



$|KM| = 1 = |KN|$ ①

$\angle KMA = \varphi = \angle KNG$ ②

$\angle MAK = 90^\circ = \angle NGK$

και άρα $KMA = KNG$ ③

Από τα ①, ②, ③

↓ *Κριτήριο
ΓΠΓ*

$\triangle KAM = \triangle KGN$

Άρα $\begin{cases} |NG| = |MA| \\ |KG| = |KA| \end{cases} \Leftrightarrow \begin{cases} -\sin(-\varphi) = \sin \varphi \\ \cos(-\varphi) = \cos \varphi \end{cases}$

Άρα

$$\sin(-\varphi) = -\sin \varphi$$

$$\cos(-\varphi) = \cos \varphi$$

$$\tan(-\varphi) = \frac{\sin(-\varphi)}{\cos(-\varphi)} = \frac{-\sin \varphi}{\cos \varphi} = -\tan \varphi$$

Άρα,

$$\tan(-\varphi) = -\tan \varphi$$

$$\cot(-\varphi) = \frac{\cos(-\varphi)}{\sin(-\varphi)} = \frac{\cos \varphi}{-\sin \varphi} = -\cot \varphi$$

Άρα,

$$\cot(-\varphi) = -\cot \varphi$$

$$\sec(-\varphi) = \frac{1}{\cos(-\varphi)} = \frac{1}{\cos \varphi} = \sec \varphi$$

Άρα,

$$\sec(-\varphi) = \sec \varphi$$

$$\csc(-\varphi) = \frac{1}{\sin(-\varphi)} = \frac{1}{-\sin \varphi} = -\csc(\varphi)$$

Άρα,

$$\csc(-\varphi) = -\csc(\varphi)$$