

Από το N και από
το M φέρω ευθείες
κάθετες στον $x'x$.

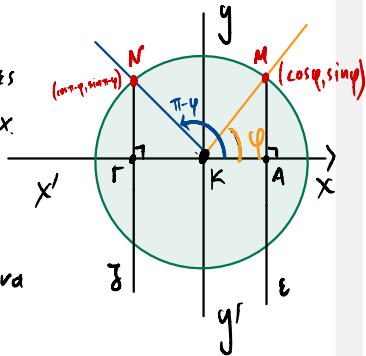
Έστω A, Γ τα
σημεία τομής

Συγκρίνω τα τρίγωνα
 $\triangle K\Gamma N$, $\triangle K\hat{A}M$

$$\left\{ \begin{array}{l} \underline{N\hat{K}\Gamma = \varphi = A\hat{K}M} \\ \underline{|\Gamma K| = 1 = |KN|} \\ N\hat{\Gamma}K = 90^\circ = M\hat{A}K \\ \text{και } \underline{\Gamma\hat{N}K = K\hat{M}A} \end{array} \right\} \xrightarrow{\text{Κριτήριο } \Gamma\Pi\Gamma} \triangle K\Gamma N = \triangle K\hat{A}M$$

$$\left. \begin{array}{l} \text{Άρα, } |\Gamma N| = |AM| \\ \text{και } |\Gamma K| = |AK| \end{array} \right\} \begin{array}{l} \sin(\pi - \varphi) = \sin \varphi \\ -\cos(\pi - \varphi) = \cos \varphi \end{array}$$

Άρα $\sin(\pi - \varphi) = \sin \varphi$
 $\cos(\pi - \varphi) = -\cos \varphi$



$$\begin{aligned} \tan(\pi - \varphi) &= \frac{\sin(\pi - \varphi)}{\cos(\pi - \varphi)} = \frac{\sin \varphi}{-\cos \varphi} = \\ &= -\frac{\sin \varphi}{\cos \varphi} = -\tan \varphi \end{aligned}$$

$$\tan(\pi - \varphi) = -\tan \varphi$$

$$\begin{aligned} \cot(\pi - \varphi) &= \frac{\cos(\pi - \varphi)}{\sin(\pi - \varphi)} = \frac{-\cos \varphi}{\sin \varphi} \\ &= -\frac{\cos \varphi}{\sin \varphi} = -\cot \varphi \end{aligned}$$

$$\cot(\pi - \varphi) = -\cot \varphi$$

$$\sec(\pi - \varphi) = \frac{1}{\cos(\pi - \varphi)} = \frac{1}{-\cos \varphi} = -\frac{1}{\cos \varphi} = -\sec \varphi$$

$$\sec(\pi - \varphi) = -\sec \varphi$$

$$\csc(\pi - \varphi) = \csc \varphi$$

Εφαρμογή

$$\begin{aligned}\sin(150^\circ) &= \sin(180^\circ - 30^\circ) \\ &= \sin\left(\pi - \frac{\pi}{6}\right) \\ &= \sin\left(\frac{\pi}{6}\right) \\ &= \frac{1}{2}\end{aligned}$$

$$\sin 150^\circ = \frac{1}{2}$$

$$\tan 150^\circ = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

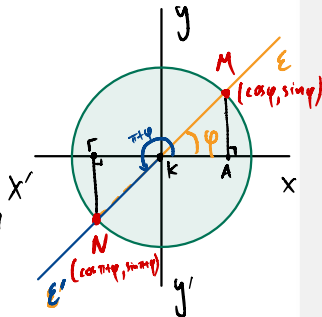
$$\begin{aligned}\cos(150^\circ) &= \cos(180^\circ - 30^\circ) \\ &= \cos\left(\pi - \frac{\pi}{6}\right) \\ &= -\cos\frac{\pi}{6} \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

$$\cos(150^\circ) = -\frac{\sqrt{3}}{2}$$

Από το Ν και
από το Μ φέρω
ευθύγραμμα τμήματα

κάθετα στον άξονα $x'x$.

Συγκρίνω τα τρίγωνα $\triangle K\Gamma N$, $\triangle K\Lambda M$



$$\hat{M} \hat{K} A = N \hat{K} \Gamma$$

(κατά κορυφήν)

$$\underline{|KM| = 1 = |KN|}$$

$$N\hat{\Gamma}_K = 90^\circ = K\hat{A}M$$

до $\Gamma \hat{N} K = K \hat{M} A$

7 Κριτήριο

$$\{ \begin{array}{c} \Gamma \\ \Pi \\ \Gamma \end{array} \} \Rightarrow K \Gamma N = K \Delta M$$

Από ισόζησα έχω

$$\begin{cases} |K\Gamma| = |KA| \\ |\Gamma N| = |AM| \end{cases} \Leftrightarrow \begin{cases} -\cos(\pi + \varphi) = \cos \varphi \\ -\sin(\pi + \varphi) = \sin \varphi \end{cases}$$

'Ezot,

$$\cos(\pi + \varphi) = -\cos \varphi$$

$$\sin(\pi + \varphi) = -\sin \varphi$$

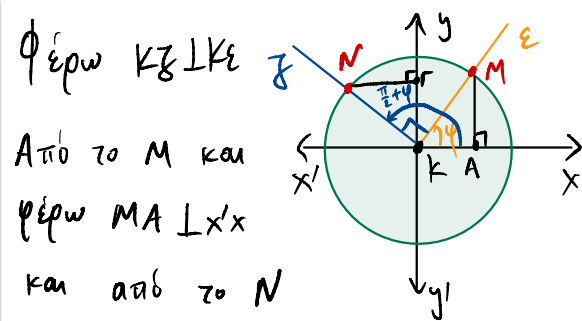
$$\tan(\pi + y) = \frac{\sin(\pi + y)}{\cos(\pi + y)} = \frac{-\sin y}{-\cos y} = \frac{\sin y}{\cos y} = \tan y$$

$$\tan(\pi + \varphi) = \tan \varphi$$

$$\cot(\pi + \varphi) = \cot \varphi$$

$$\sec(\pi + \varphi) = -\sec \varphi$$

$$\csc(\pi + \varphi) = -\csc \varphi$$



Ξυγκρίνω τα τρίγωνα $K\hat{\Gamma}N$ και $K\hat{A}M$

$$\begin{cases} |KN| = 1 = |KM| \\ \underline{M\hat{K}A = \varphi = N\hat{K}\Gamma} \text{ ("} \frac{\pi}{2} - \Gamma\hat{K}M) \\ K\hat{\Gamma}N = 90^\circ = M\hat{A}K \\ \text{Άρα } \underline{\Gamma\hat{N}K = K\hat{A}M} \end{cases}$$

Άρα $K\hat{\Gamma}N = K\hat{A}M$ από $\Pi\Gamma\Pi$

Άρα $\begin{cases} |\Gamma N| = |AM| \text{ (απέναντι της} \\ \text{γωνίας } \varphi) \\ |\Gamma K| = |KA| \text{ (πρόσκείμενη} \\ \text{της } \varphi) \end{cases}$

$$\Leftrightarrow \begin{cases} -\cos\left(\frac{\pi}{2} + \varphi\right) = \sin \varphi \\ \sin\left(\frac{\pi}{2} + \varphi\right) = \cos \varphi \end{cases}$$

$$\Leftrightarrow \begin{cases} \cos\left(\frac{\pi}{2} + \varphi\right) = -\sin \varphi \\ \sin\left(\frac{\pi}{2} + \varphi\right) = \cos \varphi \end{cases}$$

$$\tan\left(\frac{\pi}{2} + \varphi\right) = \frac{\sin\left(\frac{\pi}{2} + \varphi\right)}{\cos\left(\frac{\pi}{2} + \varphi\right)} = \frac{\cos \varphi}{-\sin \varphi} = -\cot \varphi$$

$$\tan\left(\frac{\pi}{2} + \varphi\right) = -\cot \varphi$$