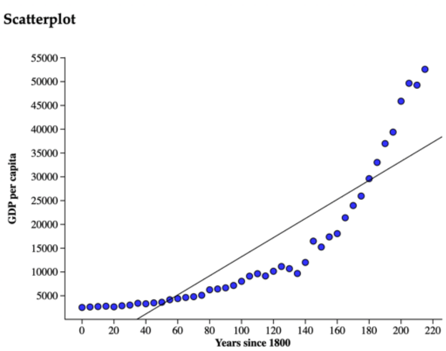
How has GDP Per Capita Changed Over Time?

Gross Domestic Product (GDP) is a measure of a country's total economic activity—the value of all goods and services produced over a given time period. GDP per capita divides this measure by the population to get a per person unit of wealth. Data about the U.S. GDP per capita is given in the spreadsheet for the years 1800 to 2020.

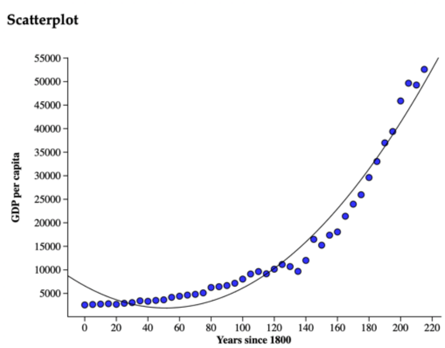
1. Does the GDP per capita seem to change by a constant difference, a constant second difference, or a constant ratio in each 5-year period? Explain.

2. Three regression models for this data and their equations are shown below. Do you think a linear model, a quadratic model, or an exponential model fits the data best? Explain.

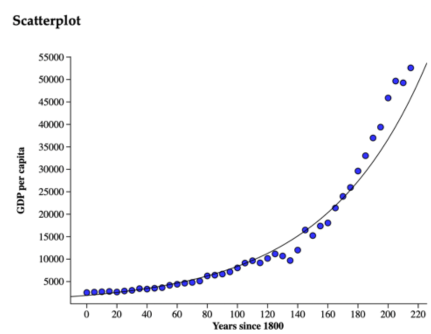
$\hat{y} = -6844.79 + 200.53x$



$\hat{y} = 6579.89 - 183.04x + 1.784x^2$



$\hat{y} = 1914.704(1.0149)^x$



3. Interpret the meaning of 1914.704 in the exponential regression equation.

4. Interpret the meaning of 1.0149 in the exponential regression equation.

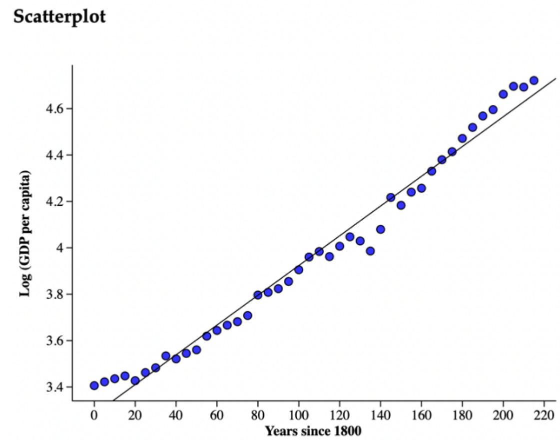
5. Use the exponential model to predict the GDP per capita for the year 1830.

6. Imagine if we took all the data values for GDP and found the log of them. Use your calculator to fill in the selected values in the table.

Years since 1800	GDP per capita	Log(GDP per capita)
0	$2545.59	
50	$3631.82	
100	$8037.57	
150	$15240.00	
200	$45886.47	

7. What do you think the scatterplot will look like when plotting Years since 1800 and Log(GDP per capita)? Explain your reasoning.

8. This scatterplot graphs Years since 1800 versus the Log(GDP per capita). What do you notice?



9. The equation for the line shown is given by $\hat{y} = 3.2821 + 0.0064x$.

    a. Interpret the meaning of 3.2821 in the linear regression equation.

    b. Interpret the meaning of 0.0064 in the linear regression equation.

    c. Use the linear model to predict the GDP per capita for the year 1830.

10. The exponential model for the original data is given by $\hat{y} = 1914.704(1.0149)^x$.

    The linear model for the transformed data is given by $\hat{y} = 3.2821 + 0.0064x$.

    How are the numbers in the transformed linear model related to the numbers of the original exponential regression?

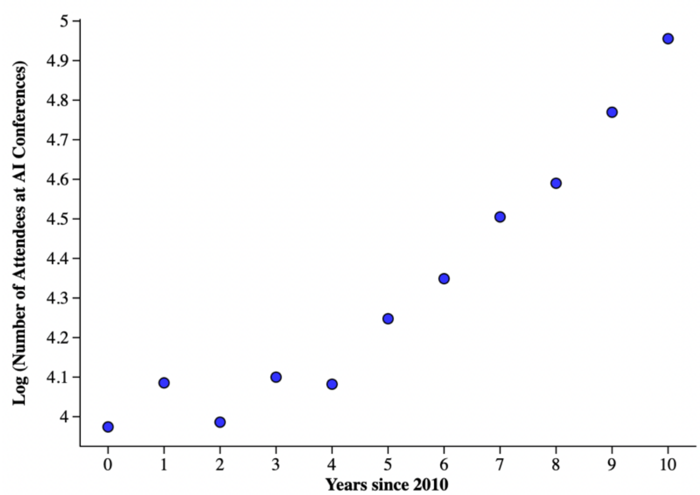
11. Why might it be helpful to transform exponential data and produce a linear model?

Lesson 5.9 – Semi-log Plots

QuickNotes

Check Your Understanding

1. Attendance at Artificial Intelligence (AI) conferences has increased over time. The scatterplot shows the Year (measured in Years since 2010) and the *log* of the number of AI conference attendees.
   a. Describe how the *log* of the number of AI attendees has changed over time.



   b. Describe how the *number* of AI conference attendees has changed over time.

   c. The equation for the regression line on the semi-log plot is $\hat{y} = 3.8525 + 0.0957x$. Write an equation that can be used to model the number of AI conference attendees $A(x)$, x years after 2010.