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You may have a Quiz on the following FRQ questions. There are three questions in this assignment. Make sure you solve them carefully at home, without looking at any notes, so that you are sure you are well prepared for both the quiz and the AP Test.

The deadline for all homework assignments is the one specified in Archie before 11:59 pm. Show all your work and justifications.

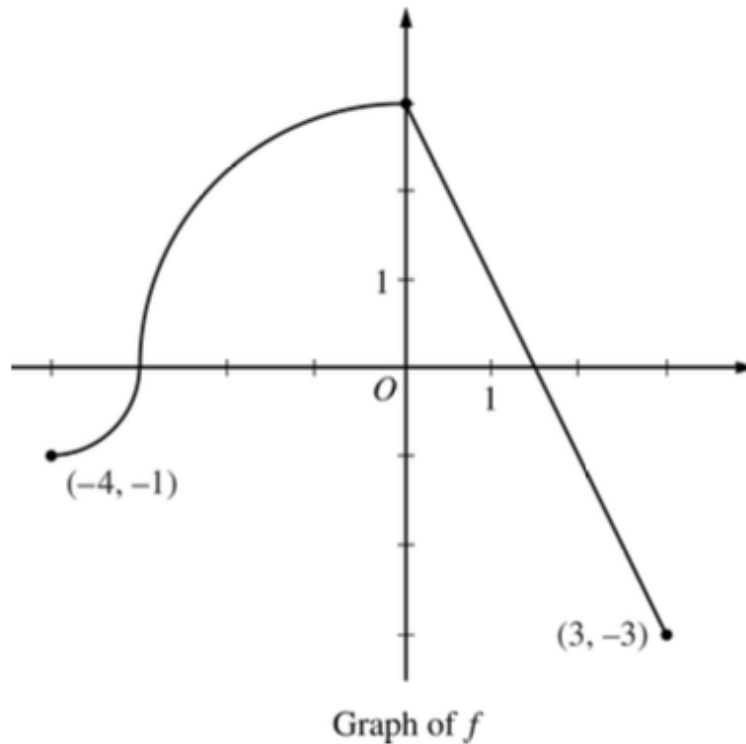
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You will need a Calculator for the following question:

$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.
- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

Do not use a Calculator for the following two questions, and evaluate any integrals or derivatives, if needed:



4. The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above. Let  $g(x) = 2x + \int_0^x f(t) dt$ .
- Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
  - Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.
  - Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
  - Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

6. Let  $f$  be a function defined by  $f(x) = \begin{cases} 1 - 2 \sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that  $f$  is continuous at  $x = 0$ .

(b) For  $x \neq 0$ , express  $f'(x)$  as a piecewise-defined function. Find the value of  $x$  for which  $f'(x) = -3$ .

(c) Find the average value of  $f$  on the interval  $[-1, 1]$ .