Chapter 7

Statement Logic: Truth Tables

The history of the discipline of logic is not a continuous, upward progression from one discovery to the next. Rather, it is characterized by episodes of discovery followed by lengthy periods in which little creative work was done. The philosopher Aristotle (384–322 B.C.E.) is generally considered the father of logic. His syllogistic logic was the first great systematic development in the field. The Stoic philosopher Chrysippus (279–206 B.C.E.) also made important contributions to logic by developing the rudiments of statement logic. For example, Chrysippus analyzed compound statements such as disjunctions and conditionals and identified the patterns of reasoning that we now refer to as *modus ponens* and *modus tollens*. But for over a thousand years after the death of Chrysippus, there were few significant advances in the field of logic. Most philosophers were content to write commentaries on the works of Aristotle and Chrysippus.

During the medieval period, there was a resurgence of interest in logic. Philosophers such as Peter Abelard (1079–1142) and William of Ockham (c. 1285–1349) made important contributions to the field. Some of the medieval logicians offered subtle insights into *modal logic* (roughly, the logic of possibility and necessity). However, at the end of the medieval period, logic again entered a time of relative neglect, so there was little creative work in the field from the middle of the 15th century until the time of the German philosopher Gottfried Wilhelm Leibniz (1646–1716). Leibniz stimulated interest in a more symbolic approach to logic.

Since the mid-19th century, the field of logic has developed at a rapid pace. We have already examined the work of John Venn (1834–1923), whose diagram method is perhaps the most intuitive means of evaluating categorical syllogisms. In this chapter, we will focus primarily on the truth table method developed by the American philosopher Charles Sanders Peirce (1839–1914).[1] However, before we can apply the truth table method, we must first learn how to translate English arguments into symbols.

7.1 Symbolizing English Arguments

In Chapter 1, we saw that arguments are valid by virtue of having a valid form. Modern logicians have developed very useful ways of symbolizing an argument's form. And as we will see, symbolizing an argument enables us to apply certain powerful techniques to determine its validity.

To symbolize statements properly, we must distinguish between atomic and compound statements. An **atomic statement** is one that does not have any other statement as a component. For example:

1. Shakespeare wrote *Hamlet.*
2. China has a large population.
3. Roses are red.

A **compound statement** is one that has at least one atomic statement as a component. For instance:

4. It is not the case that Ben Jonson wrote *Hamlet.*
5. China has a large population, and Luxembourg has a small population.
6. Either Palermo is the capital of Sicily, or Messina is the capital of Sicily.
7. If Sheboygan is in Wisconsin, then Sheboygan is in the U.S.A.
8. The Democrats win if and only if the Republicans quarrel.

We can symbolize the *atomic* statements in these compounds with capital letters, as follows:

B: Ben Jonson wrote *Hamlet.*
C: China has a large population.
L: Luxembourg has a small population.
P: Palermo is the capital of Sicily.
M: Messina is the capital of Sicily.
S: Sheboygan is in Wisconsin.
U: Sheboygan is in the U.S.A.
D: The Democrats win.
R: The Republicans quarrel.

The compounds themselves can then be written as follows (in order):

9. It is not the case that B.
10. C and L.
11. Either P or M.

12. If S, then U.
13. D if and only if R.

Note that statement (9) counts as a compound even though it has only one statement as a component. It is a compound consisting of an atomic statement and the phrase "it is not the case that."

Throughout this chapter and the next, we will use capital letters to stand for atomic statements. We will also use symbols to stand for the key logical words in our example compounds, namely, "it is not the case that," "and," "or," "if . . . then . . . ," and "if and only if." We will symbolize these English expressions by means of **logical operators**. We can sum up the symbol system as follows:*

Operator	Name	Translates	Type of Compound
~	tilde	not	negation
•	dot	and	conjunction
∨	vee	or	disjunction
→	arrow	if-then	conditional
↔	double-arrow	if and only if	biconditional

Negations

The " ~ " symbol, called the **tilde**, is used to translate the English word "not" and its stylistic variants. Take the following example:

14. Roses are not blue. (R: Roses are blue)

The **scheme of abbreviation** on the right assigns a distinct capital letter to each atomic statement in the English. Using the tilde, we can then symbolize statement (14) as follows:

15. ~R

Of course, the English language provides a number of ways of negating a statement. For example:

a. It is not the case that roses are blue.
b. It is false that roses are blue.
c. It is not true that roses are blue.
d. Roses fail to be blue.

*From a historical point of view, symbolic logic is relatively new and its notation is not yet standardized. Thus, although the symbols provided in this text are in common use, they are not the only ones in common use. Many texts employ one or more of the following alternatives: "¬" to symbolize negations; "&" to symbolize conjunctions; "⊃" to symbolize conditionals, and "≡" to symbolize biconditionals. The lack of standard notation in logic is inconvenient, but it is not difficult to move from one notation to another once the basic principles have been mastered.

Statement (15) translates each of these English expressions into symbols.

> Each of the statements below is a **negation**. The main operator is the *tilde*.
>
> ~C
>
> ~(A v B)
>
> ~(F → G)

Conjunctions

The "•" sign (called the **dot**) is used to translate the English word "and" as well as its stylistic variants. Take the following example:

16. Hobbes was born in 1588, *and* Descartes was born in 1596. (H: Hobbes was born in 1588; D: Descartes was born in 1596)

Using the scheme of abbreviation indicated, statement (16) translates into symbols as follows:

17. H • D

The statements composing a conjunction (*H* and *D* in this case) are called **conjuncts**. A partial list of stylistic variants for "and" is provided by the following set of sentences:

a. Hobbes was born in 1588, *but* Descartes was born in 1596.
b. Hobbes was born in 1588; *however*, Descartes was born in 1596.
c. *While* Hobbes was born in 1588, Descartes was born in 1596.
d. *Although* Hobbes was born in 1588, Descartes was born in 1596.
e. Hobbes was born in 1588, *yet* Descartes was born in 1596.
f. Hobbes was born in 1588; *nevertheless*, Descartes was born in 1596.
g. Hobbes was born in 1588 *even though* Descartes was born in 1596.
h. Hobbes was born in 1588 *though* Descartes was born in 1596.

(17) correctly symbolizes each of these variants. You may be thinking that such words as "but," "while," and "although" do not have quite the same connotation as "and" in ordinary English. Indeed, these words convey a sense of contrast that is lacking in "and." But bear in mind that some distortion often results when one language is translated into another. Moreover, *for the purpose of evaluating arguments for validity*, the expressions in the previous list can usually be translated adequately by means of the dot.

It should be noted, however, that the dot does not correctly translate *every* use of the English word "and." Consider the following statements:

18. Jane became pregnant and got married. (P: Jane became pregnant; M: Jane got married)
19. Jane got married and became pregnant.[2]

In ordinary conversation, these two statements do not mean the same thing. Here, the word "and" has the force of "and then." In other words, it indicates a temporal order. But the dot does not indicate any such temporal order, so the dot cannot adequately translate statements (18) and (19). Can the dot provide an adequate translation of the following statements?

20. Jack and Jill are married. (J: Jack is married; M: Jill is married)
21. Jack and Jill are persons. (J: Jack is a person; P: Jill is a person)

Statement (20) would normally be taken to mean that Jack and Jill are married *to each other*. But if we translate (20) via the dot, we get a statement that could be true even if Jack and Jill are *not* married to each other, namely, $J \bullet M$. On the other hand, (21) *can* be adequately translated by means of the dot: $J \bullet P$.

Parentheses must often be used as a form of punctuation. To understand why, consider the following statement:

22. It is not true that both Carla and Bill won. (C: Carla won; B: Bill won)

The correct symbolization is as follows:

23. $\sim(C \bullet B)$

If we remove the parentheses, we get this:

24. $\sim C \bullet B$

(24) says, "Carla did not win, and Bill won," which is not at all the meaning of (22). So, the placement of parentheses is important when translating English into symbols.

In longer statements, we may alternate parentheses and brackets, since multiple sets of parentheses can become confusing. For instance:

25. The statement "Al is old, but he is strong and healthy" is false. (O: Al is old; S: Al is strong; H: Al is healthy)

The comma indicates how the conjuncts are to be grouped:

26. $\sim[O \bullet (S \bullet H)]$

Each of the statements below is a **conjunction**. The main operator is the *dot*.

E • ~F

(G v H) • K

(L → M) • (N v O)

Disjunctions

The "v" sign (called the **vee**) is used to symbolize disjunctions. (This symbol is borrowed from the first letter of the Latin word *vel*, meaning "or.") The vee means "either . . . or . . . *or both*," which is commonly referred to as the *inclusive* sense of the word "or." The vee does *not* mean "either . . . or . . . *but not both*," which is commonly referred to as the *exclusive* sense of the word "or."[3] Consider this example:

27. Either Carol attends college or she gets a job (or both). (C: Carol attends college; J: Carol gets a job)

Statement (27) can be translated into symbols this way:

28. C v J

(28) also translates the stylistic variants of (27), such as the following:

a. Carol attends college *and/or* she gets a job.
b. Carol attends college *or* she gets a job.
c. *Either* Carol attends college *or* she gets a job.
d. Carol attends college *unless* she gets a job.

As a general rule, when symbolizing arguments containing disjunctions, assume that the word "or" is used in the inclusive sense unless this assumption renders the argument invalid. For example, consider the following argument, which has the form of a *disjunctive syllogism*:

29. Lassie is either a cat or a dog. Lassie is not a cat. So, Lassie is a dog. (C: Lassie is a cat; D: Lassie is dog)

This argument is correctly symbolized as follows:

30. C v D, ~C ∴ D

Several things should be noted here. First, a comma is used to punctuate (or separate) the premises. Second, the vee is used in the first premise even though C and D cannot in fact both be true. The argument form is valid even if the "or" is inclusive: "Either *C* or *D* (or both) are true. It is not true that C. So, *D* is true." Third, note the use of the triple-dot symbol to mark the conclusion. This is customary among logicians.

What if we need the exclusive "or" to represent an argument fairly? For example:

31. Either the Sonics won or the Bulls won. The Sonics won. So, the Bulls did not win. (S: The Sonics won; B: The Bulls won)

Intuitively, the argument is valid, but the following symbolized version is invalid:

32. $S \vee B,\ S \therefore {\sim}B$

Here's a counterexample: "Either trees are plants or flowers are plants (or both are plants). Trees are plants. So, flowers are not plants." To present the argument fairly, we need to interpret the first premise as follows:

33. Either the Sonics won or the Bulls won, but it is not true that both the Sonics won and the Bulls won.

The comma indicates that the main logical connective in (33) is the word "but," which is symbolized by the dot. The left conjunct is a disjunction ("Either the Sonics won or the Bulls won"), and the right conjunct is the negation of a conjunction ("It is not true that both the Sonics won and the Bulls won"). So, in symbols, (33) looks like this:

34. $(S \vee B) \bullet {\sim}(S \bullet B)$

This example draws attention once again to the importance of parentheses. For example, if we omit the set of parentheses on the left, we have: $S \vee B \bullet {\sim}(S \bullet B)$. But this expression is ambiguous—it might be taken to mean (34), but it could also be taken to mean this: $S \vee [B \bullet {\sim}(S \bullet B)]$. Note that the main operator in the latter statement is the vee rather than the dot. Thus, parentheses are often needed to prevent ambiguity.

With (34) as its first premise, argument (31) as a whole is symbolized as follows:

35. $(S \vee B) \bullet {\sim}(S \bullet B),\ S \therefore {\sim}B$

This argument is intuitively valid, and we will prove that it is valid later in this chapter.

Before leaving disjunctions, let us note that statements of the form "Neither *A* nor *B*" can be symbolized in two ways. For instance:

36. Neither Sue nor Fred is happy. (S: Sue is happy; F: Fred is happy)

We can symbolize statement (36) by means of the vee, as follows:

37. ~(S v F)

But we can also symbolize it by means of the dot, like this:

38. ~S • ~F

> Each of the statements below is a **disjunction**. The main operator is the *vee*.
> ~P v Q
> (R • S) v ~T
> (U → W) v ~(X • Y)

Conditionals

The "→" sign (called the **arrow**) is used to symbolize conditionals. For example:

39. If Fido is a dog, then he is an animal. (D: Fido is a dog; A: Fido is an animal)

(39) can be symbolized as follows:

40. D → A

As we observed in Chapter 1, there are many stylistic variants for if-then statements. We will use the arrow to symbolize all of them. For example, expression (40) symbolizes not only (39) but also each of the following:

a. *Given that* Fido is a dog, Fido is an animal.
b. Fido is an animal *given that* he is a dog.
c. *Assuming that* Fido is a dog, he is an animal.
d. Fido is an animal *assuming that* he is a dog.
e. *Provided that* Fido is a dog, he is an animal.
f. Fido is an animal *provided that* he is a dog.
g. *On the condition that* Fido is a dog, he is an animal.
h. Fido is an animal *on the condition that* he is a dog.
i. Fido is an animal *if* he is a dog.
j. Fido is a dog *only if* he is an animal.
k. Fido's being a dog *is a sufficient condition for* Fido's being an animal.
l. Fido's being an animal *is a necessary condition for* Fido's being a dog.

Items (k) and (l) merit comment. A *sufficient condition* is a condition that guarantees that a statement is true (or that a phenomenon will occur). For instance, *Fido's being a dog* guarantees that he is an animal. By contrast, *Fido's being an animal* does not guarantee that he is a dog, for he might be some other kind of animal. The *antecedent* (if-clause) of a true conditional statement provides a sufficient condition for the truth of the *consequent* (then-clause).

A *necessary condition* is a condition that, if lacking, guarantees that a statement is false (or that a phenomenon will not occur). Thus, *Fido's being an animal* is a necessary condition for *Fido's being a dog*, for if Fido is not an animal, then he is not a dog. The consequent (then-clause) of a true conditional statement provides a necessary condition for the truth of the antecedent (if-clause).

Each of the statements below is a **conditional**. The main operator is the *arrow*.

$\sim X \rightarrow Y$

$Z \rightarrow (A \vee B)$

$(C \bullet \sim D) \rightarrow (E \vee \sim F)$

Let us now symbolize an argument involving a conditional statement:

41. If humans have souls, then immaterial things can evolve from matter. Immaterial things cannot evolve from matter. So, humans do not have souls. (H: Humans have souls; M: Immaterial things can evolve from matter)

Using the scheme of abbreviation provided, argument (41) can be symbolized like this:

42. $H \rightarrow M, \ \sim M \ \therefore \ \sim H$

Again, we use the triple-dot symbol to mark the conclusion. Can you identify the form employed in argument (42)? It is *modus tollens*.

Before leaving our discussion of conditionals, let us note that the word "unless" can be translated by means of the arrow as well as the vee. For example:

43. We will lose unless we do our best. (L: We will lose; B: We will do our best)

As we have already seen, (43) can be symbolized as follows:

44. $L \vee B$

But it can also be symbolized by a combination of the arrow and the tilde, like this:

45. $\sim B \rightarrow L$

In other words, (43) has the same meaning as "If we do not do our best, then we will lose."

Biconditionals

The "↔" sign (called the **double-arrow**) is used to symbolize biconditionals. For example:

46. Mary is a teenager *if and only if* she is from 13 to 19 years of age. (M: Mary is a teenager; Y: Mary is from 13 to 19 years of age)

This statement may be symbolized as follows:

47. M ↔ Y

And (47) symbolizes not only (46) but also its stylistic variants, such as these:

a. Mary is a teenager *just in case* she is from 13 to 19 years of age.
b. Mary's being a teenager is *a necessary and sufficient condition for* Mary's being from 13 to 19 years of age.

> Each of the following statements is a **biconditional**. The main operator is the *double-arrow*.
> ~H ↔ J
> ~K ↔ (P v Q)
> (L • M) ↔ (N → T)

Let us now consider an example that illustrates some of the finer points of translating arguments into symbols:

48. If Dostoyevsky was right, then everything is permissible if God does not exist. But it is not true that if God does not exist, everything is permissible. Therefore, Dostoyevsky was not right. (D: Dostoyevsky was right; E: Everything is permissible; G: God exists)

The first premise may be symbolized as follows:

49. D → (~G → E)

Note that we cannot remove the parentheses from (49), for if we did, we would alter the meaning:

50. D → ~G → E

Statement (50) is ambiguous because it could be interpreted as (49) or as follows:

51. (D → ~G) → E

And these two statements have different meanings. (51) translates sentences that are difficult to put gracefully into English, such as the following:

52. If Dostoyevsky was right only if God does not exist, then everything is permitted.

53. If God doesn't exist given that Dostoyevsky was right, then everything is permitted.

This should make it clear that (51) is not an accurate translation of the first premise of the argument.

Now, let us symbolize the second premise of argument (48), that is, "It is not true that if God does not exist, everything is permissible":

54. ~(~G → E)

Note that there must be a tilde *outside* the parentheses in this case because "it is not true that" precedes the word "if" in the English statement. Moreover, we cannot drop the parentheses. If we did, we would get this:

55. ~~G → E

Statement (55) says, "If it is not true that God does not exist, then everything is permissible." In other words, since the two "nots" cancel each other out, it says, "If God exists, everything is permissible," which is not at all the meaning of the English.

Finally, let us symbolize the conclusion of argument (48), that is, "Dostoyevsky was not right":

56. ~D

Note that we use no parentheses in symbolizing the conclusion. For example, do *not* write ~(*D*). We do not put parentheses around a single statement letter because this only adds clutter. Nor do we write (~*D*). *Parentheses may be used with the dot, the vee, the arrow, and the double-arrow, but not with the tilde itself.*

Now, let's put all the pieces together. Here is the original argument and our symbolization of it:

> If Dostoyevsky was right, then everything is permissible if God does not exist. But it is not true that if God does not exist, everything is permissible. Therefore, Dostoyevsky was not right. (D: Dostoyevsky was right; E: Everything is permissible; G: God exists)
>
> *In symbols:* D → (~G → E), ~(~G → E) ∴ ~D

In closing this section, let us describe our symbolic language more precisely. The vocabulary consists of parentheses, the logical operators (namely, ~, ∨, •, →, and ↔), and statement letters (that is, capital letters A through Z). An *expression* of statement logic is *any* sequence of symbols in this vocabulary, such as (→S ∨↔(N~)). A grammatically correct symbolic expression is called a **well-formed formula** (WFF for short). To sum up what counts as a WFF, let us use the italicized, lowercase letters ***p*** and ***q*** as **statement variables**, which can stand for any statement. For instance, in the following summary, the statement variable ***p*** could stand for A, for ~B, for (C ∨ ~D), for (E • F), for (G → H), and so on. A symbolic expression is a WFF under the following conditions:

1. Capital letters (which stand for atomic statements) are WFFs.
2. If ***p*** is a WFF, then so is ~***p***.
3. If ***p*** and ***q*** are WFFs, then so is (***p*** • ***q***).
4. If ***p*** and ***q*** are WFFs, then so is (***p*** ∨ ***q***).
5. If ***p*** and ***q*** are WFFs, then so is (***p*** → ***q***).
6. If ***p*** and ***q*** are WFFs, then so is (***p*** ↔ ***q***).

Nothing counts as a WFF unless it can be demonstrated to be one by applications of the above conditions. Consider a symbolic expression that violates the above conditions:

57. (A ∨ B • C)

We cannot tell whether the main operator in (57) is the vee or the dot. Thus, (57) is ambiguous between the following:

58. ((A ∨ B) • C)
59. (A ∨ (B • C))

Both of these formulas are WFFs. Let us examine (58) in detail. Its main operator is the dot. To apply condition (3) above, replace ***p*** with (A ∨ B) and ***q*** with C. Thus, (58) is a WFF given that (A ∨ B) and C are WFFs. Condition (1) assures us that C is a WFF. Condition (4) assures us that (A ∨ B) is a WFF if A is a WFF and B is a WFF; and of course A and B are WFFs according to condition (1). Hence, (58) is itself a WFF.

Please note that the six conditions above are to be taken quite strictly. For example, condition (2) does not tell us to place parentheses around negations, so we should not do so. Thus, ~A is a WFF, but (~A) is not a WFF. Also, the above conditions tell us to use parentheses along with the dot, vee, arrow, and double-arrow. So, strictly speaking, our rules tell us to write (F → G) rather than F → G. And whenever there is a question about whether a given expression is a WFF, we can use the above conditions strictly to obtain a definitive answer. *That said, for the sake of convenience, certain abbreviations are permitted and will be routinely employed*. For example, we can drop parentheses in many cases without creating ambiguity, and we will do so to avoid clutter. Thus, we will routinely write $F \rightarrow G$ rather than $(F \rightarrow G)$, and $(A \vee B) \bullet C$ rather than $((A \vee B) \bullet C)$. Also, even though our rules make no mention of brackets, we will often alternate parentheses with brackets in long expressions, as this makes the statements a bit easier to read. For example, we will routinely write $\sim[F \bullet (G \vee H)]$ rather than $\sim(F \bullet (G \vee H))$.

The symbolic language we have developed in this section is extremely useful as a means of representing the forms of arguments. But, as with any language, practice is essential to facility. The following exercises provide you with an opportunity to practice translating English into symbols.

Exercise 7.1

Part A: Well-Formed Formulas? Which of the following symbolic expressions are well-formed formulas (WFFs)? Which are not? (In answering these questions, use the six conditions for WFFs strictly, making no allowances for routine abbreviations.)

* 1. $(A \rightarrow B \rightarrow C)$
2. $(\sim B)$
3. $(\sim(C) \rightarrow F)$
* 4. $(E \rightarrow (\sim F \rightarrow G)$
5. $\sim((H \rightarrow J) \rightarrow (K \rightarrow L)$
6. $(M \rightarrow \sim\sim N)$
* 7. $(O \rightarrow \sim(P \rightarrow R))$
8. $((Q \rightarrow S) \rightarrow T)$
9. $(\sim U \rightarrow (W))$
* 10. $\sim Z$
11. $\sim(B \bullet C)$
12. $\sim(\sim W \vee \sim Z)$
* 13. $\sim(m \leftrightarrow \sim h)$
14. $(\sim E \bullet \sim F \bullet \sim\sim G)$
15. $\sim(\sim A \rightarrow \sim R)$
* 16. $(\sim S \vee \sim R \vee \sim(T \bullet U))$
17. $(\sim P \vee Q \vee \sim R)$
18. $((L \vee M) \rightarrow \sim S)$
* 19. $(\sim(D \bullet E) \leftrightarrow (F \vee \sim G))$
20. $((\sim H \bullet \sim\sim F) \rightarrow \sim(\sim K \leftrightarrow \sim N))$

Part B: Permissible Departures from Strict Grammar What counts as a WFF is defined strictly in terms of six conditions, but in some cases, parentheses can be dropped without changing the meaning and without causing ambiguity. Also, in some cases, alternating brackets with parentheses makes a formula easier to read. Which of the following are examples of formulas that permissibly depart from a strict application of the six conditions by dropping a set of parentheses or by an appropriate use of brackets?

* 1. $E \vee \sim F$
2. $\sim G \bullet \sim H$
3. $\sim J \leftrightarrow \sim K$
* 4. $\sim L \vee [(M \rightarrow N) \rightarrow \sim O]$
5. $(\sim Q \vee \sim R \vee \sim\sim S)$
6. $(A \vee B) \leftrightarrow (C \leftrightarrow D)$
* 7. $\sim U \rightarrow \sim X$
8. $[\sim Z \bullet \sim W \vee \sim\sim Y]$
9. $\sim A \rightarrow (\sim C \rightarrow F)$
* 10. $(B \bullet E) \bullet [(G \vee H) \bullet (J \bullet K)]$

Part C: Symbolizing Translate the following statements into symbols, using the schemes of abbreviation provided.

* 1. The crops will fail unless it rains. (C: The crops will fail; R: It rains)

2. Humans are animals if they are mammals. (A: Humans are animals; M: Humans are mammals)

3. The statement "If humans are rational, then they are not animals" is false. (R: Humans are rational; A: Humans are animals)

* 4. Bats are mammals only if they nourish their young with milk. (M: Bats are mammals; N: Bats nourish their young with milk)

5. Coffee isn't good if it isn't fresh-brewed. (G: Coffee is good; F: Coffee is fresh-brewed)

6. Assuming that your test scores are high and you get your paper in on time, you will do well. (T: Your test scores are high; P: You get your paper in on time; W: You will do well)

* 7. Roberto lacks wisdom. (R: Roberto has wisdom)

8. The statement "Humans lack rationality" is false. (H: Humans have rationality)

9. Polly fails to be a parrot provided that she cannot talk and does not want a cracker. (P: Polly is a parrot; T: Polly can talk; C: Polly wants a cracker)

* 10. Neither birds nor snakes are mammals. (B: Birds are mammals; S: Snakes are mammals)

11. Given that Linda is both smart and diligent, she will do well; but Linda is not diligent. (S: Linda is smart; D: Linda is diligent; W: Linda will do well)

12. Al wins only if Ed does not win, and Ed wins only if Al does not win. (A: Al wins; E: Ed wins)

* 13. If Smith fails to win, then either Jones wins or Smith and Jones are tied. (S: Smith wins; J: Jones wins; T: Smith and Jones are tied)

14. Assuming that Julio is a bachelor, he is a man who is unmarried. (B: Julio is a bachelor; M: Julio is a man; J: Julio is married)

15. Erin's being penniless is a sufficient condition for her being miserable. (P: Erin is penniless; M: Erin is miserable)

* 16. Kareem's being tall is a necessary condition for his being on the team. (K: Kareem is tall; T: Kareem is on the team)

17. The statement "Santa does not exist" is false. (S: Santa exists)

18. We will be evicted unless we pay the rent. (E: We will be evicted; P: We pay the rent)

* 19. Although reindeer exist, Santa does not exist; but adults are not honest if Santa does not exist. (R: Reindeer exist; S: Santa exists; H: Adults are honest)

20. Paula will pass the test just in case she studies diligently. (P: Paula will pass the test; S: Paula studies diligently)

Part D: More Symbolizing Translate the following statements into symbols, using the schemes of abbreviation provided.

* 1. The picture frame is square only if it is rectangular. (S: The picture frame is square; R: The picture frame is rectangular)

2. You will not succeed if you lack common sense. (S: You will succeed; C: You have common sense)

3. If Sammy is a penguin, then Sammy is a bird that cannot fly. (P: Sammy is a penguin; B: Sammy is a bird; F: Sammy can fly)

* 4. Either you work hard or you have fun, but not both. (W: You work hard; F: You have fun)

5. Given that Bozo has a bill, Bozo is either a duck or a platypus. (B: Bozo has a bill; D: Bozo is a duck; P: Bozo is a platypus)

6. Neither penguins nor ostriches can fly. (P: Penguins can fly; O: Ostriches can fly)

* 7. If Alvin has a bill, then he is not a platypus if he has feathers. (B: Alvin has a bill, P: Alvin is a platypus; F: Alvin has feathers)

8. Neither Smith nor Jones wins if there is a tie, but Jones does not win given that Smith wins. (S: Smith wins; J: Jones wins; T: There is a tie)

9. While Miriam is both competent and hard-working, she is not interested in the job. (C: Miriam is competent; H: Miriam is hard-working; J: Miriam is interested in the job)

* 10. Given that Murphy is a bat only if he can fly, Murphy is not a bat. (B: Murphy is a bat; F: Murphy can fly)

11. Sally will pass unless her mind goes blank. (P: Sally will pass; M: Sally's mind goes blank)

12. Either Tyson wins or Holyfield wins, but not both. (T: Tyson wins; H: Holyfield wins)

13. Stella's being in Arkansas is a sufficient condition for her being in the U.S.A. (A: Stella is in Arkansas; U: Stella is in the U.S.A.)

14. Humberto's being competent is a necessary and sufficient condition for his being hired. (C: Humberto is competent; H: Humberto is hired)

15. Solomon's growing older is a necessary condition for his becoming wiser, but it is not a sufficient condition for his becoming wiser. (S: Solomon grows older; W: Solomon becomes wiser)

16. Dan's being in Pennsylvania is a necessary condition for his being in Philadelphia. (D: Dan is in Pennsylvania; P: Dan is in Philadelphia)

17. It is always wrong to kill the innocent only if it is wrong to kill an insane person in self-defense. (K: It is always wrong to kill the innocent; S: It is wrong to kill an insane person in self-defense)

18. It is not the case that if the Seahawks win, the Cowboys win. (S: The Seahawks win; C: The Cowboys win)

19. It is not always wrong to kill the innocent just in case it is not wrong to kill an insane person in self-defense. (K: It is always wrong to kill the innocent; S: It is wrong to kill an insane person in self-defense)

20. Plato's being a rational animal is a necessary and sufficient condition for his being human. (R: Plato is rational; A: Plato is an animal; H: Plato is human)

Part E: More Symbolizing Symbolize the following statements, using the schemes of abbreviation provided.

* 1. Fido is a dog only if Fido is an animal. (D: Fido is a dog; A: Fido is an animal)

2. Josey is a mammal if Josey is a cat. (M: Josie is a mammal; C: Josey is a cat)

3. Physical laws cannot be changed given that they are either necessary or eternal. (C: Physical laws can be changed; N: Physical laws are necessary; E: Physical laws are eternal)

* 4. Snakes are mammals only if snakes nourish their young with milk, but snakes do not nourish their young with milk. (M: Snakes are mammals; N: Snakes nourish their young with milk)

5. The statement "If evil exists, then God does not exist" is false. (E: Evil exists; G: God exists)

6. If Smith is guilty only if Smith's blood is on the murder weapon, then Smith is not guilty if Smith's blood is not on the murder weapon. (G: Smith is guilty; B: Smith's blood is on the murder weapon)

* 7. It is not true that if the Eiffel Tower is in Ohio, then it is in Europe. (O: The Eiffel Tower is in Ohio; E: The Eiffel tower is in Europe)

8. The defendant's having a motive is not a sufficient condition for his being guilty. (M: The defendant has a motive; G: The defendant is guilty)

9. Jane will fail unless she studies. (F: Jane will fail; S: Jane studies)

* 10. Assuming Fred is both rational and an animal, Fred is human; but Fred is not rational. (R: Fred is rational; A: Fred is an animal; H: Fred is human)

11. Senator Crockett's approval of the war is not a necessary condition for her re-election. (W: Senator Crockett approves of the war; R: Senator Crockett will be re-elected)

12. Unless we stop using fossil fuels, the earth will continue to get warmer. (S: We stop using fossil fuels; E: The earth will continue to get warmer)

* 13. Marie Curie's being a scientist is a necessary condition, but not a sufficient condition, for her being a physicist. (S: Marie Curie is a scientist; P: Marie Curie is a physicist)

14. If God exists, then evil does not exist unless God has a good reason for allowing evil. (G: God exists; E: Evil exists; R: God has a good reason for allowing evil)

15. Aaron Eckhart's being a movie star is a sufficient condition, but not a necessary condition, for his being famous. (M: Aaron Eckhart is a movie star; F: Aaron Eckhart is famous)

7.2 Truth Tables

Truth tables can be used to determine the validity (or invalidity) of a large class of arguments. In this section, we will examine the truth tables for the five basic types of compounds formed via the operators introduced in the previous section: the tilde, the dot, the vee, the arrow, and the double-arrow.

The main idea behind truth tables is that the truth value of certain compound statements is a function of the truth value of the atomic statements that make them up. A compound statement is said to be **truth-functional** if its truth value is completely determined by the truth value of the atomic statements that compose it. Let us now examine a series of truth-functional compounds.

We will again use the italicized, lowercase letters p and q as statement variables that can stand for any statement. For instance, the statement variable q can stand for A, for $\sim B$, for $\sim C \vee D$, for $E \leftrightarrow F$, and so on.

Negations

A negation has the opposite truth value of the statement negated. For example, the statement "Bertrand Russell was born in 1872" is true; so its negation, "Bertrand Russell was not born in 1872," is false. And "John F. Kennedy was born in 1872" is false; so its negation, "John F. Kennedy was not born in 1872," is true. Thus, negations are *truth-functional* compounds. We can present this in a kind of diagram, called a **truth table**, as follows:

p	$\sim p$
T	F
F	T

This truth table has two vertical **columns**, one on the left and one on the right. The column on the left gives the possible truth values for any statement p, namely, T (true) and F (false). The column on the right gives the corresponding truth values for the negation, $\sim p$. The table also has two horizontal **rows**. In the first (or top) row, p is true, so its negation is false. In the second (or bottom) row, p is false, so its negation is true.

Conjunctions

A conjunction is true if both its conjuncts are true; otherwise, it is false. Thus, one false conjunct renders an entire conjunction false. For example, "St. Augustine and Abraham Lincoln were both born in 354" is false, for although

St. Augustine was born in 354, Lincoln was not. We can sum up the relationship between the truth value of a conjunction and the truth value of its conjuncts as follows:

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

Here, the two columns on the left list all the possible truth value assignments for any two statements. Row 1 represents the situation in which both statements are true. Rows 2 and 3 represent the *two* situations in which the statements *differ* in truth value (p true, q false; and p false, q true). Finally, row 4 represents the situation in which both statements are false. The column under the dot indicates that the conjunction as a whole is true *only if* both conjuncts are true (namely, in row 1); otherwise, the conjunction as a whole is false.

Disjunctions

A disjunction (represented by the vee) is false if both its disjuncts are false; otherwise, it is true. Consider the following examples:

60. Either George Washington or John F. Kennedy was born in 2003 (or both were).
61. Either Abraham Lincoln or Andrew Jackson was born in 1809 (or both were).
62. Either Franklin D. Roosevelt or Jimmy Carter was a Democrat (or both were).

Statement (60) is false because both its disjuncts are false. (61) is true since Lincoln was born in 1809. (The statement as a whole is true even though Jackson was born not in 1809 but in 1767.) And (62) is true because both Roosevelt and Carter were Democrats. We can present these possibilities succinctly in a truth table as follows:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Again, the columns on the left represent the four possible combinations of truth values for any two statements. The column under the vee indicates that the disjunction is false only when both disjuncts are false (namely, in row 4); otherwise, the disjunction as a whole is true.

Material Conditionals

A material conditional (represented by the arrow) is false if its antecedent is true and its consequent is false; otherwise, it is true. However, English conditionals are rather complicated, and so we need to discuss the relationship between the arrow and the English if-then in some detail. Consider the following examples:

63. If some dogs are collies, then no dogs are collies.
64. If George Washington was born before Jimmy Carter, then Jimmy Carter was born before George Washington.
65. If physical objects exert a gravitational attraction on each other, then a fist-sized chunk of lead released 3 feet from the surface of the earth will always float in midair.

Each of these conditionals has a true antecedent and a false consequent, and each conditional *is itself* false. Indeed, *an English conditional is always false when its antecedent is true and its consequent is false*. As it turns out, this fact is so important as regards the validity of arguments that logicians have defined a special type of conditional, called the **material conditional**, that is false *only* when its antecedent is true and its consequent is false. The truth table for the material conditional, which is represented by the arrow, is as follows:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Again, note that the material conditional is false *only* in the situation in which the antecedent is true and the consequent is false (row 2).

Now consider the following four English sentences, which correspond to the four rows in the truth table for the material conditional:

a. If the Eiffel Tower is in France, then the Eiffel Tower is in Europe.
b. If the Eiffel Tower is in France, then the Eiffel Tower is in the U.S.A.
c. If the Eiffel Tower is in Germany, then the Eiffel Tower is in Europe.
d. If the Eiffel Tower is in Ohio, then the Eiffel Tower is in the U.S.A.

With the exception of (b), each of these conditionals is true. In (a), both antecedent and consequent are true. In (c), the antecedent is false, while the consequent is true; however, the conditional itself is true, since if the Eiffel Tower *is* in Germany, it is certainly in Europe. It may seem odd that a conditional could be true when both antecedent and consequent are false, but (d) illustrates that this can be so: If the Eiffel Tower *is* in Ohio, then of course it is in the U.S.A.

At this point, it may seem that the English if-then is truth-functional and that the truth table for the material conditional is also a truth table for the English if-then. Unfortunately, things are not that simple. Consider the following conditionals:

a. If 1 + 1 = 2, then the Eiffel Tower is in France.
b. If the Eiffel Tower is in Ohio, then it is in Europe.
c. If the Eiffel Tower is in Germany, then it is in the U.S.A.

In (a), both antecedent and consequent are true, yet the conditional as a whole seems false. At any rate, most people would hesitate to pronounce it true, since there is no relevance between the antecedent and the consequent. But if (a) is false, then English conditionals are not in general truth-functional. Obviously, if they *are* truth-functional, then any conditional with a true antecedent *and* a true consequent must be true. It appears, then, that there is a significant difference between the material conditional, defined by the preceding truth table, and ordinary English conditionals. This is borne out if we examine rows 3 and 4 of the truth table in the light of (b) and (c). (b) corresponds to row 3, since it has a false antecedent and true consequent. If we went by the truth table, we would say that (b) is true, but from the standpoint of common sense, it is false. If the Eiffel Tower *is* in Ohio, then it certainly is not in Europe. Similarly, (c) seems false. If the Eiffel Tower *is* in Germany, then it certainly is not in the U.S.A. Yet, if we go by the truth table for the material conditional, we must pronounce (c) true, since both antecedent and consequent are false.

Why are logicians so interested in the material conditional if it doesn't correspond to English conditionals? What good does it do to have a truth table for conditionals if we can see that the truth table does not give an accurate picture of the relationship between the truth value of English conditionals (in general) and the truth value of their constituent parts? As it turns out, when the truth table method is applied to arguments, it nicely corroborates our belief in the validity of such intuitive inference rules as those introduced in Chapter 1—*modus ponens*, *modus tollens*, hypothetical syllogism, disjunctive syllogism, and constructive dilemma. Moreover, it confirms our belief in the *invalidity* of such common, formal fallacies as denying the antecedent and affirming the consequent. In short, the material conditional captures that part of the meaning of the English conditional that is essential for the validity of the basic argument forms of statement logic.

Material Biconditionals

A material biconditional (represented by the double-arrow) is true when its two constituent statements have the same truth value, and it is false if the two statements differ in truth value. Thus, the truth table for the material biconditional is as follows:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Notice that the material biconditional is true when its constituent parts are both *false* (row 4) as well as when they are both true (row 1).

The truth table for the biconditional is perhaps more readily understandable if one realizes that a biconditional is in effect a conjunction of two conditionals. Consider an example:

66. Lincoln won the election *if and only if* Douglas lost the election.

Statement (66) can be broken down into two conditional statements, as follows:

67. Lincoln won the election *if* Douglas lost the election.
68. Lincoln won the election *only if* Douglas lost the election.

In standard form, (67) and (68) look like this (respectively):

69. If Douglas lost the election, then Lincoln won the election.
70. If Lincoln won the election, then Douglas lost the election.

So, (66) can be rewritten as a conjunction of two conditionals:

71. If Lincoln won the election, then Douglas lost the election; and if Douglas lost the election, then Lincoln won the election. (L: Lincoln won the election; D: Douglas lost the election)

Similar remarks could be made about any biconditional. Let us symbolize (66) and (71) and then check to see if the truth tables for these statements are alike. In symbols, (66) and (71) look like this (respectively):

72. $L \leftrightarrow D$
73. $(L \rightarrow D) \bullet (D \rightarrow L)$

Summary of Truth Tables for the Five Compounds

Negation

p	$\sim p$
T	F
F	T

Conjunction

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Let us work out the truth table for (73). The first row looks like this:

L	D	$(L \rightarrow D) \cdot (D \rightarrow L)$
T	T	T T T

When *L* and *D* are both true, then of course $L \rightarrow D$ is true, and so is $D \rightarrow L$. Hence, we place a "T" under the main operator, the dot, because both conjuncts are true. (Strictly speaking, we only need the "T" under the dot; the others are merely "scratch work" to ensure accuracy.) Now let us add the second row to the truth table:

L	D	$(L \rightarrow D) \cdot (D \rightarrow L)$
T	T	T T T
T	F	F F T

With *L* true and *D* false, $(L \rightarrow D)$ is false, while $(D \rightarrow L)$ is true. (Remember, the material conditional is false *only when* its antecedent is true and its consequent is false.) So, we have a conjunction with one false conjunct and one true conjunct. We place an "F" under the dot, since one false conjunct makes the entire conjunction false.

Next, we fill in truth values for the third row:

L	D	$(L \rightarrow D) \cdot (D \rightarrow L)$
T	T	T T T
T	F	F F T
F	T	T F F

With *L* false and *D* true, $L \rightarrow D$ is true; however, $D \rightarrow L$ is false. So, we again place an "F" under the dot, since we have one false conjunct. We can now add the fourth and final row to the truth table:

L	D	(L → D) • (D → L)		
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	T	T

With L and D both false, $L \rightarrow D$ is true, and so is $D \rightarrow L$. We place a "T" under the dot, since both conjuncts are true.

The column under the dot gives us the truth value of the entire statement, row by row. (As mentioned previously, the columns under the arrows are merely scratch work and, as such, are not essential to the truth table.) And the column under the dot is exactly like the column under the double-arrow in the truth table for the biconditional:

L	D	L ↔ D
T	T	T
T	F	F
F	T	F
F	F	T

To check your understanding of the truth-functional compounds discussed in this section, complete the following exercises.

Exercise 7.2

Part A: True or False? Determine the truth value of the following compound statements. Make the following assumptions: A is true, B is true, C is false, and D is false.

* 1. A • C
2. A ∨ C
3. ~ A
* 4. B → D
5. D → B
6. A ↔ B
* 7. C ↔ D
8. ~(A • B)
9. C ∨ D
* 10. ~(C ∨ D)
11. ~C → D
12. ~(D → A)
* 13. (A • C) → B
14. C → (A → D)
15. (C → A) → D
* 16. ~(A ↔ D)
17. ~C • ~D
18. ~(~A ↔ ~B)
* 19. (A • C) ∨ (B • D)
20. (C ∨ A) • (D ∨ B)
21. ~[A → (C ∨ B)]
* 22. (D ↔ A) ∨ (C → B)
23. (~C → A) ↔ (~A ∨ D)
24. ~B ↔ (A • C)
* 25. ~(D ∨ C) → B

Part B: More True or False Determine the truth value of the following compound statements.

* 1. It is not the case that Abraham Lincoln was born in 1997.

2. If water is H_2O, then water is not wet.

3. Either New York City is the capital of Montana, or Seattle is the capital of Montana.

* 4. Hillary Clinton is a married man if and only if Hillary Clinton is a husband.

5. If Reno is in Nevada, then Reno is in the U.S.A.

6. Either Alabama is a southern state (of the U.S.A.), or Maine is a southern state.

* 7. It is not the case that both Charlie Chaplin and George Washington are past presidents of the U.S.A.

8. If either Mozart or Beethoven was born in Korea, then it is false that both Mozart and Beethoven were born in Australia.

9. If the Taj Mahal is green, then the Taj Mahal is not invisible.

* 10. If Paris is the capital of France, then neither Seattle nor Spokane is the capital of France.

11. Samuel Clemens wrote *Huckleberry Finn* if and only if Samuel Clemens is Mark Twain.

12. If Reno is in Nevada, then Reno is in Canada.

* 13. If the Statue of Liberty is in Kentucky, then the Statue of Liberty is in the U.S.A.

14. Either Bruce Willis or Clint Eastwood is president of the U.S.A.

15. If Reno is in Nevada, then either Reno is in Canada, or Reno is in the U.S.A.

Part C: Assigning Truth Values What truth values must be assigned to the atomic statements in order to make the following compounds *false*?

* 1. $\sim P \vee Q$

2. $\sim R \rightarrow S$

3. $\sim(E \bullet G)$

* 4. $\sim(A \rightarrow B) \rightarrow C$

5. $(T \leftrightarrow \sim W) \vee T$

6. $\sim(\sim G \vee H) \rightarrow \sim K$

* 7. $(Y \rightarrow \sim Z) \vee \sim Y$

8. $(L \leftrightarrow M) \rightarrow \sim M$

9. $\sim A \rightarrow \sim(\sim B \bullet C)$

*10. $\sim(N \leftrightarrow P) \vee \sim P$

11. $\sim(\sim A \vee \sim B) \vee \sim A$

12. $(\sim C \vee E) \rightarrow \sim(E \bullet C)$

*13. $\sim(H \bullet J) \vee (K \rightarrow L)$

14. $(M \bullet N) \rightarrow \sim P$

15. $(R \bullet \sim S) \rightarrow (R \leftrightarrow Q)$

7.3 Using Truth Tables to Evaluate Arguments

We are now in a position to use truth tables to establish the validity and invalidity of arguments. Let's begin by examining an argument having the form *modus tollens*:

> 74. If Lincoln is 8 feet tall, then Lincoln is over 7 feet tall. But it is not the case that Lincoln is over 7 feet tall. It follows that Lincoln is not 8 feet tall. (L: Lincoln is 8 feet tall; S: Lincoln is over 7 feet tall)

The argument may be symbolized as follows:

> 75. $L \rightarrow S,\ \sim S\ \therefore \sim L$

First, we generate all the possible truth-value assignments for L and S. Since there are two truth values (truth and falsehood), our truth table must have 2^n rows, where n is the number of statement letters in the symbolic argument. In this case, we have just two statement letters, L and S, so our truth table will have 2^2 rows ($2^2 = 2 \times 2 = 4$). The truth-value assignments can be generated in a completely mechanical way; indeed, it is important to generate them mechanically both to avoid error and to facilitate communication. In the column nearest to the vertical line (in this case, the column under S), simply alternate Ts and Fs. In the next column to the left (in this case, the column under L), alternate couples (two Ts, followed by two Fs). Like this:

L	S	
T	T	
T	F	
F	T	
F	F	

We then write the steps of the argument out on the line at the top of the table and fill in the columns under each step of the argument, row by row. Row 1 looks like this:

L	S	$L \rightarrow S$,	$\sim S$	$\therefore \sim L$
T	T	T	F	F

As we have seen, $L \rightarrow S$ is true when L and S are both true. Of course, $\sim S$ is false when S is true, and $\sim L$ is false when L is true.

Next, we fill in truth values in row 2:

L	S	$L \rightarrow S$,	$\sim S$	$\therefore \sim L$
T	T	T	F	F
T	F	F	T	F

With its antecedent true and consequent false, $L \rightarrow S$ is false in this row of the table. Since S is false, $\sim S$ must be true. And since L is true, $\sim L$ must be false.

Now we add row 3:

L	S	$L \rightarrow S$,	$\sim S$	$\therefore \sim L$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T

The conditional premise is true when its antecedent is false and its consequent is true. Obviously, $\sim S$ is false when S is true, and $\sim L$ is true when L is false.

To complete the table, we add the fourth and final row.

L	S	$L \rightarrow S$,	$\sim S$	$\therefore \sim L$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

The conditional premise is true in row 4. (The material conditional is false *only when* its antecedent is true and its consequent is false; otherwise, it is true.) Since L and S are both false in this row, $\sim S$ and $\sim L$ are true.

Now, what does the truth table tell us about the argument? Each row in the table describes a possible situation in very abstract terms. For example, row 1 describes a situation in which both *L* and *S* are true. *L* and *S* could be about any topic—science, sorcery, celery, whatever. As long as the statements are both true, row 1 tells us that the first premise is true, the second premise is false, and the conclusion is false. *What we are looking for is a row, and hence a possible situation, in which the premises are all true but the conclusion is false.* If we can find such a row (or situation), then the argument form is invalid. Recall that validity preserves truth—if you start with truth and reason validly, you'll get a true conclusion. So, if a form of argument *can* lead from true premises to a false conclusion, that form of argument is invalid. As we look at the table for the symbolic argument (75), which has the form *modus tollens*, we see that there is no row in which all of the premises are true and the conclusion is false. This means that the argument has a valid form; hence, the argument itself is valid. And because the English argument (74) has the same form, it too is valid.

Now let's see what happens when we apply the truth table method to one of the formal fallacies. Here is an argument having the form of the fallacy of denying the antecedent:

76. If society approves of genetic engineering, then genetic engineering is morally permissible. But society does not approve of genetic engineering. Therefore, genetic engineering is not morally permissible. (S: Society approves of genetic engineering; G: Genetic engineering is morally permissible)

We translate the argument into symbols as follows:

77. $S \rightarrow G,\ \sim S\ \therefore\ \sim G$

The truth table looks like this:

S	G	S → G,	~S	∴ ~G
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Is there a row in which the premises are all true and the conclusion is false? Yes, row 3. This shows that the argument form is invalid, for it does not preserve truth. The table gives us the additional bit of information that the invalidity of the form is revealed in situations in which the antecedent of the conditional premise (i.e., *S*) is false and its consequent (i.e., *G*) is true. This gives us a strong hint about how to write an English **counterexample** that will connect what we have learned from the truth table with our intuitions as speakers of English. As you will recall from Chapter 1, a good counterexample has the following features: (a) It has the same form as the original argument, (b) its premises are *well-known* truths, and (c) its conclusion is a *well-known* falsehood. Here is a counterexample to argument (77):

> **78.** If George Washington was 8 feet tall, then he was over 2 feet tall. But Washington was not 8 feet tall. So, he was not over 2 feet tall.

Note that the conditional premise has a false antecedent but a true consequent: Washington wasn't 8 feet tall, but he was certainly over 2 feet tall. Moreover, the conditional premise as a whole is plainly true: Anyone who is 8 feet tall is certainly over 2 feet tall. And, of course, the second premise is true, while the conclusion is false. So, this English example illustrates the sort of situation described by the third row of the truth table. The pattern of reasoning is *always* invalid, since it allows for true premises and a false conclusion.

Of course, not all truth tables are as short as those we've examined thus far. Let us see what happens when we apply the truth table method to arguments having three statement letters.

> **79.** If the equatorial rain forests produce oxygen used by Americans, then either Americans ought to pay for the oxygen, or they ought to stop complaining about the destruction of the rain forests. But either it is false that Americans ought to pay for the oxygen, or it is false that Americans ought to stop complaining about the destruction of the rain forests. Therefore, it is false that the equatorial rain forests produce oxygen used by Americans. (E: The equatorial rain forests produce oxygen used by Americans; P: Americans ought to pay for the oxygen; S: Americans ought to stop complaining about the destruction of the rain forests)

Using the scheme of abbreviation provided, the argument translates into symbols as follows:

80. $E \rightarrow (P \vee S),\ \sim P \vee \sim S\ \therefore \sim E$

Now we are ready to construct a truth table. We list the statement letters *in the order in which they appear* in our symbolization: E, P, S. Since a truth table must have 2^n rows, where n is the number of statement letters, in this case we need a table with eight rows ($2^3 = 2 \times 2 \times 2 = 8$). To generate every possible combination of truth values for the three statement letters *mechanically*, we alternate Ts and Fs in the column nearest the vertical line, under S. Then we alternate couples (two Ts, followed by two Fs, etc.) in the next column to the left, under P. Finally, we alternate quadruples (four Ts followed by four Fs) in the column on the far left, under E, like this:

E	P	S	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

It's important to generate the possible truth-value combinations *in the manner indicated*, for two reasons. First, doing so will enable you to construct truth tables quickly and accurately. Second, for purposes of communication, a *standard method* of generating truth-value combinations is needed. Without a standard method, truth tables cannot readily be compared or checked for accuracy.

Next, we fill in the truth values for the premises and conclusion row by row. Check out each row of the following table. Only the circled columns are absolutely essential. The other columns are scratch work done to guarantee accuracy. For example, because the first premise is a conditional, only the column under the arrow is essential to the table. But filling in some other columns often helps one avoid errors.

E	P	S	E →	(P ∨ S),	~P	∨	~S ∴	~E
T	T	T	T	T	F	F	F	F
T	T	F	T	T	F	T	T	F
T	F	T	T	T	T	T	F	F
T	F	F	F	F	T	T	T	F
F	T	T	T	T	F	F	F	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	F	T
F	F	F	T	F	T	T	T	T

Once the table is complete, we examine it to see if there are any rows in which the premises are all true while the conclusion is false. Rows 2 and 3 meet this condition, so the argument is invalid. (An argument is invalid as long as *at least one* row meets this condition.)

Using the hints provided by row 3 of the truth table, we can construct a counterexample to argument (80):

81. If George Washington was born before Harry Truman, then either Abraham Lincoln was born before George Washington, or Abraham Lincoln was born before Harry Truman. Either it is false that Abraham Lincoln was born before

George Washington, or it is false that Abraham Lincoln was born before Harry Truman. So, it is false that George Washington was born before Harry Truman. (E: George Washington was born before Harry Truman; P: Abraham Lincoln was born before George Washington; S: Abraham Lincoln was born before Harry Truman)

Note that the counterexample matches the scenario described in row 3 of the truth table perfectly: *E* (i.e., Washington was born before Truman) is true, *P* (i.e., Lincoln was born before Washington) is false, and *S* (i.e., Lincoln was born before Truman) is true.

Truth tables can be used to evaluate for validity even when our English intuitions fail us. For example, is the following argument valid? Most people find it difficult to answer simply on the basis of logical intuition.

82. If Socrates works hard, he gets rich. But if Socrates doesn't work hard, he enjoys life. Moreover, if Socrates does not get rich, then he does not enjoy life. Hence, Socrates gets rich. (H: Socrates works hard; R: Socrates gets rich; L: Socrates enjoys life)

Using the scheme of abbreviation provided, the argument can be symbolized as follows:

83. $H \rightarrow R,\ \sim H \rightarrow L,\ \sim R \rightarrow \sim L\ \therefore R$

The truth table looks like this:

H	R	L	$H \rightarrow R,$	$\sim H \rightarrow L,$	$\sim R \rightarrow \sim L$	$\therefore R$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	F

There is no row in which the premises are true and the conclusion is false; therefore, the argument form is valid. Since argument (82) is one that most people find difficult to assess through unaided logical intuition, the fact that a truth table enables us to achieve a definitive evaluation illustrates the power of this method.

The truth table method does have an important limitation: It becomes unwieldy as arguments become longer. For instance, suppose we wish to evaluate an argument having the form of a constructive dilemma. In symbols, we have the following:

84. $A \vee B,\ A \rightarrow C,\ B \rightarrow D\ \therefore C \vee D$

Here, we have four statement letters, so we need 2^4 rows in our truth table ($2^4 = 2 \times 2 \times 2 \times 2 = 16$). The truth table looks like this:

A	B	C	D	$A \vee B$,	$A \rightarrow C$,	$B \rightarrow D$	$\therefore C \vee D$
T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	T
T	T	F	T	T	F	T	T
T	T	F	F	T	F	F	F
T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	T	T	F	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	T	F	T	T	F	T
F	T	F	T	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	F	T	T	T
F	F	T	F	F	T	T	T
F	F	F	T	F	T	T	T
F	F	F	F	F	T	T	F

The argument is valid, for there are no rows in which all the premises are true and the conclusion is false. Note that the initial truth-value assignments on the left are generated in the mechanical way previously described: alternate Ts and Fs under the letter closest to the vertical line (*D* in the table); alternate couples under the next letter to the left (C); alternate quadruples under the next letter (B); finally, alternate groups of eight. How many rows would be needed for a truth table involving five statement letters? Thirty-two ($2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$). And if six statement letters were involved, we would need a truth table with 64 rows. So, the truth table method is cumbersome when applied to arguments involving many statement letters. Nevertheless, it is a powerful method that is useful in many cases.

Check your understanding by completing the following exercises.

Exercise 7.3

Part A: Truth Tables Construct truth tables to determine whether the following arguments are valid. Make initial truth-value assignments in the mechanical fashion described in this section. That is, list statement letters *in the order in which they appear* in the argument; then alternate Ts and Fs in the column under the letter *closest* to the vertical line; alternate couples (two Ts, two Fs, etc.) under the next letter on the left; and so on.

* 1. A ∨ B, ~A ∴ B
2. F → G, F ∴ G
3. ~A ∨ ~B, ~B ∴ ~~A
* 4. ~P → ~R ∴ ~(P → R)
5. ~(X → Y) ∴ ~X → ~Y
6. E ∴ D ∨ E
* 7. A • B ∴ B
8. ~(N • L) ∴ ~N → ~L
9. (A ∨ B) • ~(A • B), A ∴ ~B
* 10. ~F • ~G ∴ ~F ↔ ~G
11. ~(S • ~R), ~R ∴ ~S
12. A ↔ B ∴ A • B
* 13. D ↔ (E ∨ C), ~D ∴ ~C
14. A → (B → C) ∴ A → (B • C)
15. N ↔ (M • L), ~L ∴ ~N
* 16. A → B, B → C ∴ A → C
17. (Q • U) → Z, ~Z ∴ ~Q
18. (E ↔ G) → H, ~H ∴ ~E ∨ ~G
* 19. A ∨ B, A → C, B → C ∴ C
20. A → C, B → D, ~C ∨ ~D ∴ ~A ∨ ~B

Part B: More Truth Tables Construct truth tables to determine whether the following arguments are valid.

* 1. A • ~B ∴ ~(A → B)
2. F → G ∴ ~F → ~G
3. ~E → ~G ∴ G → E
* 4. ~(H • K) ∴ ~H • ~K
5. A → B, B ∴ A
6. X ∨ Y, Y ∴ ~X
* 7. A ∴ (A ∨ B) • ~(A • B)
8. ~(T ↔ ~S) ∴ ~T ∨ S
9. ~F ∨ ~G ∴ ~(F ∨ G)
* 10. ~(H ↔ J) ∴ ~H ↔ ~J
11. ~(A → B) ∴ A • ~B
12. ~(N ↔ P) ∴ N → ~P
13. ~(A ↔ B) ∴ (A • ~B) ∨ (B • ~A)
14. (H • B) → S ∴ B → S
15. P → Q, S → Q, ~Q ∴ ~P • ~S
16. Z → (S ∨ G), Z ∴ S
17. ~(L ∨ M), ~M ↔ ~N ∴ ~N
18. P → (~Q ∨ R), P • ~R ∴ ~Q
19. A → (B → C) ∴ (A • B) → C
20. ~[~(D • E) ∨ (F ∨ ~D)] ∴ D • (E • ~F)

Part C: English Arguments Symbolize the following arguments. Then use truth tables to determine whether they are valid.

* 1. Not having exceeded our natural resources is a necessary condition for its being appropriate to expand our city. Unfortunately, we have exceeded our natural resources. Consequently, it is not appropriate to expand our city. (E: We have exceeded our natural resources; A: It is appropriate to expand our city)

2. Humans evolved from lower life forms given that either human life evolved from inanimate matter apart from divine causes, or God created human life via the long, slow process we call evolution. God created human life via the long, slow process we call evolution. It follows that humans evolved from lower life forms. (H: Humans evolved from lower life forms; M: Human life evolved from inanimate matter apart from divine causes; G: God created human life via the long, slow process we call evolution)

3. American foreign policy is bankrupt unless it is based on clear moral principles. American foreign policy is not based on clear moral principles just in case it is based primarily on the national interest. Unfortunately, American foreign policy is based primarily on the national interest. We may infer that American foreign policy is bankrupt. (B: American foreign policy is bankrupt; M: American foreign policy is based on clear moral principles; N: American foreign policy is based primarily on the national interest)

* 4. You won't get an A unless you do well on all the exams. Therefore, if you do well on all the exams, you will get an A. (A: You will get an A; W: You do well on all the exams)

5. There are necessary truths (i.e., truths that cannot be false under any possible circumstances). For assuming that there are no necessary truths, there are no necessary connections between premises and conclusions. But there are no valid arguments if there are no necessary connections between premises and conclusions; and there are valid arguments. (N: There are necessary truths; C: There are necessary connections between premises and conclusions; V: There are valid arguments)

6. On the condition that land mines are designed to inflict horrible suffering, they ought to be banned unless inflicting horrible suffering is sometimes justified. It is not true that inflicting horrible suffering is sometimes justified, but it is true that land mines are designed to inflict horrible suffering. Accordingly, land mines ought to be banned. (L: Land mines are designed to inflict horrible suffering; B: Land mines ought to be banned; S: Inflicting horrible suffering is sometimes justified)

* 7. The reduction of violence is a necessary and sufficient condition for making drugs legal. But more people will use drugs if drugs are made legal. And violence is not reduced if more people will use drugs. Hence, drugs are not made legal. (V: Violence is reduced; L: Drugs are made legal; P: More people will use drugs)

8. Augustine achieves heaven if Augustine is virtuous. But Augustine is happy provided that he is not virtuous. Augustine does not achieve heaven only if he is not happy. Therefore, Augustine achieves heaven. (A: Augustine achieves heaven; V: Augustine is virtuous; H: Augustine is happy)

9. Not all living things are able to feel pain. For all living things are able to feel pain only if all living things have nervous systems. But not all living things have nervous systems given that plants do not have nervous systems. And plants do not have nervous systems. (L: All living things are able to feel pain; N: All living things have nervous systems; P: Plants have nervous systems)

10. It is morally permissible for mentally superior extraterrestrials to eat humans on the condition that it is morally permissible for humans to eat animals. But either it is not morally permissible for mentally superior extraterrestrials to eat humans, or human life lacks intrinsic value. However, human life has intrinsic value. We are forced to conclude that it is not morally permissible for humans to eat animals. (E: It is morally permissible for mentally superior extraterrestrials to eat humans; H: It is morally permissible for humans to eat animals; V: Human life has intrinsic value)

7.4 Abbreviated Truth Tables

As we have seen, the truth table method is rather cumbersome when applied to arguments having more than three statement letters. But there are ways to make it less cumbersome, and we will explore one of them in this section, namely, the **abbreviated truth table method.** The essential insight behind abbreviated truth tables is this: If we can construct *one* row of a truth table, *making all the premises true while the conclusion is false*, then we have shown that the argument form in question is invalid. Here's an example:

85. If I am thinking, then my neurons are firing. Hence, if my neurons are firing, then I am thinking. (A: I am thinking; N: My neurons are firing)

Using the scheme of abbreviation provided, we may symbolize the argument as follows:

86. $A \rightarrow N \therefore N \rightarrow A$

We begin by hypothesizing that all the argument's premises are true while its conclusion is false:

	$A \rightarrow N$	$\therefore$	$N \rightarrow A$
	T		F

Now we work backward. If the conclusion is false, then *N* must be true and *A* must be false. We fill in these values uniformly throughout the argument:

A → N	∴	N → A
F T T		T F F

This truth assignment does indeed make the conclusion false and the premise true. We have in effect constructed a row in the truth table that shows the argument to be invalid: It is the row in which *A* is false and *N* is true. We add this information at the left to complete our abbreviated truth table:

A N	A → N	∴	N → A
F T	F T T		T F F

We have thus shown the argument to be invalid. And as before, our truth-functional assessment of the argument gives a strong hint about how to construct an English counterexample:

87. If Thomas Jefferson was 500 years old when he died, then he lived to be more than a year old. Therefore, if Jefferson lived to be more than a year old, then he was 500 years old when he died.

The premise of argument (87) is plainly true even though its antecedent is false. And, of course, the conclusion of the argument is false, too.

Let's try a more complicated example. Consider the following symbolic argument:

88. E ∨ S, E → (B • U), ~S ∨ ~U ∴ B

Again, we hypothesize that all the premises can be true while the conclusion is false:

E ∨ S,	E → (B • U),	~S ∨ ~U	∴	B
T	T	T		F

Then we work backward to determine the truth value of each constituent statement letter. Since we have assigned "F" to *B* in the conclusion, we must assign "F" to *B* uniformly throughout the argument. (Remember, we are in effect constructing a single row in a truth table.) This means that *B* • *U* is also false. Hence, we must assign "F" to *E*; otherwise, the second premise will be false, which contradicts our hypothesis. Now, if *E* is false, we must make *S* true; otherwise, the first premise will be false, which contradicts our hypothesis. And if *S* is true, then ~*S* is false, so we must make ~*U* true (and hence *U* false) to make the third premise true. Thus, we arrive at our abbreviated truth table:

E	S	B	U	E ∨ S, E → (B • U), ~S ∨ ~U ∴ B
F	T	F	F	FTT FT FFF FTTTF F

In this case, an argument that would require a 16-row truth table can be dealt with quickly by means of an abbreviated truth table.

The abbreviated truth table method can also be used to show that an argument is valid. Let's try it out on an old friend, disjunctive syllogism. Again, we begin by hypothesizing that the conclusion can be false while the premises are true:

	A ∨ B, ~A ∴ B
	T T F

If ~*A* is true, then *A* is false. But since *B* is also false, *A* ∨ *B* is false, contrary to our hypothesis. Thus, in trying to assign values so that the premises are all true and the conclusion is false, we are forced to contradict ourselves. This means the argument is valid. We indicate that we were forced to assign truth values inconsistently by writing "T/F" under the first premise:

	A ∨ B, ~A ∴ B	
	F (T/F) F TF F	Valid

Using an abbreviated truth table is a bit more complicated when the conclusion of the argument is false on *more than one* assignment of truth values—for example, when the conclusion is a conjunction or a biconditional. In such cases, the following principles will suffice:

> **Principle 1:** If there is *any* assignment of values in which the premises are all true and the conclusion is false, then the argument is invalid.

> **Principle 2:** If more than one assignment of truth values will make the conclusion false, then consider each such assignment; if each assignment that makes the conclusion false makes *at least one* premise false, then the argument is valid.

For instance, consider the following symbolic argument:

89. F → G, G → H ∴ ~F • H

There are three ways to make the conclusion false: (a) make both conjuncts false, (b) make the left conjunct false and the right one true, or (c) make the left conjunct true and the right one false. If we neglect this complexity, we can easily fall into error, for not every assignment that makes the conclusion false makes the premises true. For instance:

	F → G,	G → H	∴ ~F • H
	T T/F F	F T F	F T F F

With this assignment, the first premise is false. (We could make the first premise true by assigning "T" to G, but then the second premise would be false.) If we overlook the fact that other truth value assignments render the *conclusion* false, we might suppose that this abbreviated truth table shows that the argument is valid. But it does not because there is a way of assigning "F" to the conclusion that makes all the premises true, namely:

F	G	H	F → G,	G → H	∴ ~F • H
F	F	F	F T F	F T F	T F F F

And this proves that the argument form is invalid.

To show that an argument is valid, we must consider every truth-value assignment in which its conclusion is false. Consider the following example:

90. P → Q, Q → P ∴ P ↔ Q

A biconditional is false whenever its two constituent statements *differ* in truth value. So in this case, we must consider the assignment in which *P* is true and *Q* is false, *and also* the assignment in which *P* is false and *Q* is true.

	P → Q,	Q → P	∴ P ↔ Q	
	T (T/F) F	F T T	T F F	
	F T T	T (T/F) F	F F T	Valid

Here, each assignment that makes the conclusion false also makes one of the premises false (which contradicts our hypothesis that all the premises can be true while the conclusion is false). Thus, we have shown the argument to be valid.

The following exercise gives you an opportunity to practice the abbreviated truth table method.

Exercise 7.4

Part A: Abbreviated Truth Tables Use abbreviated truth tables to show that the following arguments are invalid.

* 1. A → (B → C) ∴ B → C

2. ~(E ↔ F) ∴ ~E • ~F

3. ~(G ↔ H) ∴ ~G → ~H

* 4. J → ~K ∴ ~(J ↔ K)

5. (P • Q) → R, ~R ∴ ~P

6. ~(Z • H), ~Z → Y, W → H ∴ ~W → Y
How many rows would be needed in a complete truth table for argument 6?

* 7. ~(S • H), (~S • ~H) → ~U ∴ ~U

8. (F • G) ↔ H, ~H ∴ ~G

9. ~(B → C), (D • C) v E ∴ ~B

*10. (P → ~Q) ↔ ~R, R ∴ ~P

11. S → (T → V) ∴ (S → T) → V

12. A → (B → C) ∴ A → (B • C)

*13. (Z • Y) → W ∴ Z → (Y • W)

14. ~(C v D), (~C • ~E) ↔ ~D, ~E → (C v F), S v F ∴ S
How many rows would be needed in a complete truth table for argument 14?

15. (F ↔ G) ↔ H, ~H ∴ ~F • ~G

*16. P → Q, P → R, Q ↔ R, S, S → R ∴ P • Q

17. S → (A • O), ~P v ~R, P → (S v Z), Z → (O → R) ∴ Z v ~P

18. A v (B • C), ~A ∴ (A • B) v (A • C)

*19. ~(Q v S), ~T v S, (U • W) → Q ∴ (~T • ~U) • W

20. ~J • ~K, L → J, M → K, (M → ~L) → ~(N • O) ∴ ~N

Part B: More Abbreviated Truth Tables Use abbreviated truth tables to show that the following arguments are invalid.

* 1. ~(A • B), ~A → C, ~B → D ∴ C • D

2. L ↔ (M • N), M v N, ~L ∴ ~M

3. (O ↔ P) → R, ~R ∴ ~O v P

* 4. ~(V • X) → ~Y ∴ ~[(V • X) → Y]

5. $\sim(Z \bullet H),\ \sim Z \rightarrow Y,\ W \rightarrow H\ \therefore\ \sim W \rightarrow Y$
6. $\sim X \vee (C \bullet A),\ \sim Y \vee \sim B,\ \sim Y \vee (X \vee T),\ T \rightarrow (A \rightarrow B)\ \therefore\ T \vee \sim Y$
* 7. $\sim(Z \rightarrow A),\ Z \rightarrow B,\ \sim A \rightarrow C\ \therefore\ C \bullet \sim B$
8. $B \rightarrow (C \bullet D),\ \sim E \vee \sim F,\ E \rightarrow (B \vee G),\ G \rightarrow (D \rightarrow F)\ \therefore\ G \vee \sim E$
   How many rows would be needed in a complete truth table for item 8?
9. $\sim(D \leftrightarrow E),\ \sim D \rightarrow F,\ E \rightarrow G\ \therefore\ F \bullet G$
* 10. $H \vee \sim S,\ H \rightarrow Z,\ \sim S \rightarrow P\ \therefore\ P \leftrightarrow Z$
11. $\sim[(J \bullet K) \rightarrow (M \vee N)]\ \therefore\ K \bullet N$
12. $A \rightarrow B,\ C \rightarrow \sim D,\ \sim B \vee D\ \therefore\ \sim A \leftrightarrow \sim C$
13. $\sim E \rightarrow (G \bullet A),\ \sim(P \leftrightarrow \sim L) \vee E,\ \sim(P \bullet L) \vee Q,\ \sim N \rightarrow \sim G\ \therefore\ Q \bullet A$
14. $(G \rightarrow E) \leftrightarrow S,\ \sim(S \vee H),\ \sim(P \bullet \sim H)\ \therefore\ G \bullet E$
15. $\sim(C \leftrightarrow \sim D) \vee E,\ \sim E \rightarrow (G \bullet H),\ (C \bullet D) \rightarrow K,\ \sim N \rightarrow \sim G\ \therefore\ K \bullet H$
   How many rows would be needed in a complete truth table for item 15?

Part C: Valid or Invalid? Some of the following arguments are valid, and some are invalid. Use abbreviated truth tables to determine which are valid and which are invalid.

* 1. $\sim A \vee B\ \therefore\ A \rightarrow B$
2. $F \rightarrow (G \leftrightarrow H),\ \sim F \bullet \sim H\ \therefore\ \sim G$
3. $\sim M\ \therefore\ \sim N \vee \sim M$
* 4. $A \vee (B \bullet C)\ \therefore\ (A \bullet B) \vee (A \bullet C)$
5. $P \rightarrow \sim(Q \bullet R),\ P \bullet R\ \therefore\ \sim Q$
6. $X \rightarrow Z,\ Y \rightarrow Z,\ \sim Z\ \therefore\ X \leftrightarrow Y$
7. $\sim(S \rightarrow R),\ S \rightarrow J,\ \sim R \leftrightarrow W\ \therefore\ W \rightarrow \sim J$
8. $\sim M \rightarrow O,\ \sim N \rightarrow O,\ \sim O \leftrightarrow \sim P,\ \sim P\ \therefore\ M \bullet N$
   How many rows would be needed in a complete truth table for item 8?
9. $(A \vee B) \bullet (A \vee C)\ \therefore\ A \bullet (B \vee C)$
10. $R \leftrightarrow \sim Q,\ R \vee Q,\ R \vee P\ \therefore\ (P \bullet Q) \rightarrow R$

Part D: English Arguments Translate the following English arguments into symbols, using the schemes of abbreviation provided. Use abbreviated truth tables to determine whether the arguments are valid.

* 1. If you want to mess up your life, you should drink a lot of booze. Therefore, if you don't want to mess up your life, you should not drink a lot of booze. (W: You want to mess up your life; B: You should drink a lot of booze)
2. Being undetermined is a necessary but not a sufficient condition for human behavior's being free. The laws of subatomic physics are statistical only if

human behavior is not determined. And the laws of subatomic physics are statistical. It follows that human behavior is free. (D: Human behavior is determined; F: Human behavior is free; L: The laws of subatomic physics are statistical)

3. Given that nuclear energy is needed if and only if solar energy cannot be harnessed, nuclear energy is not needed. For solar energy can be harnessed provided that funds are available; and funds are available. (N: Nuclear energy is needed; S: Solar energy can be harnessed; F: Funds are available)

* 4. If the Gulf War was about oil, and if human life is more valuable than oil, then the Gulf War was immoral. Human life is more valuable than oil, but the Gulf War was not about oil. Therefore, the Gulf War was not immoral. (G: The Gulf War was about oil; H: Human life is more valuable than oil; I: The Gulf War was immoral)

5. The rate of teenage drunk driving will decrease just in case the taxes on beer increase. The taxes on beer increase only if either the federal government or the state government will resist the liquor lobby. The state government will resist the liquor lobby, but the federal government will not. Accordingly, the rate of teenage drunk driving will not decrease. (R: The rate of teenage drunk driving will decrease; B: The taxes on beer increase; F: The federal government will resist the liquor lobby; S: The state government will resist the liquor lobby)

6. Erik attains Valhalla given that he is valiant. And Erik is depressed assuming that he is not valiant. Furthermore, Erik fails to attain Valhalla only if he is not depressed. Thus, Erik is depressed. (E: Erik attains Valhalla; V: Eric is valiant; D: Eric is depressed)

* 7. If society is the ultimate source of moral authority, then if society approves of polygamy, polygamy is right. But it is not true either that society is the ultimate source of moral authority or that society approves of polygamy. Hence, polygamy is not right. (S: Society is the ultimate source of moral authority; P: Society approves of polygamy; R: Polygamy is right)

8. Either the earth is millions of years old, or it is only 6000 years old. If the earth is millions of years old, then the traditional story of creation is a myth, and ultimate reality is nothing but atoms in motion. Now, either it is false that the earth is only 6000 years old, or it is false that ultimate reality is nothing but atoms in motion. Therefore, the traditional story of creation is a myth. (E: The earth is millions of years old; S: The earth is only 6000 years old; B: The traditional story of creation is a myth; U: Ultimate reality is nothing but atoms in motion)

9. Wittgensteinians are right if logic is embedded in language. But logic is embedded in language if and only if logic varies as language varies. And logic is language-relative if logic varies as language varies. Moreover, given that logic is language-relative, contradictions may be true in some languages.

Therefore, Wittgensteinians are right only if contradictions may be true in some languages. (W: Wittgensteinians are right; E: Logic is embedded in language; V: Logic varies as language varies; R: Logic is language-relative; C: Contradictions may be true in some languages)

10. Although most Americans approve of gun control, gun control is neither wise nor moral. For gun control is wise if and only if it prevents criminals from obtaining weapons. And gun control is moral if and only if it preserves our liberty. But it is not the case that gun control both preserves our liberty and prevents criminals from obtaining weapons. (W: Gun control is wise; M: Gun control is moral; P: Gun control prevents criminals from obtaining weapons; L: Gun control preserves our liberty)

7.5 Tautology, Contradiction, Contingency, and Logical Equivalence

Truth tables can be used to sort statements into logically significant categories: tautologies, contradictions, and contingent statements. This section explains how to identify tautologies, contradictions, and contingent statements; it also describes their main logical properties.

The atomic components of a statement are simply the atomic statements within it. For example, the atomic components of $\sim P \rightarrow Q$ are P and Q. (In the special case of an atomic statement, the atomic component is simply the statement itself. So, the atomic component of R is simply R.) A statement is a **tautology** if *it is true regardless of the truth values of its atomic components*. Equivalently, a statement is a tautology if there is a "T" beneath its main operator in every row of its truth table. The tautologies of statement logic belong to a class of statements that are true simply by virtue of their form.* Here are some examples:

91. Either it is raining or it is not raining. (R: It is raining)
92. If trees are plants, then trees are plants. (P: Trees are plants)
93. If neither atoms nor molecules exist, then atoms do not exist. (A: Atoms exist; M: Molecules exist)

These statements can be translated into symbols as follows, in order:

94. $R \vee \sim R$
95. $P \rightarrow P$

*Not all statements that are true by virtue of their form are tautologies in the sense here defined. For example, the following statement is not a tautology, but it is true by virtue of its form: "If everything is human, then something is human." We will examine statements of this type in Chapter 9, "Predicate Logic." According to many philosophers, statements that are true by virtue of their form (including tautologies) belong to a larger class of statements called *necessary truths*. Necessary truths are truths that cannot be false under any possible circumstances. Here is an example of a necessary truth that does not appear to be true simply by virtue of its form: "If Al is older than Bob, then Bob is younger than Al."

96. ~(A v M) → ~A

If we construct truth tables for these statements, then every row under the main logical operator will contain a "T":

R	R v ~R
T	T
F	T

P	P → P
T	T
F	T

A	M	~(A v M) → ~A
T	T	T
T	F	T
F	T	T
F	F	T

Tautologies have some interesting and paradoxical properties. For example, every argument whose conclusion is a tautology is valid—regardless of the content of the premises. Consider the following example:

97. The moon is made of green cheese. So, either Santa is real or Santa is not real. (M: The moon is made of green cheese; S: Santa is real)

Here is a symbolization and truth table for argument (97):

M	S	M	∴ S v ~S
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	T

As you can see, there is no row in which the premise is true while the conclusion is false, and so the argument is valid. This may seem puzzling because intuitively the premise is irrelevant to the conclusion. But the argument does satisfy our definition of validity, for since the conclusion is a tautology, it is impossible for the conclusion to be false while the premise is true.

A statement is a **contradiction** if *it is false regardless of the truth values of its atomic components*. Equivalently, a statement is a contradiction if there is an "F" beneath its main operator in every row of its truth table. The contradictions of statement logic belong to a class of statements that are false simply by virtue of their form. Here are two examples:

98. Ants exist, and yet they do not exist. (A: Ants exist)

99. If lemons are yellow, then they are not blue; but lemons are both blue and yellow. (Y: Lemons are yellow; B: Lemons are blue)

In symbols, we have this:

100. A • ~A
101. (Y → ~B) • (B • Y)

If we construct a truth table for a contradiction, then every row under the main logical operator will contain an "F":

A	A • ~A
T	F
F	F

Y	B	(Y → ~B) • (B • Y)
T	T	F
T	F	F
F	T	F
F	F	F

Like tautologies, contradictions have some interesting logical properties. For example, any argument that has a contradiction among its premises is a valid argument. For instance:

102. Atoms exist, and yet they do not exist. So, God exists. (A: Atoms exist; G: God exists)

Here is the truth table:

A	G	A • ~A	∴ G
T	T	F	T
T	F	F	F
F	T	F	T
F	F	F	F

Note that there is no row in which the premise is true and the conclusion is false; hence, the argument is valid. This may seem strange, but the argument does satisfy our definition of validity. It is impossible for the conclusion to be false while the premise is true (because it is impossible for the premise to be true). Notice, however, that all arguments having a contradiction among their premises are *unsound*, since contradictions are always false.

We can go a step further here: Any argument with logically inconsistent premises will be valid yet unsound. If the premises of an argument are inconsistent, then if we form a conjunction of the premises, that conjunction will be a contradiction. Here is an example:

103. P → Q, ~P → Q, ~Q ∴ R

If we form a conjunction of the premises, the argument looks like this:

P	Q	R	(P → Q) • [(~P → Q) • ~Q]			∴ R
T	T	T	T	F	F	T
T	T	F	T	F	F	F
T	F	T	F	F	T	T
T	F	F	F	F	T	F
F	T	T	T	F	F	T
F	T	F	T	F	F	F
F	F	T	T	F	F	T
F	F	F	T	F	F	F

The truth table reveals that the conjunction is a contradiction. Again, the point is that, paradoxically, every argument with inconsistent premises is valid. (Of course, all such arguments are *unsound* due to having one or more false premises.) How will you know if the premises of an argument are inconsistent? Here's how: There will be no row in the truth table in which all of the premises are true. For instance:

M	N	L	M ↔ N,	M • ~N	∴ L
T	T	T	T	F	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

Because there is no row in which all the premises are true, the premises are inconsistent and the argument is valid.

A statement is **contingent** if *it is true on some assignments of truth values to its atomic components and false on others*. Equivalently, a statement is contingent if there is a "T" beneath its main operator in at least one row of its truth table *and* an "F" beneath its main operator in at least one row of its truth table. Here is an example:

104. P • (P → R)

The truth table looks like this:

P	R	P • (P → R)
T	T	T
T	F	F
F	T	F
F	F	F

Contingent statements have important logical relations to both tautologies and contradictions. For example, any argument that has a tautology as its premise but a contingent statement as its conclusion is invalid. (The premise will be true in every row of the truth table, while the conclusion will be false in at least one row.) And suppose that the premises of an argument, when made into a conjunction, form a contingent statement. Then, if the conclusion of the argument is a contradiction, the argument is invalid. (The conclusion will be false in every row, while the premise will be true in at least one row.)

Notice that tautologies and contradictions place limitations on the method of abbreviated truth tables that was introduced in the previous section. For instance, if an argument has a tautology as its conclusion, then there is no way to assign truth values so that the conclusion is false. One way to deal with such a case is to use a complete truth table to prove that the conclusion is a tautology (which simultaneously proves that the argument is valid). Similarly, if at least one premise is a contradiction (or if the premises are inconsistent), then there is no way to assign values so that the premises are all true, and a complete truth table may be needed to establish this, as in the case of argument (102). However, a complete truth table is not always needed in such cases, for consider the following (admittedly odd) argument:

105. B • ~B ∴ B

We can deal with this argument by means of an abbreviated truth table:

	B • ~B	∴ B	
	F (T/F) T	F	Valid

Since there is only one way to make *B* false, and it forces an assignment of "F" to the premise, the abbreviated truth table works in this case.

Logically equivalent statements, such as *P* ∨ ~*Q* and ~*Q* ∨ *P*, validly imply each other. More formally, two statements are **logical equivalent** if *they agree in truth value regardless of the truth values assigned to their atomic components*. In other words, two statements are logically equivalent if, in a truth table constructed for both statements, the same truth value occurs beneath the main operators of the two statements in each row. For example:

A	B	A → B	~A ∨ B
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Note that the columns under the arrow and the vee are exactly the same, row by row.

The concept of logical equivalence has an important relationship to the concept of a tautology—namely, if a biconditional statement is a tautology, then its two constituent statements (joined by the double-arrow) are logically equivalent. For instance, consider the following tautology:

F	G	(F → G)	↔	(~G → ~F)
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

From the fact that (*F* → G) ↔ (~G → ~*F*) is a tautology, we may infer that the following two statements are logically equivalent:

106. F → G
107. ~G → ~F

Note also that in the truth table, the same truth value occurs beneath the main operators of the two statements (i.e., the arrows) in each row.

To sum up, truth tables can be used to sort statements into logically significant categories: tautologies, contradictions, and contingent statements. And, as we have seen, each of these types of statements has important logical properties. In addition, we have seen how truth tables can be used to show that statements are logically equivalent.

To check your understanding of these concepts, complete the following exercises.

Exercise 7.5

Part A: Tautologies, Contradictions, and Contingent Statements Use truth tables to determine whether the following statements are tautologies, contradictions, or contingent statements.

* 1. $\sim A \rightarrow (A \rightarrow B)$

2. $\sim F \rightarrow G$

3. $\sim S \leftrightarrow S$

* 4. $B \rightarrow (A \rightarrow B)$

5. $F \rightarrow [\sim(F \bullet G) \rightarrow \sim G]$

6. $A \rightarrow [(A \rightarrow B) \rightarrow B]$

* 7. $P \rightarrow (P \rightarrow Q)$

8. $(A \leftrightarrow B) \rightarrow (\sim A \bullet \sim B)$

9. $\sim P \bullet \sim(\sim P \vee \sim Q)$

*10. $(R \bullet \sim R) \rightarrow S$

11. $B \rightarrow [\sim(A \bullet B) \rightarrow A]$

12. $\sim(F \rightarrow G) \bullet G$

13. $\sim(N \leftrightarrow M) \rightarrow (N \bullet \sim M)$

14. $[A \rightarrow (B \vee C)] \rightarrow [(A \rightarrow B) \vee (A \rightarrow C)]$

15. $(\sim P \leftrightarrow P) \rightarrow Q$

16. $\sim(K \rightarrow L) \rightarrow (K \bullet \sim L)$

17. $[A \rightarrow (B \rightarrow C)] \rightarrow [(A \rightarrow B) \rightarrow (A \rightarrow C)]$

18. $(H \rightarrow J) \rightarrow \sim(H \bullet \sim J)$

19. $(\sim Z \bullet \sim W) \rightarrow (Z \leftrightarrow W)$

20. $[(A \rightarrow B) \rightarrow A] \rightarrow A$

Part B: Logical Equivalence Use truth tables to prove that the following pairs of statements are logically equivalent. It will be useful to know these particular equivalences as we move on to the material in the next chapter.

* 1.	$\sim(A \bullet B)$	$\sim A \vee \sim B$
2.	$\sim(F \vee G)$	$\sim F \bullet \sim G$
3.	$P \bullet (Q \vee R)$	$(P \bullet Q) \vee (P \bullet R)$
* 4.	$S \rightarrow U$	$\sim S \vee U$
5.	Q	$\sim\sim Q$
6.	$P \vee (Q \bullet R)$	$(P \vee Q) \bullet (P \vee R)$
7.	$F \leftrightarrow G$	$(F \bullet G) \vee (\sim F \bullet \sim G)$
8.	$A \vee B$	$B \vee A$
9.	$K \bullet K$	K
10.	$U \leftrightarrow Z$	$(U \rightarrow Z) \bullet (Z \rightarrow U)$

Part C: English Arguments Symbolize the following arguments. Then use truth tables to determine whether they are valid. Most of these arguments illustrate important logical properties of tautologies, contradictions, or contingent statements.

* 1. Grass is green. So, if Clinton wins, then Clinton wins. (G: Grass is green; W: Clinton wins)

2. Light is both a wave and a particle. But if light is a wave, then it is not a particle. So, physicists are profoundly mistaken. (W: Light is a wave; P: Light is a particle; M: Physicists are profoundly mistaken)

3. Either unicorns exist or unicorns do not exist. Therefore, trees exist. (U: Unicorns exist; T: Trees exist)

* 4. Pain is an illusion if and only if it is not an illusion. It follows that everything is an illusion. (P: Pain is an illusion; E: Everything is an illusion)

5. If it is wet, then it is wet if it is raining. Consequently, either Sasquatch exists or Sasquatch fails to exist. (W: It is wet; R: It is raining; S: Sasquatch exists)

6. Human behavior is determined only if human behavior is not free. Human behavior is determined; nevertheless, it is free. Therefore, life is but a dream. (D: Human behavior is determined; F: Human behavior is free; L: Life is but a dream)

7. The sky's being colored is a necessary condition for its being blue. It follows that the sky's being blue is a sufficient condition for its being colored. (C: The sky is colored; B: The sky is blue)

8. It is not the case both that electrons exist and that electrons do not exist. Hence, electrons exist. (E: Electrons exist)

9. If ultimate reality is divine, then it can be described in human language if and only if it cannot be described in human language. So, ultimate reality is not divine. (U: Ultimate reality is divine; L: Ultimate reality can be described in human language)

10. If you do not accept me as I am, then you do not love me. You accept me as I am if and only if you accept me as a violent criminal. You love me, but you do not accept me as a violent criminal. Consequently, you do not love me. (A: You accept me as I am; L: You love me; C: You accept me as a violent criminal)

Notes

1. C. S. Peirce, "On the Algebra of Logic: A Contribution to the Philosophy of Notation," *American Journal of Mathematics* 7 (1885): 180–202. The credit for the invention and development of truth tables should probably be spread around a bit. The idea occurs informally in Gottlob Frege's *Begriffsschrift* (Halle, Germany: L. Nebert, 1879). And the Austrian philosopher Ludwig Wittgenstein developed truth tables independently in his famous *Tractatus Logico-philosophicus* (London: Routledge & Kegan Paul, 1922).

2. The example is borrowed from Wesley Salmon, *Logic*, 3rd ed. (Englewood Cliffs, NJ: Prentice-Hall, 1984), p. 39.
3. Some logicians deny that the word "or" has two meanings. These logicians hold that the word "or" always means "either . . . or . . . *or both*," but they would agree that in many contexts both disjuncts cannot in fact be true—for instance, "Either all trees are plants or some trees are not plants." I do not wish to enter into the dispute about whether "or" has two meanings. I will only venture to say that I have found it pedagogically useful to speak of an *inclusive* and *exclusive* sense of the word "or," and thus I have freely done so in this book.