

Chapter 5

Categorical Logic: Statements

In this chapter and the next, we will explore a traditional approach to logic that was first developed by Aristotle (384–322 B.C.E.). This approach to logic focuses on categorical arguments, that is, arguments whose validity depends centrally on the relationships among classes, sets, or categories. Historically, this approach to logic has been of tremendous importance in Western culture. It was the dominant approach to logic in the medieval and early modern periods and is still quite useful today.

5.1 Standard Forms of Categorical Statements

To understand categorical arguments, we must first understand categorical statements. A **categorical statement** is a statement that relates two classes or categories. A class is a collection or set of things. Here are some examples of categorical statements:

1. All ducks are animals.
2. No humans are horses.
3. Some soldiers are cowards.
4. Some subatomic particles are not electrons.

Statement (1) says that every member of the class of ducks is a member of the class of animals. Statement (2) says that the class of humans and the class of horses have no members in common. Statement (3) says that some (i.e., at least one) member of the class of soldiers is a member of the class of cowards. And statement (4) says that some (i.e., at least one) member of the class of subatomic particles is not a member of the class of electrons.

There are four different **standard forms** of categorical statements, traditionally labeled **A, E, I, and O**. This labeling stems from the medieval period when logic was studied in Latin. The letters are vowels from the Latin words *affirmo* and *nego*, meaning (respectively) “I affirm” and “I deny.” To be in standard form, the elements of a categorical statement must appear in the following order:

- 1. quantifier (i.e., the word “all,” “no,” or “some”)
- 2. subject term (i.e., a word or phrase that names a class or category)
- 3. copula (“are” or “are not”)
- 4. predicate term (i.e., a word or phrase that names a class or category)

The four standard forms are as follows:

Name	Form	Example
A	All S are P.	All trees are plants.
E	No S are P.	No plants are animals.
I	Some S are P.	Some trees are oaks.
O	Some S are not P.	Some trees are not oaks.

The letter S stands for the subject term, and the letter P stands for the predicate term. For example, the word “trees” is the subject term in “All trees are plants,” while the word “plant” is the predicate term. And in the statement “Some trees are not oaks,” the word “trees” is the subject term, while the word “oaks” is the predicate term.

A term must denote (or refer to) a class or category; as such, a term is a noun or nounlike expression. Three brief comments may help clarify what counts as a term. First, it is important to realize that a term need not be a single word; indeed, it may be a rather long expression. For example, “All people who write best-selling novels are famous authors” is an **A** statement in standard form—the subject term is “people who write best-selling novels,” and the predicate term is “famous authors.” Second, note that a proper name is not a term, for a proper name denotes a specific individual rather than a class. Hence, “Some philosopher is Socrates” is not a standard-form categorical statement, since “Socrates” denotes a specific individual and not a class (i.e., collection of things). Third, adjectives are not terms. Thus, “All paintings are beautiful” is not in standard form because “beautiful,” being an adjective, does not denote a class; however, “All paintings are beautiful things” is in standard form because “beautiful things” is a nounlike expression that denotes the class containing all the things that are beautiful.

To be in standard form, a statement must *strictly* possess one of the listed forms. For example, is “Every tree is a plant” an **A** statement in standard form? No. To put it into standard form, we must replace “Every” with “All,” “is” with

“are,” “tree” with “trees,” and so on, to arrive at “All trees are plants.” We will discuss how to put statements into standard form shortly, but for now, note that the four forms are to be interpreted quite strictly.

Quality and Quantity

Every categorical statement has a **quality**, affirmative or negative. If a statement affirms that one class is wholly or partially included in another class, then the statement’s quality is *affirmative*. If a statement denies that one class is wholly or partially included in another, its quality is *negative*. Every categorical statement also has a **quantity**, universal or particular. *Universal* statements refer to all members of the class denoted by the subject term. *Particular* statements refer to only some members of the class denoted by the subject term.

An **A** statement (“All S are P”) is a **universal affirmative** statement. A universal affirmative statement says that all members of class S are members of class P. Thus, “All wives are women” says that all members of the class of wives are also members of the class of women.

An **E** statement (“No S are P”) is a **universal negative** statement. A universal negative statement says that no members of class S are members of class P. In other words, a universal negative says that classes S and P have no members in common. Thus, “No men are women” says that no members of the class of men are members of the class of women.

An **I** statement (“Some S are P”) is a **particular affirmative** statement. A particular affirmative statement says that some members of class S are members of class P. Thus, “Some animals are carnivores” says that some members of the class of animals are also members of the class of carnivores. Here, it is important to note that for present purposes, the word “some” means “at least one.” In ordinary English, “some” occasionally has the force of “some but not all.” But this is not the meaning assigned to “some” in logic. It is especially important to bear in mind that “Some S are P” does not imply that “Some S are not P.” (For example, “Some dogs are animals” does not imply that “Some dogs are not animals.”)

An **O** statement (“Some S are not P”) is a **particular negative** statement. A particular negative statement says that some members of class S are not members of class P. Thus, “Some mammals are not land animals” says that at least one member of the class of mammals is not a member of the class of land animals.

At this point we can sum up what has been said about categorical statements in the following table:

Name	Form	Quantity	Quality
A	All S are P.	universal	affirmative
E	No S are P.	universal	negative
I	Some S are P.	particular	affirmative
O	Some S are not P.	particular	negative

Putting Categorical Statements into Standard Form

When categorical statements appear in ordinary English, they are often not in standard form. But the logical tools we will discuss here are designed to apply to statements in standard form. So, let us now consider some techniques for putting categorical statements into standard form.

First, as we have already noted, when a statement fails to be in standard form simply because its predicate is an adjective, we can add an appropriate noun. Thus, to put "All humans are rational" into standard form, we can write "All humans are rational animals" or "All humans are rational things."

Second, when the elements of a standard-form statement are all present but not in the right order, we merely rearrange the elements. Thus, to put "Rubies are all gems" into standard form, we simply write, "All rubies are gems."

Third, when a statement contains a verb other than "are," we can add "are" and shift the original verb into the predicate. For example, to put "All fish swim" into standard form, we can write, "All fish are swimmers." And to put "All criminals should be punished" into standard form, we may write, "All criminals are people who should be punished." Similarly, if the verb "to be" is in the past tense or future tense, we can add "are" and relocate the tensed verb in the predicate. Thus, to put "All workers were tired" into standard form, we may write, "All workers are people who were tired." And "No persons who confess will be prosecuted" may be rewritten as "No persons who confess are persons who will be prosecuted."

Fourth, each of the basic types of categorical statements has common stylistic variants. A **stylistic variant** of a categorical statement is just another way of saying the same thing. Let's first consider stylistic variants of **A** statements ("All S are P"). For example, each of the following is a stylistic variant of "All cats are mammals":

- Every cat is a mammal.
- Each cat is a mammal.
- Any cat is a mammal.
- If anything is a cat, then it is a mammal.
- Things are cats only if they are mammals.
- Only mammals are cats.

To put any of these statements into standard form, we simply write, "All cats are mammals."

Take special note of the word "only" in the preceding list. Be aware that "Only P are S" means "All S are P," but it does *not* mean "All P are S." For example, "Only mammals are cats" means "All cats are mammals" (which is true), but it does not mean "All mammals are cats" (which is false). By contrast, "Only cats are mammals" (which is false) means "All mammals are cats," but it does not mean "All cats are mammals" (which is true).

Summary of Stylistic Variants

Universal Affirmative: All S are P.

- Every S is a P.
- Each S is a P.
- Any S is a P.
- If anything is an S, then it is a P.
- Things are S only if they are P.
- Only P are S.

Universal Negative: No S are P.

- Nothing that is an S is a P.
- A thing is an S only if it is not a P.
- If anything is an S, then it is not a P.
- Nothing is an S unless it is not a P.

Particular Affirmative: Some S are P.

- There are S that are P.
- At least one S is a P.
- There exists an S that is a P.
- Something is both an S and a P.

Particular Negative: Some S are not P.

- At least one S is not a P.
- Not all S are P.
- Not every S is a P.
- Something is an S but not a P.
- There is an S that is not a P.

Next, consider stylistic variants of **E** statements ("No S are P"). For example, each of the following is a stylistic variant of "No whales are humans":

- Nothing that is a whale is a human.
- A thing is a whale only if it is not human.
- If anything is a whale, then it is not a human.
- Nothing is a whale unless it is not a human.

To put any of these into standard form, write, "No whales are humans."

Now, consider stylistic variants of **I** statements ("Some S are P"). For example, each of the following is a stylistic variant of "Some fish are sharks":

- There are fish that are sharks.
- At least one fish is a shark.
- There exists a fish that is a shark.
- Something is both a fish and a shark.

To put any of these into standard form, we simply write, "Some fish are sharks."

Finally, consider stylistic variants of **O** statements ("Some S are not P"). For example, each of the following is a stylistic variant of "Some fish are not sharks":

At least one fish is not a shark.
 Not all fish are sharks.
 Not every fish is a shark.
 Something is a fish but not a shark.
 There is a fish that is not a shark.

To put any of these into standard form, we simply write, "Some fish are not sharks."

Prior to the late 19th century, many logicians thought that all valid arguments could be analyzed in terms of classes or categories. From this perspective, the four standard forms of categorical statements are the basic elements of deductive logic. While logicians no longer hold that all valid arguments can be expressed in terms of categorical statements, many important logical insights still can be derived from the study of the logic of categorical statements.

The following exercise gives you some practice in working with categorical statements.

Exercise 5.1

Part A: Categorical Statements Name the form of each of the following categorical statements (A, E, I, or O). Identify the subject and predicate terms in each case. Then state the quantity (universal or particular) and quality (affirmative or negative).

- * 1. All hungry cannibals are dangerous people.
- 2. No Ohioans are Texans.
- 3. Some diamonds are not valuable objects.
- * 4. No green vegetables are minerals.
- 5. Some outlaws are heroes.
- 6. All equilateral triangles are geometrical figures.
- * 7. Some poems are not sonnets.
- 8. No junk-food addicts are people with healthy diets.
- 9. Some scoundrels are people who have been mistreated.
- * 10. Some numbers are not odd numbers.
- 11. No pacifists are warmongers.
- 12. All mammals are cats.
- * 13. Some celebrities are highly moral people.
- 14. Some criminals are evil people.
- 15. All people who intentionally direct violence at noncombatants for political purposes are terrorists.
- * 16. No odd numbers are even numbers.

- 17. Some bank robbers are well-trained professionals.
- 18. All unreported crimes are lamentable events.
- * 19. Some art critics who like Picasso are snobs.
- 20. Some wealthy people are not nice people.
- 21. No sedimentary rocks are volcanic rocks.
- * 22. All individuals who lie frequently are deeply unhappy people.
- 23. No losers are winners.
- 24. Some people who voted for Gore are intelligent people.
- * 25. No photons are objects visible to the naked eye.
- 26. Some ancient gods are not morally perfect deities.
- 27. All people who worship money are lunatics.
- * 28. Some literature professors who love Tolstoy are not good lecturers.
- 29. No created entities are things that have always existed.
- 30. No humans who are truly happy are people who never work.

Part B: Standard Forms Which of the following are categorical statements in standard form? Which are not? (Remember, there are just four standard forms: "All S are P," "No S are P," "Some S are P," and "Some S are not P.") If a statement is already in standard form, simply name the form (A, E, I, or O) and indicate what the quantity and quality are. If a statement is not already in standard form, rewrite it so that it is; then name the form and indicate the quantity and quality.

- * 1. No human being can swim across the Atlantic Ocean.
- 2. Every kangaroo is a marsupial.
- 3. At least one car is not a Ford.
- * 4. At least one person is a nerd.
- 5. Nothing that is a spider is an insect.
- 6. All ancient Greeks worshiped Zeus.
- * 7. There exists a poem that is not a sonnet.
- 8. There are saints who are reformed criminals.
- 9. Not all politicians are liars.
- * 10. Some morally virtuous human beings are atheists.
- 11. Each patriotic American loves justice.
- 12. No Vikings were wimps.
- * 13. Not every animal that can fly is a bird.

14. All people who have committed murder deserve death.
15. Some good-looking people are snobs.
- * 16. Shawnees were all skillful trackers.
17. Nothing is a fool unless it is not a sage.
18. Masai warriors are all superb athletes.
- * 19. No people who are unlucky are happy.
20. All college students who listened to Jimi Hendrix opposed the war in Vietnam.
21. Nothing is a snake unless it is not a mammal.
- * 22. Only reptiles are lizards.
23. If anything is a chimpanzee, then it is not a fish.
24. Not every bright green stone is an emerald.
- * 25. Things are birds only if they have feathers.
26. Only diamonds are gems.
27. World-class athletes all train vigorously.
- * 28. Something is a painting but not a masterpiece.
29. Only physical objects are quarks.
30. The voters will all be disappointed.
- * 31. There exists a mountain that is beautiful.
32. If anything is a slug, then it is not intelligent.
33. Soldiers who served under General George Patton all saw combat.
- * 34. At least one tree is ugly.
35. Nothing that is an odd number is divisible by 2.
36. Things are beautiful only if they are pleasant to behold.
- * 37. At least one animal is vicious.
38. If anything is a bad-tempered person, then it is a curmudgeon.
39. Only red things are scarlet.
- * 40. If anything is a sibling that is female, then it is a sister.
41. Dogs over 15 years of age are all old.
42. A thing is a tragedy only if it is not a fortunate event.
- * 43. Of the living survivors of the Nazi prison camps, some were tortured.
44. Hindus all believe in reincarnation.
45. No prisoners will be mistreated.

- * 46. At least one soldier will be wounded.
47. Whole numbers between 1 and 5 are in some cases even numbers.
48. Of the living veterans of World War I, none were generals.
- * 49. Not every person who chooses not to fight is a coward.
50. There exists an animal that is a dog.

5.2 The Traditional Square of Opposition

We will now begin to discuss the logical relations between categorical statements. In this section and the next, we will focus on immediate inferences. An inference is said to be **immediate** when a conclusion is drawn from only one premise.

What are the logical relationships between standard-form categorical statements *having the same subject and predicate terms*? For example:

- A All dogs are collies.
- E No dogs are collies.
- I Some dogs are collies.
- O Some dogs are not collies.

Let us refer to categorical statements having the same subject term and the same predicate term as **corresponding statements**. Logicians in the Aristotelian tradition offer the following theses regarding the logical relationships between corresponding statements.

First, corresponding **A** and **O** statements are contradictories. Two statements are **contradictories** if they cannot both be true and they cannot both be false. (In other words, if one is true, the other must be false; and if one is false, the other must be true.) For example, "All dogs are collies" contradicts "Some dogs are not collies." Therefore, given that all dogs are collies, we can immediately conclude that it is false that some dogs are not collies. And given that some dogs are not collies, we can immediately conclude that not all dogs are collies.

Similarly, corresponding **E** and **I** statements are contradictories. For example, "No dogs are collies" contradicts "Some dogs are collies." Therefore, given that no dogs are collies, we can immediately conclude that it is false that some dogs are collies. And given that some dogs are collies, we can immediately conclude that it is false that no dogs are collies.

Second, corresponding **A** and **E** statements are contraries. Two statements are **contraries** if they cannot both be true but they can both be false. (For instance, the following two statements are contraries: "The Taj Mahal is white all over" and "The Taj Mahal is blue all over." Although the Taj Mahal is, in fact, white, it could be painted, say, green, in which case both of these statements would be false.) For example, corresponding **A** and **E** statements such as

"All dogs are collies" and "No dogs are collies" are contraries. These statements can both be false if some (but not all) dogs are collies. But if one of these statements is true, the other must be false.

One exception to the Aristotelian view of contraries should be noted at this point, namely, the case in which **A** or **E** statements are necessary truths. A **necessary truth** is one that cannot be false under any possible circumstances, for example, "All triangles are three-sided figures" or "No triangles are circles." Clearly, if a statement is necessarily true, then it cannot be false; but two statements are contraries only if they can *both* be false. So, the Aristotelian thesis regarding contraries clearly does not hold in the special case in which an **A** or **E** statement is a necessary truth.

Third, corresponding **I** and **O** statements are subcontraries. Two statements are **subcontraries** if they cannot both be false but they can both be true. For example, "Some dogs are collies" and "Some dogs are not collies" are subcontraries.

One exception to the Aristotelian view of subcontraries should be noted at this point, namely, the case in which **I** or **O** statements are necessarily false. A statement is **necessarily false** if it cannot be true in any possible circumstances, for example, "Some circles are triangles" or "Some triangles are not three-sided figures." Clearly, if a statement is necessarily false, then it cannot be true, but two statements are subcontraries only if they both can be true. So, the Aristotelian thesis regarding subcontraries clearly does not hold in the special case in which an **I** or **O** statement is necessarily false.

Fourth, **A** statements logically imply their corresponding **I** statements. For example, the following argument is valid according to Aristotelians:

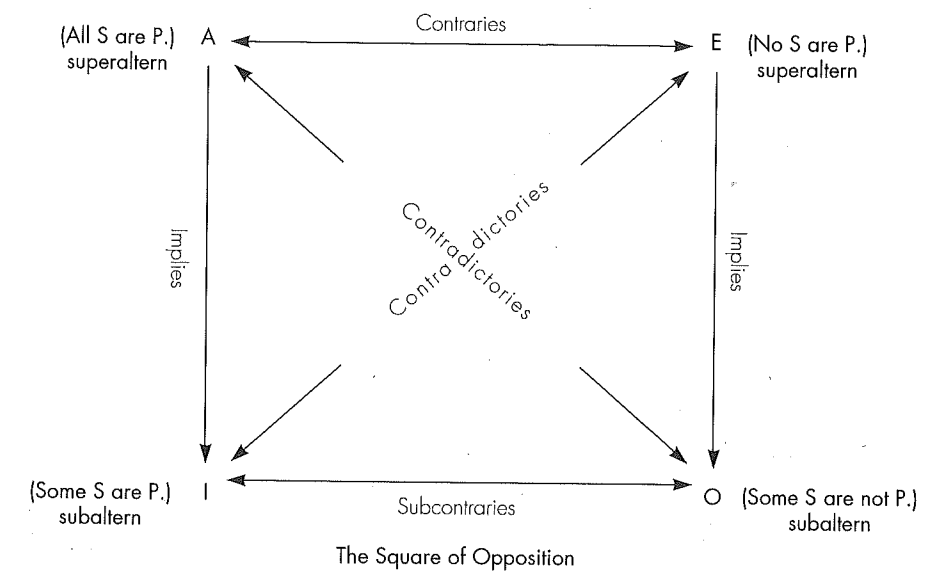
1. All ants are insects.
- So, 2. Some ants are insects.

Similarly, **E** statements logically imply their corresponding **O** statements. For example, the following argument is valid according to Aristotelians:

1. No ants are antelopes.
- So, 2. Some ants are not antelopes.

The logical relationship between a universal statement and its corresponding particular statement is called **subalternation**. The universal statement is called the **superaltern**, while the particular statement is called the **subaltern**. Thus, "All ants are insects" is a *superaltern* with "Some ants are insects" as its *subaltern*; and "No ants are antelopes" is a *superaltern* with "Some ants are not antelopes" as its *subaltern*. A *superaltern* implies its corresponding *subaltern*. This means that if the *superaltern* is true, the *subaltern* is true. But it also means that if the *subaltern* is false, then the *superaltern* is false (since any statement that implies a false statement is itself false). However, a *subaltern* does not imply its corresponding *superaltern* (e.g., "Some dogs are collies" does not imply "All dogs are collies").

All of these logical relationships can be pictured in a single diagram called the Traditional Square of Opposition, as shown here.



The Traditional Square of Opposition forms a helpful picture of a number of important logical relationships between corresponding statements.

1. Suppose an **A** statement (All S are P) is true; then
 - a. its corresponding **E** statement or contrary (No S are P) is false.
 - b. its corresponding **I** statement or subaltern (Some S are P) is true.
 - c. its corresponding **O** statement or contradictory (Some S are not P) is false.
2. Suppose an **E** statement (No S are P) is true; then
 - a. its corresponding **A** statement or contrary (All S are P) is false.
 - b. its corresponding **O** statement or subaltern (Some S are not P) is true.
 - c. its corresponding **I** statement or contradictory (Some S are P) is false.

Note: When the truth or falsehood of one statement, X, logically implies *neither* the truth *nor* the falsehood of another statement, Y, let us say that the truth value of Y is *not guaranteed* by the truth value of X.

3. Suppose an **I** statement (Some S are P) is true; then
 - a. its corresponding **E** statement or contradictory (No S are P) is false.
 - b. the truth value of the corresponding **A** statement (All S are P) is not guaranteed.
 - c. the truth value of the corresponding **O** statement (Some S are not P) is not guaranteed.

4. Suppose an **O** statement (Some S are not P) is true; then
- its corresponding **A** statement or contradictory (All S are P) is false.
 - the truth value of the corresponding **E** statement (No S are P) is not guaranteed.
 - the truth value of the corresponding **I** statement (Some S are P) is not guaranteed.

The following exercises provide you with an opportunity to deepen your understanding of the logical relationships embodied in the Traditional Square of Opposition.

◆ Exercise 5.2

Part A: Logical Relationships Give the names of the logical relations that hold between the following pairs of corresponding categorical statements. (In the case of subalternation, indicate which statement is the superaltern and which is the subaltern, in the order in which they appear.) If the pair of statements does not exemplify any of the logical relations discussed in this section, simply write "None."

- * 1. All roses are red flowers./No roses are red flowers.
- 2. All souls are immortal substances./Some souls are immortal substances.
- 3. Some people are jerks./Some people are not jerks.
- * 4. No Apaches are Shawnees./Some Apaches are Shawnees.
- 5. No emeralds are plants./Some emeralds are not plants.
- 6. Some people who believe in ghosts are smart people./Some people who believe in ghosts are not smart people.
- * 7. Some radical skeptics are profoundly miserable people./All radical skeptics are profoundly miserable people.
- 8. No truths are statements worth dying for./All truths are statements worth dying for.
- 9. All atoms are physical objects./Some atoms are not physical objects.
- * 10. Some odd numbers are numbers that can be divided by 2 (without remainder)./Some odd numbers are not numbers that can be divided by 2 (without remainder).
- 11. Some gems are not amethysts./Some gems are amethysts.
- 12. All liars are harmful people./No liars are harmful people.
- * 13. Some leaders are followers./Some leaders are not followers.
- 14. All positive whole numbers between 4 and 6 are odd numbers./No positive whole numbers between 4 and 6 are odd numbers.
- 15. Some dinosaurs are not rational animals./No dinosaurs are rational animals.

Part B: Immediate Inferences Which of the following immediate inferences are valid according to Aristotelian logicians? Which are not valid?

- * 1. All cougars are carnivores. So, it is false that some cougars are not carnivores.
- 2. All legal treaties are promises. Hence, it is not the case that no legal treaties are promises.
- 3. Some mosquitoes are evil beings. Therefore, it is not true that no mosquitoes are evil beings.
- * 4. All self-absorbed people are boring people. It follows that some self-absorbed people are boring people.
- 5. Some lawyers are shysters. So, all lawyers are shysters.
- 6. No five-star generals are humble people. Consequently, it is false that some five-star generals are humble people.
- * 7. Some heavenly bodies are not planets. Thus, it is false that all heavenly bodies are planets.
- 8. Some grapes are seedless fruit. So, some grapes are not seedless fruit.
- 9. It is false that all geometrical figures are trapezoids. Accordingly, some geometrical figures are not trapezoids.
- * 10. No fossils are traces of the missing link. Therefore, it is not true that all fossils are traces of the missing link.
- 11. No humans are morally perfect beings. It follows that some humans are not morally perfect beings.
- 12. All boxers who admire Hurricane Carter are winners. So, some boxers who admire Hurricane Carter are winners.
- * 13. Some persons who have landed on the moon are not women. Hence, no persons who have landed on the moon are women.
- 14. It is false that some grizzly bears are herbivores. It follows that some grizzly bears are not herbivores.
- 15. It is false that some wealthy Americans are political radicals. Thus, no wealthy Americans are political radicals.
- * 16. It is false that all dinosaurs are animals that existed prior to the Cambrian explosion. Accordingly, some dinosaurs are not animals that existed prior to the Cambrian explosion.
- 17. It is false that some Christians are not theists. Therefore, some Christians are theists.
- 18. All bureaucrats are spies. So, some bureaucrats are spies.
- * 19. Some Senators are lovers of justice. It follows that some Senators are not lovers of justice.
- 20. It is false that no astrologers are scientists. Consequently, some astrologers are scientists.

Part C: Generalizing Recall that when the truth value of one statement X logically implies neither the truth nor the falsehood of another statement Y, we say the truth value of Y is *not guaranteed* by X.

- * 1. Suppose an **A** statement is false. What can be logically inferred regarding the truth or falsehood of its corresponding **E**, **I**, and **O** statements?
- 2. Suppose an **E** statement is false. What can be logically inferred regarding the truth or falsehood of its corresponding **A**, **I**, and **O** statements?
- 3. Suppose an **I** statement is false. What can be logically inferred regarding the truth or falsehood of its corresponding **A**, **E**, and **O** statements?
- * 4. Suppose an **O** statement is false. What can be logically inferred regarding the truth or falsehood of its corresponding **A**, **E**, and **I** statements.

Part D: Standard Form Put the premise and conclusion of each of the following arguments into standard form. Then indicate whether each argument is valid.

- * 1. If anything is a capitalist, then it is not a hero. So, at least one thing is a capitalist but not a hero.
- 2. Only gods are immortal beings. Hence, there exist immortal beings that are gods.
- 3. Every misguided moralist is a menace to society. Consequently, a thing is a menace to society only if it is a misguided moralist.
- * 4. Things are positrons only if they are smaller than atoms. Therefore, not all positrons are smaller than an atom.
- 5. If anything is a falsehood, then it is not beneficial. So, only beneficial things are falsehoods.
- 6. There exists an athlete who can run the mile in under 4 minutes. Hence, not all athletes can run the mile in under 4 minutes.
- * 7. Nothing is an acid unless it is not a base. Therefore, each acid is a base.
- 8. Only acts that conform to the Ten Commandments are right acts. So, an act is right only if it does not conform to the Ten Commandments.
- 9. Nothing that is a categorical argument is valid. Hence, not every categorical argument is valid.
- * 10. Any person who is kept awake for over a week will go crazy. Thus, only persons who are kept awake for over a week will go crazy.
- 11. A thing is a wicked act only if it is an act committed with malice aforethought. Therefore, at least one wicked act is an act committed with malice aforethought.
- 12. Some people who defended slavery were plantation owners. Thus, not all persons who defended slavery were plantation owners.

- * 13. No person who invented the airplane died flying an airplane. It follows that only persons who died flying an airplane invented the airplane.
- 14. Colonels are all authoritarian people. So, nothing that is a colonel is an authoritarian person.
- 15. Only chemicals that turn blue litmus paper red are acids. Thus, there exists an acid that is a chemical that turns blue litmus paper red.

5.3 Further Immediate Inferences

In this section, we will discuss some important types of immediate inferences *beyond* those associated with the Traditional Square of Opposition. In particular, we will discuss conversion, obversion, and contraposition.

Conversion

The **converse** of a standard-form categorical statement is formed simply by interchanging its subject and predicate terms. Here are four examples of statements together with their converses:

Statement	Converse
A All dogs are animals.	All animals are dogs.
E No plants are animals.	No animals are plants.
I Some plants are trees.	Some trees are plants.
O Some plants are not trees.	Some trees are not plants.

Conversion is the inference from a categorical statement to its converse. Conversion is valid for **E** and **I** statements. For example, both of the following arguments are valid:

- 5. No plants are animals. So, no animals are plants.
- 6. Some plants are trees. So, some trees are plants.

In fact, every **E** statement is logically equivalent to its converse. Two statements are **logically equivalent** if each validly implies the other. For example, "No plants are animals" implies (and is implied by) "No animals are plants." Similarly, every **I** statement is logically equivalent to its converse. For example, "Some plants are trees" implies (and is implied by) "Some trees are plants."

But, as regards **A** and **O** statements, conversion is not a valid form of argument. The following examples should suffice to indicate why:

- 7. All dogs are animals. So, all animals are dogs.
- 8. Some plants are not trees. So, some trees are not plants.

Both arguments move from a true premise to a false conclusion and hence are invalid.

However, logicians in the Aristotelian tradition have endorsed an inference called **conversion by limitation**. In conversion by limitation, we switch the subject and predicate terms of an A statement and change the quantity from universal to particular. Here is an example:

1. All seaweeds are plants.
- So, 2. Some plants are seaweeds.

The general pattern of the inference is as follows:

1. All S are P.
- So, 2. Some P are S.

To understand conversion by limitation, it may be helpful to note that we can move logically from "All S are P" to its subaltern "Some S are P," and then (since conversion is always valid for I statements) we can switch the subject and predicate terms to get "Some P are S." Thus, while *conversion* itself is not a valid argument form as applied to A statements, logicians in the Aristotelian tradition endorse conversion by limitation.

We can picture what has been said about conversion in the following table:

Standard Form	Converse
A All S are P.	*All P are S. Converse by limitation: Some P are S.
E No S are P.	No P are S.
I Some S are P.	Some P are S.
O Some S are not P.	*Some P are not S.

The asterisk (*) indicates those cases in which the inference from the standard form statement to its converse is not a valid form of argument.

Obversion

The concept of an obverse requires a bit of explanation. First, each class has a complement. The **complement** of a class X is the class containing all things that are not a member of X. For instance, the complement of the class of trees is the class containing all nontrees, that is, everything that is not a tree (horses, hawks, humans, hamburgers, and so on).

Second, each term has a term-complement. The **term-complement** is the word or phrase that denotes the class complement. For instance, the term-complement of "dogs" is "nondogs," which denotes the class containing everything that is not a dog. And the term-complement of "nondogs" is simply "dogs," which denotes the class containing everything that is not a nondog. (Note: Do

not confuse term-complements with contrary terms. For instance, the term-complement of "winner" is not "loser" but "nonwinner," and the class of non-winners includes players who tie, nonplayers, and losers.)

When a term consists of more than one word, care must be taken in forming the term-complement. For example, what is the term-complement of "wild dogs"? Is it, "non-wild dogs"? No. The term-complement must denote a class that includes *everything* outside the class denoted by the term. So, in this case the term-complement is "things that are not wild dogs," which denotes a class that includes both tame dogs and nondogs in general. (For example, it includes wild geese, since they are not wild dogs.) Similarly, the term-complement of "good gymnast" would not be "nongood gymnast" but "things that are not good gymnasts." And the class of *things that are not good gymnasts* includes not only all the mediocre and poor gymnasts but also all the nongymnasts (atoms, apples, airplanes, etc.).

The **obverse** of a statement is formed by (a) changing its quality (from affirmative to negative, or vice versa) and (b) replacing the predicate term with its term-complement. Here are four examples:

Statement	Obverse
A All trees are plants.	No trees are nonplants.
E No cats are trees.	All cats are nontrees.
I Some trees are oaks.	Some trees are not nonoaks.
O Some trees are not oaks.	Some trees are nonoaks.

Obversion is the inference from a categorical statement to its obverse. Obversion is always valid. In fact, every standard-form categorical statement is logically equivalent to its obverse. For example, "No boxers are wimps" implies (and is implied by) "All boxers are nonwimps."

We can picture what has been said about obversion in the following table:

Standard Form	Obverse
A All S are P.	No S are non-P.
E No S are P.	All S are non-P.
I Some S are P.	Some S are not non-P.
O Some S are not P.	Some S are non-P.

The inference from a standard-form categorical statement to its obverse is always valid, and vice versa.

Contraposition

The **contrapositive** of a statement is formed by (a) replacing its subject term with the term-complement of its predicate term and (b) replacing the predicate term with the term-complement of its subject term. Here are four examples:

Statement	Contrapositive
A All cats are mammals.	All nonmammals are noncats.
E No bats are elephants.	No nonelephants are nonbats.
I Some plants are weeds.	Some nonweeds are nonplants.
O Some plants are not weeds.	Some nonweeds are not nonplants.

Note: In every case, we switch the subject and predicate terms, then replace each with its term-complement.

Contraposition is the inference from a statement to its contrapositive. Contraposition is valid for **A** and **O** statements. Thus, the following arguments are valid:

9. All rubies are stones. So, all nonstones are nonrubies.
10. Some trees are not elms. So, some nonelms are not nontrees.

It is interesting to note that these same results can be achieved by a sequence of obversions and conversions. For instance, consider the inference from an **A** statement to its contrapositive:

- Step 1: All S are P.
- Step 2: No S are non-P. [obverse of Step 1]
- Step 3: No non-P are S. [converse of Step 2]
- Step 4: All non-P are non-S. [obverse of Step 3]

Note: An **A** statement and its contrapositive are logically equivalent. Thus, "All collies are dogs" implies (and is implied by) "All nondogs are noncollies." Similarly, an **O** statement and its contrapositive are logically equivalent. Thus, "Some dogs are not collies" implies (and is implied by) "Some noncollies are not nondogs."

As regards **E** and **I** statements, contraposition is not a valid form of argument. For example, the following argument is invalid:

11. No dogs are trees. So, no nontrees are nondogs.

Argument (11) is plainly invalid. Its premise is true, but its conclusion is false. (Examples of nontrees include stones, steers, and stereoscopes; therefore, some nontrees are nondogs.) The following argument involves an inference from an **I** statement to its contrapositive and is also invalid:

12. Some animals are nondogs. So, some dogs are nonanimals.

Here, the premise is true. (The mere fact that there are cats is proof of that.) But the conclusion is false, so the argument is invalid.

While contraposition is not a valid form of argument as applied to **E** statements, Aristotelian logicians endorse an inference called **contraposition by limitation**.

Memory Device

1. **OBVERSION** is the odd rule: It works on all four standard forms.

How about conversion and contraposition? Look at the *middle* vowels:

2. **CONVERSION** works on **E** and **I** statements.

3. **CONTRAPOSITION** works on **A** and **O** statements.

tation. To obtain the contrapositive *by limitation* of an **E** statement, we (a) replace the subject term with the term-complement of the predicate term, (b) replace the predicate term with the term-complement of the subject term, and (c) change the quantity from universal to particular. Here's an example:

1. No flags are rags.
- So, 2. Some nonrags are not nonflags.

It is interesting to note that we can move from (1) to (2) by way of inferences previously discussed:

- Step 1. No flags are rags.
- Step 2. Some flags are not rags. [subaltern of Step 1]
- Step 3. Some flags are nonrags. [obverse of Step 2]
- Step 4. Some nonrags are flags. [converse of Step 3]
- Step 5. Some nonrags are not nonflags. [obverse of Step 4]

The general pattern of inference in contraposition by limitation is as follows:

1. No S are P.
- So, 2. Some non-P are not non-S.

We can sum up what has been said about contraposition in the following table:

Standard Form	Contraposition
A All S are P.	All non-P are non-S.
E No S are P.	*No non-P are non-S. <i>Contrapositive by limitation:</i> Some non-P are not non-S.
I Some S are P.	*Some non-P are non-S.
O Some S are not P.	Some non-P are not non-S.

The asterisk (*) indicates those cases in which the inference from the standard-form statement to its contrapositive is not a valid form of argument.

The following exercises will test your understanding of immediate inferences.

Summary Table: Conversion, Obversion, and Contraposition	
Standard Form	Converse
A All S are P.	*All P are S. <i>Converse by limitation:</i> Some P are S.
E No S are P.	No P are S.
I Some S are P.	Some P are S.
O Some S are not P.	*Some P are not S.
<i>Note:</i> An asterisk indicates that the form of argument is not valid.	
Standard Form	Obverse
A All S are P.	No S are non-P.
E No S are P.	All S are non-P.
I Some S are P.	Some S are not non-P.
O Some S are not P.	Some S are non-P.
Standard Form	Contrapositive
A All S are P.	All non-P are non-S.
E No S are P.	*No non-P are non-S. <i>Contrapositive by limitation:</i> Some non-P are not non-S.
I Some S are P.	*Some non-P are non-S.
O Some S are not P.	Some non-P are not non-S.
<i>Note:</i> An asterisk indicates that the form of argument is not valid.	

Exercise 5.3

Part A: Term-Complements Rewrite the following categorical statements, replacing each term with its term-complement.

- * 1. No brown bears are herbivores.
- 2. All corporals are nongenerals.
- 3. Some large birds are eagles.
- * 4. Some unhappy entities are not people.
- 5. No things that are not humans are rational animals.
- 6. All drinkers are nondrivers.
- * 7. No great women are men.
- 8. Some athletes are poor losers.

- 9. Some nonsmokers are not healthy people.
- * 10. Some nonmetals are chemicals.

Part B: Conversion Form the converse of each of the following statements. Then indicate whether conversion, as applied to the type of categorical statement in question, is a valid form of argument.

- * 1. No magnates are maggots.
- 2. All miracles are acts of God.
- 3. Some rectangles are nonsquares.
- * 4. Some explosives are not bombs.
- 5. All demons are angels.
- 6. No lovers are loners.
- * 7. All forgeries are copies.
- 8. No roaches are coaches.
- 9. Some Africans are not Kenyans.
- * 10. Some leopards are nontigers.

Part C: Obversion Form the obverse of each of the following statements. (Recall that obversion always results in a valid inference.)

- * 1. All shar-peis are dogs.
- 2. No platypi are vegetarians.
- 3. Some prime ministers are women.
- * 4. Some heroes are not martyrs.
- 5. All shamans are priests.
- 6. No tulips are weeds.
- * 7. All colonels are objects weighing at least 100 pounds.
- 8. Some logicians are septuagenarians.
- 9. No giants are things less than 10 feet tall.
- * 10. No serigraphs are sculptures.

Part D: Contraposition Form the contrapositive of each of the following statements. Then indicate whether contraposition, as applied to the type of categorical statement in question, is a valid form of argument.

- * 1. All cynics are pessimists.
- 2. Some plants are nonroses.
- 3. Some dramas are not comedies.
- * 4. Some noncollies are not nondogs.
- 5. All photons are things that travel at the speed of light.
- 6. No red oaks are elms.
- * 7. All things that can run at more than 50 miles an hour are cats.
- 8. Some non-Fords are not nonautomobiles.
- 9. All college students are entities having IQs of at least 100.
- * 10. All great white sharks are nonguppies.

Part E: Inferences from A Statements Assuming that "All ideologues are fools" is true, what is implied regarding the truth or falsehood of the following statements? (If neither the truth nor the falsehood of the statement is implied, simply write "not guaranteed.") *Note:* To get the correct answer, you may need to make a series of inferences from the assumed statement.

- * 1. All fools are ideologues.
- 2. No ideologues are nonfools.
- 3. Some ideologues are fools.
- * 4. All nonfools are nonideologues.
- 5. No nonfools are ideologues.
- 6. Some fools are ideologues.
- * 7. No fools are nonideologues.
- 8. Some ideologues are not fools.
- 9. No ideologues are fools.
- * 10. Some ideologues are not nonfools.
- 11. Some fools are not nonideologues.
- 12. All nonideologues are nonfools.
- * 13. Some nonfools are nonideologues.
- 14. No fools are ideologues.
- 15. No nonideologues are nonfools.

Part F: Inferences from E Statements Assuming that "No psychiatrists are optimists" is true, what is implied regarding the truth or falsehood of the following statements? (If neither the truth nor the falsehood of the statement is implied, simply write "not guaranteed.") *Note:* To get the correct answer, you may need to make a series of inferences from the assumed statement.

- * 1. All psychiatrists are nonoptimists.
- 2. No optimists are psychiatrists.
- 3. Some psychiatrists are not optimists.
- * 4. No nonoptimists are nonpsychiatrists.
- 5. All optimists are nonpsychiatrists.
- 6. Some optimists are not psychiatrists.
- * 7. All psychiatrists are optimists.
- 8. All nonpsychiatrists are optimists.
- 9. Some psychiatrists are optimists.
- * 10. Some nonoptimists are not nonpsychiatrists.
- 11. No nonpsychiatrists are optimists.
- 12. Some optimists are nonpsychiatrists.
- * 13. No nonoptimists are psychiatrists.
- 14. Some optimists are psychiatrists.
- 15. Some psychiatrists are nonoptimists.

Part G: Inferences from I Statements Assuming that "Some chemicals are poisons" is true, what is implied regarding the truth or falsehood of the following statements? (If neither the truth nor the falsehood of the statement is implied, simply write "not guaranteed.") *Note:* To get the correct answer, you may need to make a series of inferences from the assumed statement.

- * 1. Some poisons are chemicals.
- 2. No chemicals are poisons.
- 3. Some chemicals are not nonpoisons.
- * 4. Some nonchemicals are nonpoisons.
- 5. No poisons are chemicals.
- 6. All chemicals are poisons.
- * 7. Some nonpoisons are nonchemicals.
- 8. No nonpoisons are nonchemicals.
- 9. Some chemicals are not poisons.
- * 10. Some nonchemicals are not nonpoisons.
- 11. All poisons are chemicals.
- 12. Some chemicals are nonpoisons.
- * 13. Some nonchemicals are poisons.
- 14. Some nonpoisons are chemicals.
- 15. All chemicals are nonpoisons.

Part H: Inferences from O Statements Assuming that "Some celebrities are not saints" is true, what is implied regarding the truth or falsehood of the following statements? (If neither the truth nor the falsehood of the statement is implied, simply write "not guaranteed.") *Note:* To get the correct answer, you may need to make a series of inferences from the assumed statement.

- * 1. No celebrities are saints.
- 2. All celebrities are saints.
- 3. Some celebrities are nonsaints.
- * 4. Some saints are not celebrities.
- 5. Some nonsaints are not noncelebrities.
- 6. Some celebrities are saints.
- * 7. Some nonsaints are celebrities.
- 8. All nonsaints are noncelebrities.
- 9. No celebrities are nonsaints.
- * 10. Some noncelebrities are not nonsaints.
- 11. All celebrities are nonsaints.
- 12. No saints are celebrities.
- * 13. Some celebrities are not nonsaints.
- 14. Some noncelebrities are saints.
- 15. No nonsaints are celebrities.