Chapter 6

Categorical Logic: Syllogisms

Categorical syllogisms are arguments composed entirely of categorical statements. Every categorical syllogism has two premises and one conclusion, and every categorical syllogism contains exactly three terms. For example:

1.
   1. All human acts are behaviors caused by genes.
   2. All altruistic acts are human acts.

   So, 3. All altruistic acts are behaviors caused by genes.

"Human acts," "altruistic acts," and "behaviors caused by genes" are the terms in this syllogism. Categorical syllogisms are important and useful forms of argument. Many a long argument can be interpreted as a series of categorical syllogisms. And it is often revealing to try to express the main steps in a complex argument as a syllogism. Very often the attempt to "boil an argument down" into a categorical syllogism will reveal significant strengths and weaknesses.

6.1 Standard Form, Mood, and Figure

Before we can develop logical tools for evaluating categorical syllogisms, we must first develop a clear terminology for talking about them. Consider the following syllogism:

2.
   1. All astronomers are scientists.
   2. Some astrologers are not scientists.

   So, 3. Some astrologers are not astronomers.

Note that one of the terms, namely, "scientists," occurs once in each premise. The **middle term** of a categorical syllogism is the term that occurs once in each premise. The **major term** of a categorical syllogism is the predicate term of the conclusion. Thus, "astronomers" is the major term of syllogism (2). The **minor term** of a categorical syllogism is the subject term of the conclusion. So, "astrologers" is the minor term of syllogism (2).

Just as there are standard forms for categorical statements, there is a standard form for categorical syllogisms. Our logical tools are designed to apply to syllogisms in standard form, so it is important to be able to put a syllogism into standard form. A categorical syllogism is in **standard form** when these conditions are met:

a. The premises and the conclusion are categorical statements in standard form ("All S are P," "No S are P," "Some S are P," or "Some S are not P").
b. The first premise contains the major term.
c. The second premise contains the minor term.
d. The conclusion is stated last.

The **major premise** of a categorical syllogism is the premise containing the major term, and the **minor premise** is the premise containing the minor term. So, when a categorical syllogism is in standard form, the first premise is the major premise and the second premise is the minor premise.

Which of the following categorical syllogisms are in standard form?

3. 1. All oaks are trees.
2. All trees are plants.
So, 3. All oaks are plants.

4. 1. All trees are plants.
2. All oaks are trees.
So, 3. All oaks are plants.

5. 1. All trees are plants.
2. Only trees are oaks.
So, 3. All oaks are plants.

Only syllogism (4) is in standard form. Syllogism (3) is not in standard form because the minor premise comes first. And syllogism (5) is not in standard form because its second premise is not a categorical statement in standard form. We can put (5) into standard form by rewriting "Only trees are oaks" as "All oaks are trees."

The logical form of a categorical syllogism is determined by its mood and figure. The **mood** of a categorical syllogism *in standard form* is determined by the kinds of categorical statements involved and the order in which they appear. For example,

6. 1. All psychiatrists are physicians.
2. Some psychologists are not physicians.
So, 3. Some psychologists are not psychiatrists.

The mood of this syllogism is **AOO**. That is, the first premise is an **A** statement, the second premise is an **O** statement, and the conclusion is an **O** statement. What is the mood of the following syllogism?

7. 1. No birds are mammals.
2. All bats are mammals.
So, 3. No bats are birds.

The mood is **EAE**. That is, the first premise is an **E** statement, the second premise is an **A** statement, and the conclusion is an **E** statement. Since the mood involves the *sequence* of the statements as well as their types, be sure that a syllogism is in standard form when trying to identify its mood.

Two syllogisms can have the same mood and yet differ in logical form. The following syllogism has the same mood as (7), but it differs in logical form:

8. 1. No mammals are birds.
2. All mammals are bats.
So, 3. No bats are birds.

We can bring out the difference in form by using letters to stand for terms. Let "S" stand for the minor term (the subject term of the conclusion), "P" for the major term (the predicate term of the conclusion), and "M" for the middle term. (Recall that the middle term occurs once in each premise but does not occur in the conclusion.) Then arguments (7) and (8) have the following forms, respectively:

No P are M.	No M are P.
All S are M.	All M are S.
So, no S are P.	So, no S are P.

In the Aristotelian scheme, (7) and (8) are said to differ in **figure**. Figure is specified by the position of the middle term. There are four possible figures, which can be diagrammed as follows:

First Figure	Second Figure	Third Figure	Fourth Figure
M–P	P–M	M–P	P–M
S–M	S–M	M–S	M–S
So, S–P	So, S–P	So, S–P	So, S–P

In the first figure, the middle term is the subject term of the major premise and the predicate term of the minor premise. In the second figure, the middle term is the predicate term of both premises. In the third figure, the middle term is the subject term of both premises. In the fourth figure, the middle term is the predicate term of the major premise and the subject term of the minor premise.

The form of a syllogism is completely specified by its mood and figure. The Aristotelian approach works out which combinations of mood and figure result in valid forms and which result in invalid forms. For example, argument (7) is a syllogism in the *second figure* having the mood **EAE**; this form is valid. On the other hand, argument (8) is a syllogism in the *third figure* having the mood **EAE**; this form is invalid. Thus, according to Aristotelian logic, validity is determined by mood and figure.

How many different forms of categorical syllogisms are there? Two hundred and fifty-six. As we have seen, there are four kinds of categorical statements and three categorical statements per categorical syllogism. Thus, there are $4^3 = 4 \times 4 \times 4 = 64$ possible moods (**AAA, AAE, AAI, AAO, AEA,** etc.). Moreover, there are four different figures, and $64 \times 4 = 256$. Out of all these possibilities, ancient and modern logicians agree that the following 15 forms are valid:

First figure: AAA, EAE, AII, EIO

Second figure: EAE, AEE, EIO, AOO

Third figure: IAI, AII, OAO, EIO

Fourth figure: AEE, IAI, EIO

It is not necessary to memorize this list of valid forms. It is much more important to learn how to test categorical syllogisms for validity, and we shall begin to do that in the next section. By the way, according to logicians in the Aristotelian tradition, an additional nine forms are valid:

First figure: AAI, EAO

Second figure: AEO, EAO

Third figure: AAI, EAO

Fourth figure: AEO, EAO, AAI

We will discuss why logicians in the Aristotelian tradition accept these additional nine forms as valid—and why many modern logicians do not—in section 6.4. For the moment, you may wish to note that all the forms in the additional set of nine involve an inference from two universal premises to a particular conclusion. These forms do not test valid using the method we shall discuss in the next two sections.

The following exercise will help you deepen your understanding of the concepts introduced in this section.

Exercise 6.1

Part A: Standard Form Which of the following categorical syllogisms are *in standard form*? Which are not? If a syllogism is not in standard form, rewrite it so that it is.

* 1.
1. Some works of art are books.
2. All novels are books.

So, 3. Some works of art are novels.

2.
1. Some poems are not masterpieces.
2. No limericks are masterpieces.

So, 3. Some limericks are not poems.

3.
1. All movies are films.
2. Some documentaries are not movies.

So, 3. Some documentaries are not films.

* 4.
1. All sculptures are beautiful.
2. Some beautiful things are paintings.

So, 3. Some sculptures are not paintings.

5.
1. Some short stories are not interesting.
2. Every famous short story is interesting.

So, 3. Some short stories are not famous short stories.

6.
1. Some artists are millionaires.
2. No millionaires are poor.

So, 3. At least one artist is not poor.

* 7.
1. All sadists are mean.
2. All art critics are mean.

So, 3. All art critics are sadists.

8.
1. All metaphors are figures of speech.
2. All metaphors are words.

So, 3. All figures of speech are words.

9.
1. All opera singers are cool people.
2. No rock singers are opera singers.

So, 3. No rock singers are cool people.

* 10.
1. Some ballerinas are clumsy dancers.
2. All people who hate music are clumsy dancers.

So, 3. Some people who hate music are ballerinas.

11.
1. Some comedians are poets.
2. Some comedians are prophets.

So, 3. Some poets are prophets.

12.
1. Every author is insightful.
2. All novelists are authors.

So, 3. All insightful people are novelists.

* 13. 1. No aspiring actor is a saint.
2. At least one aspiring actor is not an egoist.
So, 3. Some egoists are saints.

14. 1. Some musicians are not classically trained musicians.
2. All jazz musicians are musicians.
So, 3. Some jazz musicians are not classically trained musicians.

15. 1. Only artists are alive.
2. No ancient Greek poets are alive.
So, 3. No artists are ancient Greek poets.

Part B: Mood and Figure Specify the mood and figure of the following forms. Then use the list of valid forms provided in this section to determine whether the forms are valid.

* 1. 1. Some P are M.
2. All S are M.
So, 3. Some S are P.

2. 1. Some M are P.
2. Some M are S.
So, 3. Some S are P.

3. 1. All P are M.
2. No M are S.
So, 3. No S are P.

* 4. 1. No M are P.
2. Some M are not S.
So, 3. Some S are P.

5. 1. No M are P.
2. All S are M.
So, 3. No S are P.

6. 1. No P are M.
2. All M are S.
So, 3. No S are P.

* 7. 1. All P are M.
2. Some S are M.
So, 3. Some S are P.

8. 1. Some P are not M.
2. Some S are not M.
So, 3. Some S are not P.

9. 1. Some M are not P.
2. All M are S.
So, 3. Some S are not P.

* 10. 1. Some M are P.
2. All S are M.
So, 3. Some S are not P.

11. 1. All P are M.
2. Some S are not M.
So, 3. Some S are not P.

12. 1. No M are P.
2. Some S are M.
So, 3. Some S are not P.

* 13. 1. All P are M.
2. All S are M.
So, 3. All S are P.

14. 1. All M are P.
2. All M are S.
So, 3. All S are P.

15. 1. All M are P.
2. Some S are M.
So, 3. Some S are P.

Part C: Putting Syllogisms into Standard Form Put the following syllogisms into standard form. Then specify the mood and figure. Finally, use the list of valid forms to determine whether the syllogisms are valid.

* 1. Every cowboy loves horses. Not all farmers love horses. It follows that at least one farmer is not a cowboy.

2. Nothing is a rodeo unless it is not an opera. Each opera includes singers. So, nothing that is a rodeo includes singers.

3. Not everyone who loves country music is a rodeo star. For at least one rock star loves country music; and no rock stars are rodeo stars.

* 4. No cowards ride bulls; therefore, some fools are not cowards since at least one bull rider is a fool.

5. No cowgirls are city slickers; hence, no city slicker is a talented rider, because only talented riders are cowgirls.

6. There exists a drifter who is a sheriff; for at least one sheriff is a gunslinger, and a thing is a gunslinger only if it is a drifter.

* 7. Only good guys are cowboys in white outfits. A thing is a cattle rustler only if it is not a good guy. It follows that no cowboys in white outfits are cattle rustlers.

8. Every barkeep who serves rotgut is a bad guy. Therefore, at least one person who won't live long is a bad guy since at least one barkeep who serves rotgut won't live long.

9. Only westerns are worth seeing because only good movies are worth seeing, and only westerns are good movies.

* 10. At least one bronco is not hard to ride, for all bulls are hard to ride, and some broncos are not bulls.

11. Nothing is a wealthy landowner unless it is not a buckaroo. A thing is a cattle baron only if it is a wealthy landowner. Hence, nothing that is a buckaroo is also a cattle baron.

12. If anything is a bounty hunter, then it is not a sodbuster. At least one outlaw is a sodbuster; therefore, not every outlaw is a bounty hunter.

* 13. A thing is a trail boss only if it is not a hired hand. There exists a rancher who is a hired hand. Consequently, at least one rancher is a trail boss.

14. Not all bandits will be hanged. After all, at least one bandit is not a horse thief, and every horse thief will be hanged.

15. Not all sheep ranchers are fast guns; hence, not all honest citizens are fast guns since only honest citizens are sheep ranchers.

Part D: Constructing Syllogisms Write your own syllogisms with forms as specified below. Then use the list of valid forms provided in this section to determine whether the syllogisms are valid.

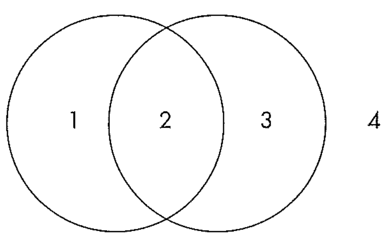
1. First figure: **EIO**
2. Second figure: **AEE**

3. Third figure: IAI
4. Fourth figure: EAE
5. First figure: AAE
6. Second figure: EIO
7. Third figure: OAO
8. Fourth figure: IAI
9. First figure: EEA
10. Second figure: AOO

6.2 Venn Diagrams and Categorical Statements

In this section and the next, we will examine a method for establishing the validity and invalidity of categorical arguments. This method was discovered around 1880 by the English logician John Venn. Venn's method involves the use of a special type of picture or diagram.

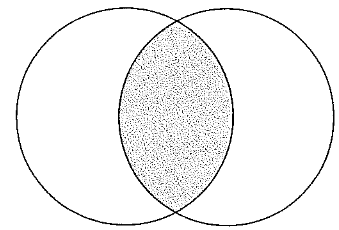
A Venn diagram consists of overlapping circles. Each circle stands for a class. Since each categorical statement has two terms, with each term denoting a class, the Venn diagram for a single categorical statement involves just two overlapping circles. For example, in the following diagram, the circle on the left denotes the class of dogs and the circle on the right denotes the class of animals. The numerals (1 through 4) are not normally part of the diagram but are added here temporarily to enable us to refer to the separate areas of the diagram.



Area 1 stands for things that are dogs but not animals. Of course, in reality this area is empty since there are no dogs that are not animals. Area 2 (the area of overlap between the circles) stands for things that are both dogs and animals, that is, all the dogs. Area 3 stands for things that are animals but not dogs, such as cats, crickets, and kangaroos. Area 4 stands for things that are neither dogs nor animals, such as neutrons, nickels, and numbers.

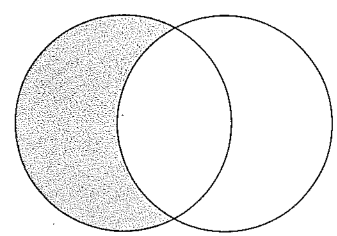
To construct Venn diagrams, we indicate that the various areas of the diagram either contain objects or are empty. To show that an area contains at least one object, we use an "x." To show that an area is empty, we shade it in. If an

area does not contain an "x" and is not shaded in, we simply have no information about it. Thus, to diagram a universal negative statement, such as "No dogs are cats," we indicate that the area of overlap between the two circles is empty by shading it in, as follows:



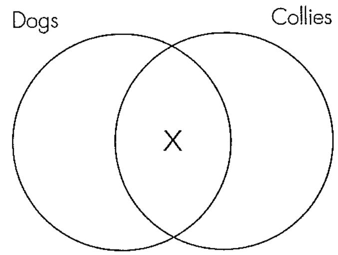
This sort of diagram, with shading in the area of overlap, is the sort you will always use for a universal negative statement. However, as previously noted, the English language provides various ways of saying that "No S are P," such as "If anything is an S, then it is not a P" and "Nothing that is an S is a P." When you encounter such stylistic variants in a syllogism, simply rewrite the statement in "No S are P" form and use a diagram similar to the one just shown. One last thing: Notice that the preceding diagram does not say that there are any dogs, nor does it say that there are any cats. It simply says that nothing belongs to the class (or set) of things that are both dogs and cats.

Universal affirmatives have the form "All S are P," and they say that the members of set S are also members of set P, or in other words, that S has no members that are not members of P. Thus, the diagram for "All dogs are animals" looks like this:



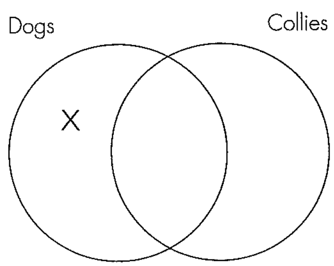
Notice that this diagram does not say that there are any dogs, nor does it say that there are any animals. It simply says, "If there are any dogs, then they are animals" (or "Anything you put in the dog-circle has to go in the animal-circle"). This is the sort of shading you will always use when diagramming a universal affirmative statement. However, as previously noted, the English language contains numerous stylistic variants for "All S are P," such as "Every S is a P," "If anything is an S, then it is a P," and "Only P are S." When you encounter these stylistic variants, rewrite the statement into "All S are P" form and use a diagram similar to the one just shown.

Particular affirmatives have the form "Some S are P," and these say that sets S and P have at least one member in common. The diagram for "Some dogs are collies" looks like this:



This diagram asserts that there exists at least one dog that is a collie. ("Something is in the dog-circle *and* in the collie-circle.") This is the type of diagram you will always use for particular affirmative statements. However, as previously noted, the English language contains a number of stylistic variants for "Some S are P," including "At least one S is a P" and "There are S that are P." When you encounter these variants, simply rewrite the statement into "Some S are P" form and use a diagram similar to the one just shown.

Particular negatives have the form "Some S are not P." These statements say that set S has at least one member that does not belong to set P. The diagram for "Some dogs are not collies" looks like this:



The diagram asserts that there exists at least one dog that is not a collie. ("Something is in the dog-circle but *not* in the collie-circle.") This is the type of diagram you will always use for particular negative statements. But as previously noted, the English language contains a number of stylistic variants for "Some S are not P," such as "Not all S are P" and "At least one S is not a P." When you

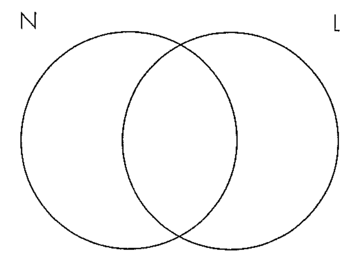
encounter these variants, simply rewrite the statement in "Some S are not P" form and use a diagram similar to the one just shown.

Now that we know how to diagram the four relevant types of categorical statements, we can use Venn diagrams to evaluate arguments for validity. We begin with short arguments involving just one premise. To determine whether an argument is valid, we proceed as follows. First, we diagram the premise. Second, we look to see whether our diagram of the premise tells us that the conclusion is true. This works because if an argument is valid, the content of the conclusion is contained, at least implicitly, in the premise(s).

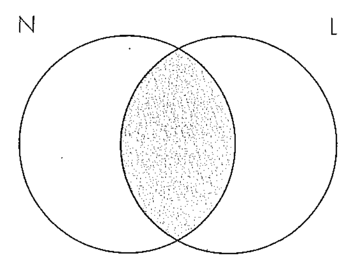
Let's start with some examples of conversion:

9. 1. No Namibians are Libyans.
So, 2. No Libyans are Namibians.

To apply the Venn method, we draw two overlapping circles and label them. To cut down on writing, we will use capital-letter abbreviations to label our circles. In our current example, let's use "N" for "Namibians" and "L" for "Libyans." (Normally, we will use the first letter of the term to label the relevant circle.) As regards *single-premise* arguments, let us follow the convention of labeling the circle on the left with the *subject term* of the premise and the circle on the right with the *predicate term* of the premise, like this:



Now we diagram the premise:



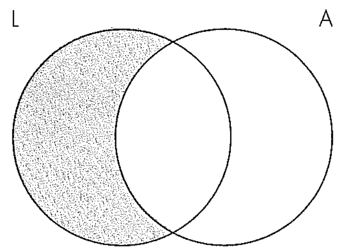
Next we look to see whether the diagram of the premise tells us that the conclusion is true. In this case, the answer is yes, for the shading indicates that no

Libyan is a Namibian. So, the diagram tells us that the argument is valid, which is as it should be since conversion is valid for E statements.

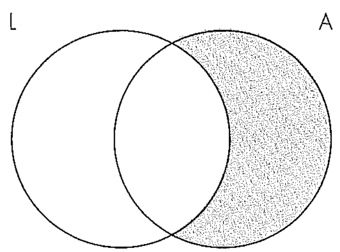
Let's consider a second short argument:

10. 1. All Liberians are Africans.
So, 2. All Africans are Liberians.

As before, we draw two overlapping circles, label them, and diagram the premise:



Now we look to see whether the diagram of the premise tells us that the conclusion is true. In this case, it does not. A diagram of the conclusion would have to shade in area 3, the part of the circle that stands for Africans who are not Liberians, like this:

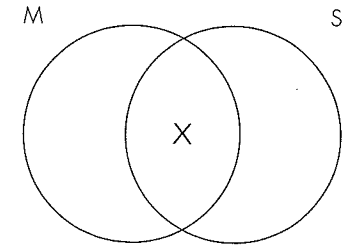


Thus, our diagram of the premise tells us that the argument is not valid. And this is as it should be since argument (10) is plainly invalid. In general, to determine whether premises make a conclusion true, it helps to compare a diagram of the premises with a separate diagram of the conclusion. For an argument to be valid, a diagram of the premises must contain the essential content of the conclusion, but this does not mean that the diagram of the premises will always look exactly like the diagram you would produce if you simply set out to diagram the conclusion.

Consider another example of conversion:

11. 1. Some Moroccans are Spanish speakers.
So, 2. Some Spanish speakers are Moroccans.

Again, we diagram the premise:

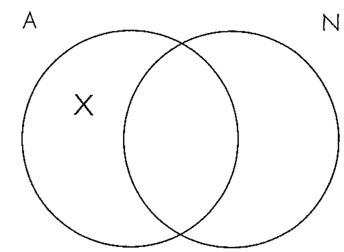


And now we look to see whether our diagram of the premise tells us that the conclusion of the argument is true. In this case, the answer is yes, the "x" tells us that some members of the class of Spanish speakers are also members of the class of Moroccans. So, our Venn diagram indicates that the argument is valid.

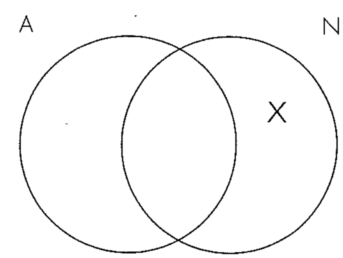
Consider one last example of conversion:

12. 1. Some Africans are not Nigerians.
So, 2. Some Nigerians are not Africans.

As before, we diagram the premise:



Does the diagram tell us that the conclusion is true? No, for this to be so, an "x" would have to appear in area 3, like this:

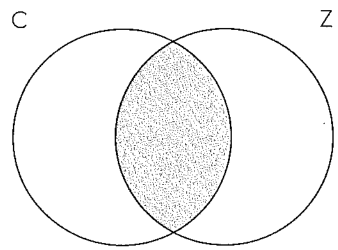


Consequently, the Venn method tells us that argument (12) is invalid.

Now let's consider an example of obversion:

13. 1. No Cameroonians are Zimbabweans.
So, 2. All Cameroonians are non-Zimbabweans.

The diagram of the premise is as follows:



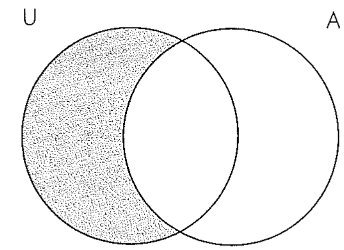
Does the diagram tell us that the conclusion is true? Yes, for it indicates that if there are any Cameroonians, they are not Zimbabweans (i.e., they must be in the part of the Cameroonian-circle that lies outside the Zimbabwean-circle). In the case of obversion, the conclusion of the argument is always logically equivalent to the premise, so a diagram of the premise is bound to contain the essential content of the conclusion.

The above diagram raises the issue of dealing with negative terms, such as "non-Zimbabweans." To avoid certain complications, we will *not* label the circles in our diagrams with negative terms. But we will be forced at times (as in the above case) to understand what sort of diagram is or would be needed where a negative term is involved. The essential question is, "In order for the statement to be true, where would the diagram need to be shaded or marked with an 'x'?" Sometimes it helps to experiment with a separate diagram on a sheet of scratch paper—do some shading or "x-ing" and ask, "Does that tell me that the statement is true?" A good working knowledge of obversion and contraposition is also helpful since it will help you identify statements that are (and are not) logically equivalent to the one containing a negative term. Let's consider some cases.

Here is an example of contraposition:

14. 1. All Ugandans are Africans.
So, 2. All non-Africans are non-Ugandans.

The diagram of the premise looks like this:

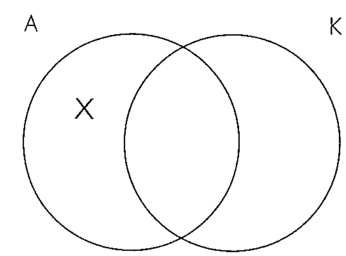


An A statement and its contrapositive are logically equivalent; so the diagram of the premise is bound to contain the essential content of the conclusion, but the term-complements ("non-Africans" and "non-Ugandans") may make this less than obvious. To understand the diagram, note that "All non-Africans are non-Ugandans" says that if anything is outside of the circle labeled "Africans," then it is outside the circle labeled "Ugandans." And the diagram does indeed contain this information.

Here's another example of contraposition:

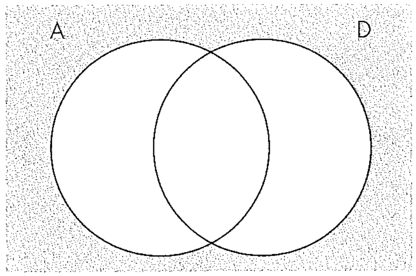
15. 1. Some Africans are non-Kenyans.
So, 2. Some Kenyans are non-Africans.

The diagram of the premise is as follows:



Does the diagram tell us that the conclusion is true? No, for there is no "x" in area 3, the area that stands for Kenyans who are non-Africans.

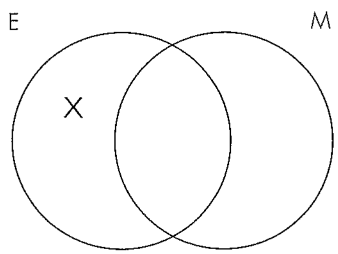
Now consider an especially difficult case involving negative terms, "No nonanimals are nondogs." (If you recall the table on contraposition, you know that this is *not* logically equivalent to "No dogs are animals.") To diagram "No nonanimals are nondogs," we have to shade in the area of overlap between "nonanimals" and "nondogs." In a diagram in which the circles are labeled "animals" and "dogs," the area of overlap is the area *outside* of the circles, so the diagram looks like this:



In closing this section, let's examine contradictories from the standpoint of the Venn method. Recall that if two statements are contradictories, then if one is true, the other must be false (and vice versa). Consider the following argument:

16. 1. Some Egyptians are not Moslems.
 So, 2. It's false that all Egyptians are Moslems.

The premise may be diagrammed as follows:



Does the diagram tell us that the conclusion of the argument is true? Yes. A diagram of "All Egyptians are Moslems" would declare area 1 of the diagram empty, but the "x" in area 1 tells us it isn't empty. Thus, given the content of the premise, it's false that all Egyptians are Moslems. Accordingly, the Venn method nicely confirms the Aristotelian view of contradictories.

The following exercise will give you some practice in using the Venn method.

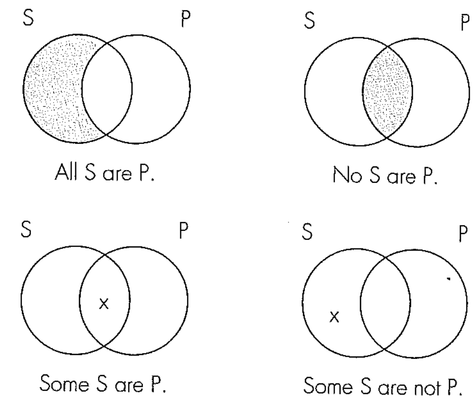
Exercise 6.2

Part A: Venn Diagrams and Standard Form Put the following categorical statements into standard form, then construct a Venn diagram for each of them. Label the left circle of the diagram with an abbreviation of the subject term and the right circle with an abbreviation of the predicate term.

* 1. At least one ancient philosopher believed in the unreality of change.

2. Not every act of killing is an act of murder.

Summary of Diagrams of Categorical Statements



3. If anything is a divine being, then it is not limited.

* 4. Only tax-dodgers deserve harsh treatment from the IRS.

5. At least one current musical hit will not be a hit next year.

6. Only people who believe in life after death are people who believe in reincarnation.

* 7. A thing is a chlorofluorocarbon only if it is not good for the ozone layer.

8. At least one corporation is cheating the government.

9. Only violent offenders should be incarcerated.

* 10. Nothing is a physical entity unless it is not spiritual.

Part B: Venn Diagrams and Arguments Draw two Venn diagrams for each of the following arguments, one for the premise and one for the conclusion. First draw a Venn diagram of the premise, labeling the left circle with an abbreviation of the subject term and the right circle with an abbreviation of the predicate term. Second, draw a Venn diagram for the conclusion, but label the circles as before (with the left circle labeled by an abbreviation of the subject term of the *premise* and the right circle labeled with an abbreviation of the predicate term of the *premise*). Finally, indicate whether the argument is valid.

* 1. Some chairs are not thrones. So, some thrones are not chairs.

2. All scallops are mollusks. Hence, no scallops are nonmollusks.

3. All minds are brains. Therefore, all nonbrains are nonminds.

* 4. Some married persons are persons who have attachment disorders. Thus, some persons who have attachment disorders are married persons.

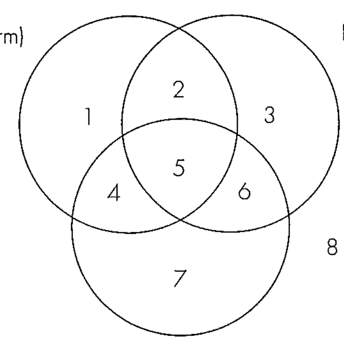
5. No laypersons are priests. It follows that all laypersons are nonpriests.

6. Some political philosophers are egalitarians. Accordingly, some non-egalitarians are things that are not political philosophers.
* 7. No elephants are beetles. Consequently, no nonbeetles are nonelephants.
8. No Pickwickian interpretations are obvious interpretations. So, no obvious interpretations are Pickwickian interpretations.
9. Some rays are devilfish. Hence, some devilfish are not nonrays.
* 10. Some wines are not merlots. Therefore, some nonmerlots are not nonwines.
11. All acts of torture are immoral acts. It follows that all immoral acts are acts of torture.
12. Some vipers are not copperheads. Consequently, some vipers are noncopperheads.
* 13. Some mammals are edentulous animals. Thus, all mammals are edentulous animals.
14. Some boring events are colloquia. Accordingly, some boring events are not colloquia.
15. Some theists are not predestinarians. Therefore, no theists are predestinarians.

6.3 Venn Diagrams and Categorical Syllogisms

The Venn method can be applied to categorical syllogisms. In fact, because the Venn method gives us a visual representation of the logic, many find it an especially insightful and intuitive way of testing syllogisms for validity.

To apply the Venn method to a categorical syllogism, we first check to see if the syllogism is in standard form. If it is, we can proceed immediately to construct a diagram. If the syllogism is not in standard form, we rewrite it so that it is. Next, since there are three terms in every categorical syllogism, with each term denoting a class, we need a diagram with three overlapping circles to represent the various possible relationships among the classes, as shown here:

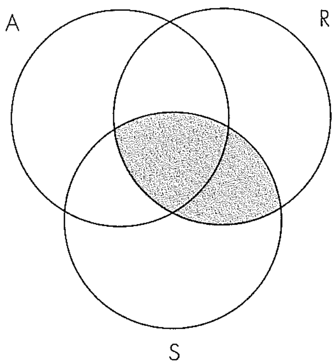


To ensure uniformity in our diagrams, we always let the circle in the middle stand for the class of things denoted by the *middle term* of the syllogism (the term that appears in both premises). To cut down on writing, we will ordinarily label this circle with an abbreviation of the middle term. The circle in the upper left stands for the class of things denoted by the *minor term* (the subject term of the conclusion). To cut down on writing, we will ordinarily label this circle with an abbreviation of the minor term. And the circle in the upper right stands for the set of things denoted by the *major term* (the predicate term of the conclusion). To cut down on writing, we will ordinarily label this circle with an abbreviation of the major term. The numerals (1 through 8) are not normally part of a Venn diagram, but they are added here temporarily to enable us to refer to the separate areas of the diagram. Notice that there are eight areas (counting the region outside the circles). Each area represents a possible relationship among the three sets or classes. For example, if we placed an "x" in area 5, we would be saying that at least one thing belongs to all three of the sets or classes. If we shaded in area 5, we would be saying that no object belongs to all three sets. If we placed an "x" in area 8, we would be saying that at least one thing is not a member of any of the three classes in question. If we shaded in areas 4 and 5, we would be saying that nothing that belongs to the set denoted by the middle term also belongs to the set denoted by the minor term.

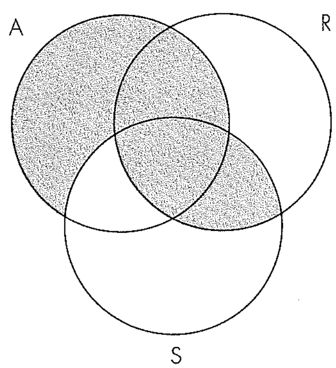
To determine whether a syllogism is valid, we proceed as follows. First, we diagram the premises. Second, we look to see whether our diagram of the premises tells us that the conclusion is true. This works because if an argument is valid, the content of the conclusion is contained, at least implicitly, in the premises. Accordingly, we do not diagram the conclusion itself on these three circles (for doing so could easily confuse us about exactly what information was provided by the premises). Consider the following example:

17. 1. No rocks are sentient things.
2. All animals are sentient things.
So, 3. No animals are rocks.

We set up our diagram and label the circles as just prescribed: An abbreviation of the middle term ("S" for "sentient things") labels the middle circle; an abbreviation of the minor term ("A" for "animals") labels the circle in the upper left; and an abbreviation of the major term ("R" for "rocks") labels the circle in the upper right. Next, we diagram the first premise:



In diagramming the first premise, we focus on the two circles representing rocks and sentient things since only those classes are mentioned in the first premise. Next, we diagram the second premise:



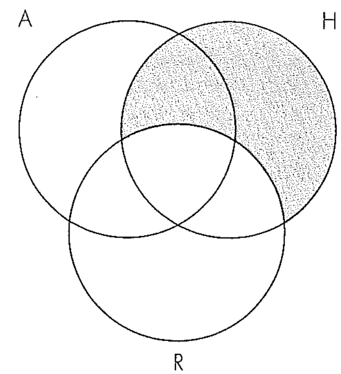
In diagramming the second premise, we need pay attention only to the circles representing animals and sentient things.

Having diagrammed the premises, we must now check to see whether the content of the conclusion has also been diagrammed in the process. In other words, does our diagram tell us that no animal is a rock? Yes, it does, for the areas of overlap between the circles representing these two classes are shaded in. Therefore, the argument is valid.

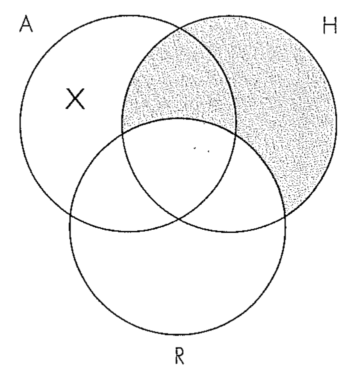
Our first example involved only universal statements. Let us now consider a syllogism involving a particular negative statement:

18. 1. All humans are rational beings.
2. Some animals are not rational beings.
So, 3. Some animals are not humans.

First, we set up our diagram, label the three circles (as prescribed previously), and diagram the first premise:



Once again, notice that we focus on only two circles at a time as we diagram the premises. Next, we diagram the second premise:



The "x" lies within the A-circle (which represents the class of animals) but outside the R-circle (which represents the class of rational beings). Why did we put our "x" in area 1 and not in area 2 of the diagram? We couldn't put it in area 2 because area 2 is shaded in. That is, the information in the first premise tells us that area 2 is empty. Thus, if we put an "x" in area 2, we would in effect be saying that the premises of the argument are logically inconsistent—that is, that the first premise says area 2 is empty, while the second premise says area 2 is not empty. Clearly, this would misrepresent the content of the two premises because they are consistent.

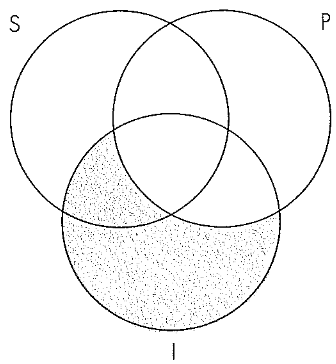
Now, we examine the diagram to see whether, in the process of diagramming the premises, we have diagrammed the content of the conclusion. Does the diagram tell us that some animals are not humans? Yes. The "x" lies within the animal-circle but outside the human-circle.

In diagramming argument (18), we diagrammed the universal premise before we diagrammed the particular premise. *When a syllogism contains both universal and particular premises, always diagram the universal premise first.* Otherwise, you may run into obstacles in constructing your diagram. To illustrate, try to diagram the particular premise of argument (18) prior to diagramming the universal premise. (You won't know whether to put the "x" in area 1 or area 2.)

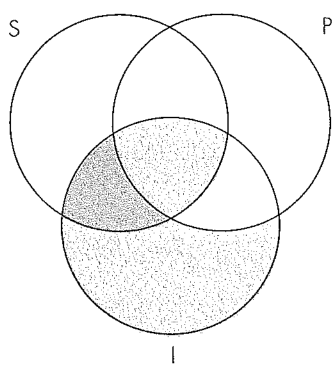
How does the Venn diagram method apply to invalid arguments? Consider the following syllogism:

19. 1. All immoral persons are psychologically disturbed persons.
2. No saints are immoral persons.
So, 3. No saints are psychologically disturbed persons.

We draw and label the circles and then diagram the first premise:



Next, we diagram the second premise:



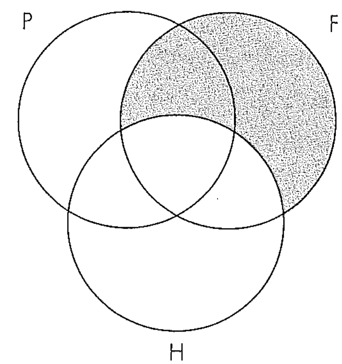
Notice that area 4 has been shaded in twice, since the diagram for each premise declares area 4 empty. This is not a problem; it merely means that our diagram is redundant as regards the emptiness of area 4.

Now that the premises are diagrammed, the crucial question is this: Does our diagram tell us that the conclusion of the argument is true? In other words, does it tell us that no saints are psychologically disturbed? The answer, of course, is that it does not. Area 2 has not been declared empty. Thus, the diagram leaves open the possibility that some saints are psychologically disturbed persons. This means that the premises do not guarantee the truth of the conclusion, and so the argument is invalid.

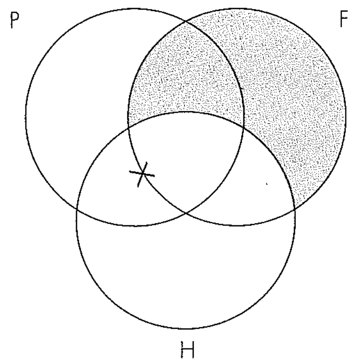
Let us now consider an example that brings out a slight complication in the Venn technique.

20. 1. All famous actors are highly successful people.
2. Some highly successful people are people of average intelligence.
So, 3. Some people of average intelligence are famous actors.

We diagram the universal premise first:



Now, when we try to diagram the second (or particular affirmative) premise, we see that the "x" could go in either area 4 or area 5. The premises do not contain more specific information than that. We indicate this by putting an "x" *precisely* on the line separating the two areas, like this:

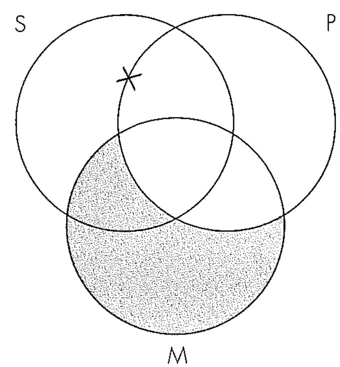


Now, for the argument to be valid, the premises must tell us that either area 2 or area 5 contains an object. But our diagram for the first premise declares that area 2 is empty. And our diagram for the second premise does not *assure* us that area 5 contains an object—it may or it may not. The "x" straddles areas 4 and 5, so the premises do not definitely say that the "x" belongs in area 4, nor do they say that the "x" belongs in area 5. Hence, the argument is invalid.

Obviously, the Venn method can be used to test argument forms as well as arguments. Can you construct a Venn diagram for the following form?

21. 1. All M are P.
2. Some S are not M.
So, 3. Some S are not P.

The diagram looks like this:



Note that the "x" straddles areas 1 and 2. The premises tell us that there is an "S" in at least one of these areas, but the premises do not tell us to place an "x" in area 1. And the argument form is valid *only if* the premises tell us to place an "x" within the S-circle but outside the P-circle. So, the argument form is not valid.

The following exercise provides you with an opportunity to use the Venn method on argument forms and categorical syllogisms.

Exercise 6.3

Part A: Argument Forms Construct Venn diagrams to determine whether the following argument forms are valid. In labeling the circles of your diagrams, remember that "M" labels the circle in the middle, "S" labels the circle in the upper left, and "P" labels the circle in the upper right.

* 1. 1. All M are P.
2. Some M are not S.
So, 3. Some S are not P.

2. 1. No P are M.
2. Some M are not S.
So, 3. Some S are P.

3. 1. No M are P.
2. Some M are S.
So, 3. Some S are not P.

* 4. 1. Some P are M.
2. Some S are M.
So, 3. Some S are P.

5. 1. All P are M.
2. Some M are not S.
So, 3. Some S are not P.

6. 1. All P are M.
2. Some S are not M.
So, 3. Some S are not P.

* 7. 1. No P are M.
2. Some M are S.
So, 3. Some S are P.

8. 1. All P are M.
2. No S are M.
So, 3. No S are P.

9. 1. No P are M.
2. No S are M.
So, 3. No S are P.

* 10. 1. All M are P.
2. No S are M.
So, 3. No S are P.

Part B: Categorical Syllogisms Use Venn diagrams to determine the validity of the following categorical syllogisms. If a given syllogism is not in standard form, be sure to put it into standard form before constructing your diagram. In labeling the circles of your diagrams, use an abbreviation of the *middle term* to label the circle in the middle, use an abbreviation of the *minor term* to label the circle in the upper left, and use an abbreviation for the *major term* to label the circle in the upper right.

* 1. Only Greeks are Athenians. At least one human is not an Athenian. Therefore, not all humans are Greeks.

2. Every animal is sentient. And each sentient thing is a rights-holder. Hence, if anything is an animal, then it is a rights-holder.

3. No serial killer is good because every serial killer is evil, and no evil thing is good.

* 4. Every wicked person is self-deceived, for all liars are wicked, and every liar is self-deceived.

5. Every person without a conscience is happy; hence, at least one criminal is happy since not every criminal has a conscience.

6. All those who have faith are virtuous. But there are highly moral people who do not have faith. Therefore, not all highly moral people are virtuous.

* 7. No human is omniscient. Something is both divine and human. So, at least one divine being is not omniscient.

8. Only wars are great evils. Some wars are ordained by God. Hence, at least one great evil is ordained by God.

9. Not every hobby is worth doing well. But anything worth doing is worth doing well. Therefore, at least one hobby is not worth doing.

*10. If anything is a mental event, then it is not a brain event. For only physical events are brain events, and no mental events are physical.

11. Some philosophical views are not worth considering. Every philosophical view has been held by a genius. Thus, some views that have been held by geniuses are not worth considering.

12. No wicked person is utterly without a conscience. But all wicked persons are deeply confused individuals. Hence, no deeply confused individual is utterly without a conscience.

*13. Only metaphorical statements are similarity statements. And every statement is a similarity statement. Accordingly, a thing is a statement only if it is metaphorical.

14. Contrary to what traditional Western morality says, some acts of suicide are morally permissible. For all morally permissible acts are ones that conform to the categorical imperative, and some acts of suicide conform to the categorical imperative.

15. Only acts that maximize utility are obligatory. Not all acts that maximize utility are prescribed by the Ten Commandments. Therefore, at least one act prescribed by the Ten Commandments is not obligatory.

*16. Not every act is free, since every act foreknown by God is nonfree and some acts are foreknown by God.

17. Only acts approved of by God are moral. Some acts of killing are approved of by God. Hence, some acts of killing are moral.

18. No human is omnipotent. All divine beings are omnipotent. Therefore, no human is divine.

*19. Only persons who have inner conflicts are unhappy. At least one successful comedian is unhappy. We may conclude that some successful comedians are persons who have inner conflicts.

20. At least one tycoon is a person who has walked over others to get to the top. Every person who has walked over others to get to the top is evil. It follows that at least one tycoon is evil.

21. Some trees are maples. Some trees are oaks. So, some oaks are maples.

22. No balalaika is a banjo. Some balalaikas are beautiful. Hence, some beautiful things are not banjos.

23. Each tyrant is mendacious. If anything is a tyrant, then it is a liar. Consequently, all liars are mendacious.

24. Every aphorism is an apothegm. Each epigram is an aphorism. Accordingly, only apothegms are epigrams.

25. Every Saint Bernard is a large dog. Not all large dogs are brown. So, not all brown dogs are Saint Bernards.

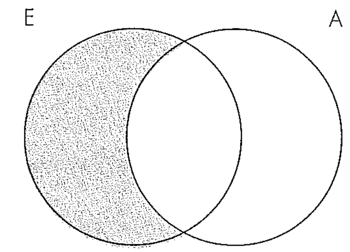
6.4 The Modern Square of Opposition

We now come to some points of disagreement between the Aristotelian logical tradition and modern logic. By "modern logic" we here refer to the tradition of logic developed by 19th- and early 20th-century thinkers such as George Boole (1815–1864), John Venn (1834–1923), Charles Sanders Peirce (1839–1914), Gottlob Frege (1848–1925), and Bertrand Russell (1872–1970).

Consider the following argument:

22. 1. All Egyptians are Africans.
So, 2. Some Egyptians are Africans.

The diagram of the premise looks like this:



Does the diagram tell us the conclusion is true? No. An "x" would need to appear in the area of overlap between the two circles in order for the conclusion to be true. Note that argument (22) is an example of subalternation. Thus, our diagram in effect tells us that the Venn method (as presented in the two previous sections) does not confirm the Aristotelian view of subalternation. What's going on?

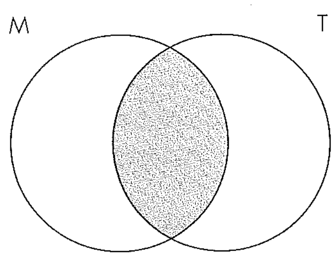
The Aristotelian and modern traditions agree that particular statements have existential import. Categorical statements have **existential import** if (and only if) they imply that their subject terms denote nonempty classes. For example, "Some Egyptians are Africans" implies that there *exists* at least one Egyptian (that is, the class of Egyptians has at least one member and so is not empty). Similarly, "Some Rwandans are not marathoners" implies that there *exists* at least one Rwandan. And since the Venn diagrams for particular statements require an "x" within the circle labeled by the subject term, the Venn method has built into it the view that particular statements have existential import.

But modern logicians understand universal statements to have a conditional (if-then) aspect. From this perspective, "All Egyptians are Africans" is equivalent to "If anything is an Egyptian, then it is an African." So understood, "All Egyptians are Africans" does not imply that there are any Egyptians. After all, "If anything is a unicorn, then it is an animal" is surely a true statement, but it does not imply that there actually are any unicorns. (If it did imply that there are unicorns, then it wouldn't be true.) Similarly, "No Namibians are Libyans" is equivalent to "If anything is a Namibian, then it is not a Libyan," and so understood, it does not imply that there are any Namibians. This modern understanding of universal statements is built into the Venn method as here presented. And this seemingly small point about universal statements has a series of ramifications for the Traditional Square of Opposition.

For example, consider the Aristotelian thesis that corresponding **A** and **E** statements are contraries (that is, they cannot both be true but can both be false). If the Aristotelians are right, the following argument is valid:

23. 1. No Malawians are Tanzanians.
So, 2. It is false that all Malawians are Tanzanians.

A Venn diagram of the premise is as follows:

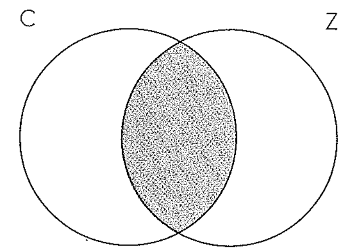


Does the diagram tell us that the conclusion is true? No. Think about it like this. First, mentally delete "It is false that" in the conclusion, so you have "All Malawians are Tanzanians," which would require shading in area 1, that is, declaring area 1 empty. Now, to make "All Malawians are Tanzanians" false, we need to do the opposite, that is, place an "x" in area 1 (thus declaring that some Malawians are not Tanzanians). So, the diagram tells us the conclusion is true only if an "x" appears in area 1, but no such "x" appears. Hence, the Venn method does not preserve the Aristotelian view that corresponding **A** and **E** statements are contraries.

What about subcontraries? Logicians in the Aristotelian tradition claim that corresponding **I** and **O** statements are subcontraries (that is, they can both be true but cannot both be false). If this Aristotelian claim about subcontraries is correct, the following argument is valid:

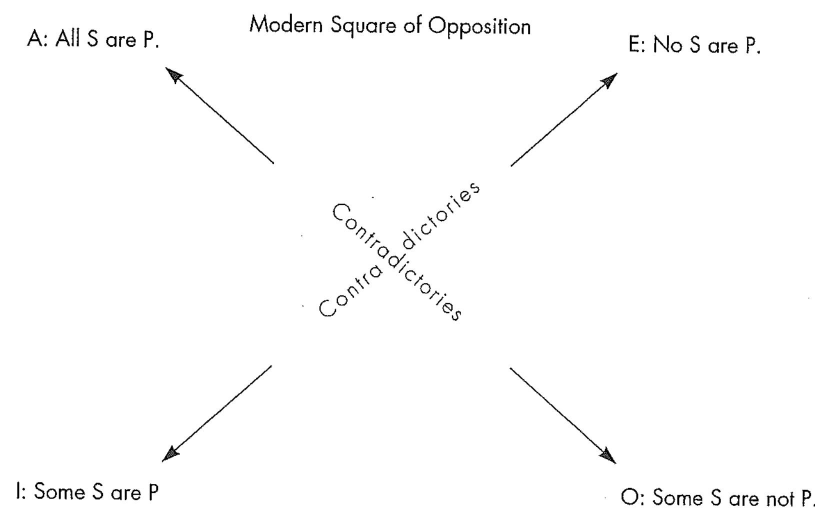
24. 1. It is false that some Chadians are Zambians.
So, 2. Some Chadians are not Zambians.

Now the premise here is equivalent to "It's false that something is both a Chadian and a Zambian," which requires us to shade in the area of overlap. (Think about it like this: "Some Chadians are Zambians" would require an "x" in the area of overlap, so to make that statement false, we do the opposite: shade the area in.) The Venn diagram of the premise looks like this:



Does the diagram tell us that the conclusion is true? No, for there is no "x" in area 1. So, the Venn method does not preserve the Aristotelian view that corresponding **I** and **O** statements are subcontraries.

At this point, what's left of the Traditional Square of Opposition? Just the theses regarding contradictories, namely, *corresponding* **A** *and* **O** *statements are contradictories, and corresponding* **E** *and* **I** *statements are contradictories*. This is sometimes called the Modern Square of Opposition and may be represented pictorially as follows:



You may wish to stop a moment and compare this to the Traditional Square of Opposition discussed in section 5.2. The relationships along the sides of the Traditional Square have vanished (subalternation, contraries, subcontraries); only the diagonal relationships are retained (the contradictories).

The modern approach has at least one advantage over the Aristotelian approach that deserves special mention here. Consider the following pairs of statements:

25. All unicorns are animals. Some unicorns are not animals.
26. All ideal societies are perfectly just. Some ideal societies are not perfectly just.
27. No perfect vacuums are spaces through which sound can be transmitted. Some perfect vacuums are spaces through which sound can be transmitted.

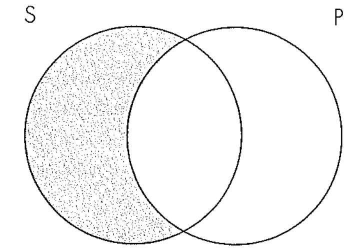
Are these pairs contradictories? Oddly enough, they are not if we take an Aristotelian approach. For in each case, the subject term refers to an empty class. (Unicorns do not exist, and there are no ideal societies or perfect vacuums in existence.) It follows that the above pairs are not contradictories given the Aristotelian approach. For according to Aristotelians, all of the above statements have existential import. Thus, "All unicorns are animals" implies that at least one unicorn exists. And "Some unicorns are not animals" also implies that at least one unicorn exists. But any statement that implies a falsehood is itself false. Therefore, both "All unicorns are animals" and "Some unicorns are not animals" must be declared false by Aristotelians. But if two statements are contradictories, then if one is false, the other must be true. So, Aristotelians cannot preserve the thesis that corresponding **A** and **O** statements are always contradictories. And the same goes for corresponding **E** and **I** statements.

How does the modern approach differ? From this perspective, "All unicorns are animals" is equivalent to saying, "If anything is a unicorn, then it is an animal." This if-then statement seems to be true, and it doesn't imply that unicorns exist. But "Some unicorns are not animals" does imply that at least one unicorn exists, and so it is false. In this way, the modern approach preserves the thesis that corresponding **A** and **O** statements are contradictories *even when their subject terms refer to empty classes*. Similarly, if there are no unicorns, then "No unicorns are animals" is true, while "Some unicorns are animals" is false. So the modern approach also preserves the thesis that corresponding **E** and **I** statements are contradictories, *even when their subject terms refer to empty classes*.

Note that the differing understanding of universal statements affects the evaluation of conversion by limitation and contraposition by limitation. Consider conversion by limitation, which has this form:

28. 1. All S are P.
So, 2. Some P are S.

A diagram of the premise looks like this:

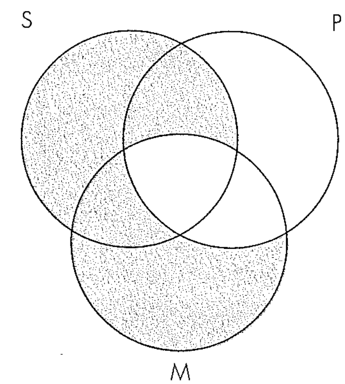


Does the diagram tell us that the conclusion is true? No, for there is no "x" in the area of overlap between the two circles. So, the Venn method tells us that conversion by limitation is not valid.

The differing understanding of universal statements also affects the assessment of certain syllogistic forms. In fact, nine syllogistic forms judged valid by Aristotelians are judged invalid via the modern approach. In every case, these syllogistic forms move from two universal premises to a particular conclusion. As an example, consider syllogisms in the first figure with the mood **AAI**:

29. 1. All M are P (e.g., all perfect spouses are spouses).
2. All S are M (e.g., all perfect husbands are perfect spouses).
So, 3. Some S are P (e.g., some perfect husbands are spouses).

A Venn diagram for (29) looks like this:

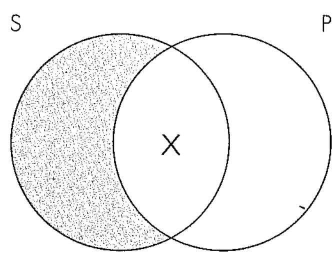


Does the diagram tell us that the conclusion is true? No, for there is no "x" in the area of overlap between the S-circle and the P-circle. Indeed, given the modern understanding of universal statements as having a conditional (if-then) aspect, no categorical syllogism with two universal premises and a particular conclusion is valid.

These disagreements between the Aristotelian and modern traditions in logic may seem a bit troubling. But note that the disagreement has its source in the very specific issue of whether universal categorical statements have existential import. Let us suppose that the modern logicians are correct in claiming that universal categorical statements do not have existential import. Still, in many actual cases, if an arguer appears to move from a universal categorical statement to a particular one, we can reasonably assume that the arguer has not stated all of his or her premises explicitly. (For practical purposes, it is often unnecessary to state every premise of an argument.) For example, the move from "All politicians are liars" to "Some politicians are liars" is valid if we add the premise "At least one politician exists." And in many cases, when someone asserts a premise of the form "All S are P," he or she is reasonably assuming that *there are some Ss*. Consider the following argument form:

30. 1. All S are P.
 2. At least one S exists.
 So, 3. Some S are P.

The second premise makes explicit the assumption that the class denoted by the subject term of the first premise is not empty. A diagram of the premises looks like this:



The second premise tells us we need an "x" within the S-circle, but the "x" can go only in the area of overlap between the two circles given our diagram of the first premise. (Here as elsewhere it is important to diagram universal premises before diagramming particular ones.) Thus, the diagram of the premises tells us that the conclusion is true and the argument is valid. By making unstated premises explicit, we can thus accept the modern view of universal statements while recognizing an insight in the traditional Aristotelian view. In this case, an inference akin to superalternation ("All S are P, so some S are P") is valid if we add that there are *Ss* (i.e., if we add that the subject term of "All S are P" denotes a nonempty class).

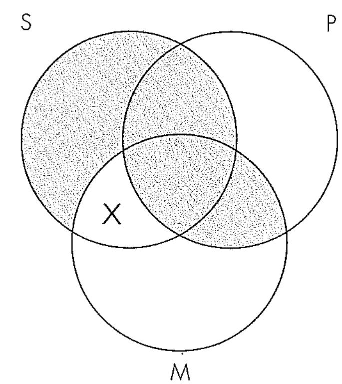
A similar point can be made about the syllogistic forms that Aristotelians regard as valid but modern logicians reject as invalid. For example, consider syllogisms in the second figure with **EAO** as their mood:

31. 1. No P are M.
 2. All S are M.
 So, 3. Some S are not P.

Here we need only add a premise to the effect that the subject term of the second premise denotes a nonempty class:

32. 1. No P are M.
 2. All S are M.
 3. At least one S exists.
 So, 4. Some S are not P.

Of course, (32) does not have the form of a categorical syllogism since it has three premises instead of two, but we can easily diagram it, as long as we remember to diagram the universal premises first:



Premise 3 tells us to place an "x" within the S-circle, and the "x" can go only in area 4 given our diagrams for the universal premises. Clearly the diagram of the premises tells us that the conclusion is true, so argument form (32) is valid. Again, the point is that by adding unstated premises when it is reasonable to do so, we can in effect honor insights from both the Aristotelian and the modern traditions. Incidentally, arguments with unstated premises or conclusions are called *enthymemes*, and we shall take a closer look at them in the next section.

The following exercise gives you an opportunity to apply the concepts discussed in this section.

Exercise 6.4

Part A: Argument Forms Use Venn diagrams to test the following argument forms for validity. (Do not supply unstated premises.)

* 1. 1. No M are P.
2. All S are M.
So, 3. Some S are not P.

2. 1. All P are M.
2. No S are M.
So, 3. Some S are not P.

3. 1. All M are P.
2. All M are S.
So, 3. Some S are P.

* 4. 1. All M are P.
2. All M are S.
3. At least one M exists.
So, 4. Some S are P.

5. 1. No M are P.
2. All M are S.
So, 3. Some S are not P.

6. 1. No M are P.
2. All M are S.
3. At least one M exists.
So, 4. Some S are not P.

* 7. 1. No S are P.
So, 2. Some non-P are not non-S.

8. 1. All P are M.
2. No M are S.
So, 3. Some S are not P.

9. 1. All P are M.
2. All M are S.
So, 3. Some S are P.

* 10. 1. No S are P.
2. At least one S exists.
So, 3. Some non-P are not non-S.

Part B: Testing Arguments Use Venn diagrams to test the following arguments for validity. (Do not supply unstated premises.)

* 1. 1. All people who never make mistakes are admirable people.
2. All ideal humans are people who never make mistakes.
So, 3. Some ideal humans are admirable people.

2. 1. No plants are animals.
2. All weeds are plants.
So, 3. Some weeds are not animals.

3. 1. No perfect circles are perfect squares.
2. Some perfect circles are objects of beauty.
So, 3. Some objects of beauty are not perfect squares.

* 4. 1. All persons who advocate the use of overwhelming nuclear force are persons who lack moral sensibility.
2. All persons who advocate the use of overwhelming nuclear force are persons who should not serve as world leaders.
3. At least one person who advocates the use of overwhelming nuclear force exists.
So, 4. Some persons who should not serve as world leaders are persons who lack moral sensibility.

5. 1. All lions are cats.
2. All cats are mammals.
So, 3. Some mammals are lions.

6. 1. No terrorists who use nuclear weapons are good people.
2. All terrorists who use nuclear weapons are people who mean well.
3. At least one terrorist who uses nuclear weapons exists.
So, 4. Some people who mean well are not good people.

* 7. 1. All sycophants are flatterers.
2. All flatterers are disgusting persons.
3. At least one flatterer exists.
So, 4. Some digusting persons are sycophants.

8. 1. Some highly educated people are sybarites.
2. All sybarites are poor role models.
So, 3. Some poor role models are highly educated people.

9. 1. No oaks are elms.
2. All oaks are trees.
3. At least one elm exists.
So, 4. Some trees are not elms.

* 10. 1. No members of the IRA are members of the IRS.
So, 2. It is false that all members of the IRA are members of the IRS.

11. 1. All great inventors are slightly odd people.
2. At least one great inventor exists.
So, 3. Some slightly odd people are great inventors.

12. 1. No anarchists are Republicans.
2. At least one anarchist exists.
So, 3. Some non-Republicans are not nonanarchists.

* 13. 1. All scarlet things are red things.
So, 2. It is false that no scarlet things are red things.

14. 1. No cities are nations.
2. At least one city exists.
So, 3. Some cities are not nations.

15. 1. All people with perfect memories are people who remember everything.
So, 2. Some people with perfect memories are people who remember everything.

* 16. 1. It is false that some Germans are Zoroastrians.
So, 2. Some Germans are not Zoroastrians.

17. 1. It is false that some vampires are living things.
2. At least one vampire exists.
So, 3. Some vampires are not living things.

18. 1. All people who do not care about social justice are heartless people.
So, 2. It is false that no people who do not care about social injustice are heartless people.

* 19. 1. No kangaroos are karate experts.
So, 2. Some kangaroos are not karate experts.

20. 1. It is false that some tyrants are not humans.
2. At least one tyrant exists.
So, 3. Some tyrants are humans.

6.5 Enthymemes

An **enthymeme** is an argument with an unstated premise or an unstated conclusion. If we use the more general term "step" to refer to premises and/or conclusions, we can say that an enthymeme is an argument with one or more unstated steps. Unstated steps are also referred to as *missing* or *implicit* steps.

Enthymemes are common both in ordinary discourse and in academic writing. Why? Because many statements are presumed to be known to one's audience, and so it may be unnecessary to make these statements explicit. When evaluating an enthymeme, fairness and charity demand that we fill in the missing step(s). Here's an example:

33. All Mozambicans are Africans. Hence, no Mozambicans are Asians.

A Venn diagram will show that (33), taken as it stands, is invalid. But obviously there is an unstated premise here, "No Africans are Asians" (or its converse, "No Asians are Africans"). And if we add this statement (or its converse) to the argument, we get a valid categorical syllogism:

34. 1. No Africans are Asians.
2. All Mozambicans are Africans.
So, 3. No Mozambicans are Asians.

Any kind of argument can have an unstated step, but we will here focus on categorical syllogisms. When we fill in missing steps in an argument, we must adhere to the principles of fairness and charity. This means that, to the extent possible, added steps should be true (or at least plausible) and should make the argument valid.

In order to evaluate an enthymematic categorical syllogism for validity, we proceed in three stages. First, identify the missing step. The missing step may be a major premise, a minor premise, or the conclusion. Second, put the syllogism into standard form. Third, apply the Venn method.

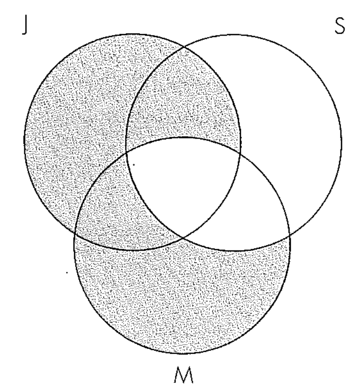
How do we identify the missing step? The missing step will contain two terms that have been used only once each in the enthymeme. If the missing step is a premise, the form (**A**, **E**, **I**, or **O**) and order of the terms should be chosen with an eye toward the information needed to arrive at the conclusion. If the missing step is a conclusion, the form and order of terms should be chosen with an eye toward what the information in the premises would show to be true. For example, consider the following argument:

35. If anything is a moral judgment, then it is a subjective opinion. It follows that judgments about the wrongness of theft are subjective opinions.

The missing step here is a minor premise: "All judgments about the wrongness of theft are moral judgments." Putting the syllogism into standard form, we get:

36. 1. All moral judgments are subjective opinions.
2. All judgments about the wrongness of theft are moral judgments.
So, 3. All judgments about the wrongness of theft are subjective opinions.

And now we can readily apply the Venn method:



The diagram indicates that the argument is valid. The premise we have added seems to be true, but of course the first premise of the argument is highly controversial.

When adding steps to an enthymeme, it is not always possible to add a statement that is true (or plausible) and that makes the argument valid. Consider the following example:

37. All Ohioans are Americans. Hence, no Ohioans are socialists.

The missing step here is presumably "No Americans are socialists" (or its converse). Putting the syllogism into standard form, we get:

38. 1. No Americans are socialists.
2. All Ohioans are Americans.
So, 3. No Ohioans are socialists.

Premise (1) is needed to make the argument valid, but (1) is false, since some Americans are socialists. When a false or doubtful premise is needed to make an argument valid, we bring out an important weakness in the argument by making the premise explicit, for in doing so we have identified an important reason for doubting the soundness of the argument.

In some cases, the context forces us to make a hard choice between adding a blatantly false premise and adding a premise that makes the argument invalid. For example, what is the missing step in the following argument?

39. All Lutherans are Christians. Therefore, all Protestants are Christians.

The missing step seems to be either "All Protestants are Lutherans" or "All Lutherans are Protestants." If we add "All Protestants are Lutherans" to the argument, then it is valid but has a blatantly false premise since not all Protestants are Lutherans (for example, Presbyterians and Baptists are Protestants). If we add "All Lutherans are Protestants" to the argument, then we add a true premise but one that makes the argument invalid. Obviously, the argument is deeply flawed. It is up to us to choose whether to present that flaw as a flaw in the logical structure or as a false premise. When faced with this sort of choice, let us adopt the practice of adding a step that makes the argument valid, leaving the truth value of the added step as a matter for discussion. So, in standard form, argument (39) looks like this:

40. 1. All Lutherans are Christians.
2. All Protestants are Lutherans.
So, 3. All Protestants are Christians.

Sometimes the conclusion of an argument is left unstated.

41. Every politician who wants to win slings mud, and every politician wants to win!

When we put the argument into standard form, we make the conclusion explicit:

42. 1. All politicians who want to win are mudslingers.
2. All politicians are politicians who want to win.
So, 3. All politicians are mudslingers.

We add "All politicians are mudslingers" instead of "All mudslingers are politicians" because the former (unlike the latter) is guaranteed true by a diagram of the premises.

The following exercise will give you some practice in dealing with enthymemes.

Exercise 6.5

Part A: Enthymemes Identify the missing step in each of the following arguments. Then put the argument into standard form. Finally, use a Venn diagram to check the argument for validity.

* 1. No certainty should be rejected. So, no self-evident propositions should be rejected.

2. Every virtue is beneficial. Therefore, no vice is a virtue.

3. Only rational animals are humans. It follows that no ducks are humans.

* 4. Atoms are indestructible because every simple substance is indestructible.

5. Every envious person wants others to fail. Consequently, no good person is envious.

6. Nothing is a liar unless it is not praiseworthy. At least one liar is a human being. Draw your own conclusion!

* 7. Only scientific statements are rational. It follows that aesthetic judgments are never rational.

8. All verdicts rendered in courts of law are relative since all value judgments are relative.

9. Perfect beings have every virtue. Therefore, at least one god is not a perfect being.

* 10. Some beliefs about aliens are not rational, for all rational beliefs are proportioned to the available evidence.

11. No matter of faith is provable. At least one belief about life after death is a matter of faith. Draw your own conclusion!
12. No bears are wolves, so some grizzlies are not wolves.
*13. Every vice is harmful. Accordingly, every vice is a form of laziness.
14. Every composite substance is a substance that has parts. Hence, no soul is a composite substance.
15. Every evil thing is to be avoided. But at least one evil thing is pleasurable. Draw your own conclusion.

Part B: More Enthymemes The following enthymemes are paraphrased or slightly altered versions of arguments found in the writings of the philosopher Gottfried Wilhelm Leibniz. (These are arguments Leibniz *discusses;* some he endorses and some he does not.) Identify the missing step in each of the arguments. Then put the argument into standard form. Finally, use a Venn diagram to check the argument for validity.

*1. Every event that is foreseen by God is predetermined. Consequently, every event is predetermined.
2. Every event that is predetermined is necessary. Every event is predetermined. Draw your own conclusion!
3. Every event is necessary. Hence, every sin is necessary.
*4. No necessary event can be avoided. Only events that can be avoided are justly punished. Therefore, . . .
5. Every being that is omnipotent and perfectly good creates the best of all possible worlds. Hence, some deity creates the best of all possible worlds.
6. All composites are aggregates of simple substances. At least one composite exists. Draw your own conclusion.
*7. Nothing that has no parts can come apart. So, nothing that has no parts can be annihilated.
8. If anything has the power to help everyone but helps only some, then it is not fair. Thus, some deity is not fair.
9. Any being who punishes people who do their best is an unjust being. No perfectly good being is an unjust being. Draw your own conclusion.
*10. Every event is caused by a deity. It follows that every sin is caused by a deity.

6.6 Sorites and Removing Term-Complements

We have seen that ordinary language throws the logician many "curve balls" (i.e., complications). This section concerns two additional complications. First, we will discuss categorical syllogisms that are linked in a chain. Second, we will

discuss categorical arguments that can be put into standard form through the elimination of term-complements.

Sorites

The term "sorites" (so-rī-teez) comes from the Greek word *soros*, meaning "heap" or "pile." Roughly put, a sorites is a "heap" of syllogisms. More precisely, a **sorites** is a chain of syllogisms in which the final conclusion is stated but the subconclusions are unstated. Here is an example:

43. 1. All statements about beauty are statements known through the senses.
2. All statements known through the senses are empirical statements.
3. No mathematical statements are empirical statements.
4. All geometrical statements are mathematical statements.
So, 5. No geometrical statements are statements about beauty.

Premises (1) and (2) validly imply the following subconclusion:

Subconclusion 1: All statements about beauty are empirical statements.

When combined with premise (3), subconclusion 1 validly implies:

Subconclusion 2: No mathematical statements are statements about beauty.

Premise (4) and subconclusion 2 validly imply (5), the conclusion of the argument. Thus, the sorites is a chain of three valid categorical syllogisms.

In general, sorites are easier to evaluate when they are in standard form. A sorites in **standard form** has these features:

a. Each statement in the argument is in standard form ("All S are P," "No S are P," "Some S are P," or "Some S are not P").
b. The predicate term of the conclusion occurs in the first premise.
c. Each term appears twice, in two different statements.
d. Each premise (except the first) has a term in common with the immediately preceding premise.

Is argument (43) in standard form? Yes. This is perhaps easier to see by looking at the form of the argument. Using obvious abbreviations, the form of the argument is this:

44. 1. All B are K.
2. All K are E.
3. No M are E.
4. All G are M.
So, 5. No G are B.

When evaluating a sorites, we follow a three-step process. First, check to see if the sorites is in standard form. If it is, proceed to the next step. If it isn't, put it into standard form. To cut down on writing, we will freely use abbreviations for the terms as we put a sorites into standard form. Second, identify the subconclusions. Third, test each syllogism in the chain for validity, using a Venn diagram. If each syllogism in the chain is valid, the sorites is valid. If any of the syllogisms in the chain are invalid, the entire sorites is invalid on the principle that a chain is no stronger than its weakest link.

To illustrate this three-step process, let us evaluate the following sorites:

45. No cruel and unusual punishments are proportioned to the offense. Every just punishment is deserved. Moreover, all deserved punishments are proportioned to the offense. Therefore, no cruel and unusual punishment is a just punishment.

First, we put the sorites into standard form, using obvious abbreviations for the various terms (J: just punishments; D: deserved punishments; P: punishments that are proportioned to the offense; C: cruel and unusual punishments). Incidentally, it often helps to begin with the conclusion and then work backward, making sure that the first premise contains the predicate term of the conclusion:

46. 1. All J are D.
2. All D are P.
3. No C are P.
So, 4. No C are J.

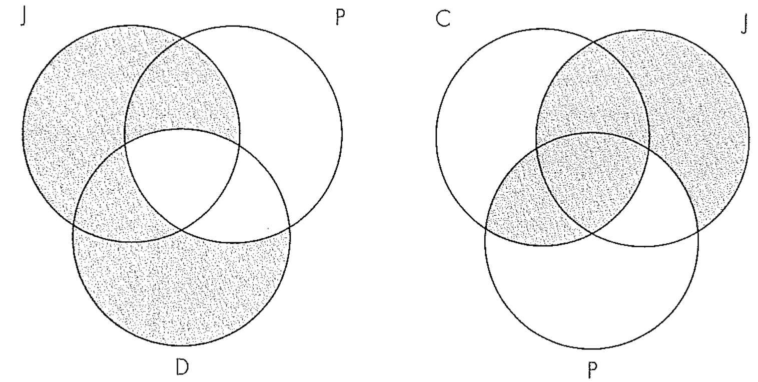
Premises (1) and (2) imply that all just punishments are punishments that are proportioned to the offense. Using our abbreviations:

Subconclusion 1: All J are P.

Let us adopt the practice of writing a subconclusion beside the last premise from which it is derived, like this:

47. 1. All J are D.
2. All D are P. Sub 1: All J are P.
3. No C are P.
So, 4. No C are J.

Subconclusion 1 and premise (3) combine to yield (4), the conclusion of the argument. So, this sorites is a chain of two categorical syllogisms. And we now apply the Venn method to each of the two categorical syllogisms:



The diagram on the left evaluates the inference from premises (1) and (2) to subconclusion 1. The diagram on the right evaluates the inference from subconclusion 1 and premise (3) to the final conclusion of the argument, (4). Since both of these syllogisms are valid, the sorites itself is valid.

Removing Term-Complements

Consider the following argument:

48. 1. All cats are nondogs.
2. Some mammals are dogs.
So, 3. Some mammals are not cats.

This argument has four terms: "cats," "nondogs," "mammals," and "dogs." Hence, it is not a categorical syllogism in standard form. And if you try to evaluate this argument with a Venn diagram, questions arise regarding the label for the middle circle. You must label it either "dogs" or "nondogs." If you label the middle circle "dogs," then your diagram for the first premise cannot be the usual diagram for an **A** statement. If you label the middle circle "nondogs," then your diagram for the second premise cannot be the usual diagram for an **I** statement. (Try it!)

Note, however, that "dogs" and "nondogs" are term-complements. In arguments involving term-complements, we can often reduce the number of terms by applying conversion, obversion, and/or contraposition. For example, the obverse of "All cats are nondogs" is "No cats are dogs." Thus, we can rewrite argument (48) as:

49. 1. No cats are dogs.
2. Some mammals are dogs.
So, 3. Some mammals are not cats.

We have reduced the number of terms in the argument from four to three by removing a term-complement. Argument (49) is a categorical syllogism in standard form, and the Venn diagram for it is entirely routine.

Consider yet another argument:

50. 1. No nonanimals are mammals.
2. All nonmammals are nondogs.
So, 3. All dogs are animals.

Is this argument a categorical syllogism in standard form? No, it has six terms instead of three. But we can put it into standard form by a series of immediate inferences. First, we can convert the first premise to obtain "No mammals are nonanimals." Then, applying obversion, we get "All mammals are animals." Second, we can apply contraposition to premise (2) to get "All dogs are mammals." With these changes, the argument looks like this:

51. 1. All mammals are animals.
2. All dogs are mammals.
So, 3. All dogs are animals.

Argument (51) is a categorical syllogism in standard form. Moreover, by removing the term-complements in argument (50), we have made it easier to understand.

We can often put an argument into standard form by removing term-complements via conversion, obversion, and/or contraposition. And we are free to remove term-complements as long as the changes we make to each statement produce a *logically equivalent* statement. (Remember, two statements are logically equivalent *if and only if* they validly imply each other.) However, if the changes we make produce any statement that is *not* logically equivalent to the original, then we have significantly changed the meaning of the statement—in which case we are working with a different argument from the original one. When removing term-complements, the permissible changes are as follows:

Conversion

No S are P.	No P are S.
Some S are P.	Some P are S.

Contraposition

All S are P.	All non-P are non-S.
Some S are not P.	Some non-P are not non-S.

Obversion

All S are P.	No S are non-P.
No S are P.	All S are non-P.
Some S are P.	Some S are not non-P.
Some S are not P.	Some S are non-P.

In each case, the statement on the left is logically equivalent to the statement on the right, and vice versa.

In attempting to remove term-complements, the most common error involves misapplications of conversion and contraposition. When you apply conversion to **A** and **O** statements, you do not in general wind up with logically equivalent statements. So, *never* use conversion on **A** and **O** statements when removing term-complements. Similarly, when you apply contraposition to **E** and **I** statements, you do not in general wind up with logically equivalent statements. So, *never* apply contraposition to **E** and **I** statements when removing term-complements. On the other hand, since a statement and its obverse are *always* logically equivalent, we are always free to apply obversion when we are removing term-complements.

It is often convenient to remove term-complements in order to put a sorites into standard form. Here's an example:

52. 1. No F are G.
2. All H are G.
3. All non-H are non-K.
So, 4. All K are non-F.

This sorites is not in standard form because the terms F, H, and K do not appear twice (the term-complements appear instead). But we can easily put this sorites into standard form by applying contraposition to premise (3) and obversion to the conclusion:

53. 1. No F are G.
2. All H are G.
3. All K are H.
So, 4. No K are F.

Again, by removing term-complements, we have not only put the argument into standard form but also made it easier to grasp and to evaluate.

The following exercise gives you practice in removing term-complements and in evaluating sorites.

Exercise 6.6

Part A: Removing Term-Complements The following categorical arguments (and argument forms) all have more than three terms. Reduce the terms to three in each case by removing term-complements via applications of conversion, obversion, and/or contraposition. To cut down on writing, use capital letters to abbreviate English terms.

* 1. 1. All M are non-P.
2. No S are non-M.
So, 3. No S are P.

2. 1. All logicians are nonpoets.
2. Some logicians are not nondreamers.
So, 3. Some dreamers are not poets.

3. 1. All non-P are non-M.
2. Some S are M.
So, 3. Some S are not non-P.

* 4. 1. All nonphysicians are nonsurgeons.
2. All physicians are nonchiropractors.
So, 3. No chiropractors are surgeons.

5. 1. Some non-P are not non-M.
2. No M are non-S.
So, 3. Some S are not P.

6. 1. All mesomorphs are nonectomorphs.
2. All nonectomorphs are things that are not slight persons.
So, 3. No slight persons are mesomorphs.

* 7. 1. No M are non-P.
2. All non-M are non-S.
So, 3. All S are P.

8. 1. No nonairplanes are jets.
2. All nonjets are non-737s.
So, 3. All 737s are airplanes.

9. 1. All M are non-P.
2. Some S are M.
So, 3. Some non-P are not non-S.

* 10. 1. Some subatomic particles are mesons.
2. No mesons are entities not subject to the strong nuclear force.
So, 3. Some entities subject to the strong nuclear force are subatomic particles.

Part B: Standard Form Put the following sorites into standard form, removing term-complements whenever possible. Then identify the unstated subconclusions and use Venn diagrams to test the sorites for validity.

* 1. 1. Some A are B.
2. All non-D are non-C.
3. No B are non-C.
So, 4. Some D are A.

2. 1. Some F are G.
2. All non-H are non-G.
3. All H are non-E.
So, 4. Some non-E are not non-F.

3. 1. Some K are non-L.
2. No M are non-N.
3. All N are L.
So, 4. Some K are not M.

* 4. 1. Some A are B.
2. All non-E are non-D.
3. All B are D.
4. All E are non-C.
So, 5. Some A are not C.

5. 1. All F are non-M.
2. All non-D are non-B.
3. All D are M.
So, 4. All F are non-B.

6. 1. All U are non-T.
2. All S are T.
3. All non-S are non-R.
4. All non-R are non-P.
So, 5. All P are non-U.

* 7. 1. All non-B are non-A.
2. No C are B.
3. Some D are non-C.
So, 4. Some D are not A.

8. 1. No R are non-S.
2. All T are V.
3. All non-R are non-Q.
4. All P are Q.
5. No T are S.
So, 6. All V are non-P.

9. 1. All F are G.
2. No G are non-H.
3. No J are H.
4. Some K are J.
So, 5. Some non-F are not non-K.

* 10. 1. Some B are A.
2. All non-D are non-C.
3. All B are C.
So, 4. Some D are A.

Part C: Sorites Put the following sorites into standard form. To cut down on writing, use capital letters as abbreviations for the various terms. Then identify the unstated subconclusions and use Venn diagrams to test the sorites for validity.

* 1. No theorists who hold that life evolved from nonlife are people who have solid evidence for their views. Every advocate of chemical evolution is a theorist who holds that life evolved from nonlife. At least one Darwinian is a person who has solid evidence for his or her views. Consequently, not all Darwinians are advocates of chemical evolution.

2. No voters who reject trickle-down economics are Republicans, for every voter who rejects trickle-down economics is a person who opposes tax cuts for the wealthy; all Democrats are non-Republicans; and only Democrats are persons who oppose tax cuts for the wealthy.

3. All folks who know that the oil reserves are running low are people who favor the development of alternative sources of energy. Every person who favors the development of alternative sources of energy is a voter who favors a tax increase. All well-informed citizens are folks who know that the oil reserves are running low. Not every highly educated person is a voter who favors a tax increase. Therefore, not all highly educated people are well-informed citizens.

* 4. Each and every dualist is a moral realist. Nothing is an emotivist unless it is not a moral realist. Only dualists are Zoroastrians. At least one positivist is an emotivist. It follows that some non-Zoroastrians are not nonpositivists.

5. Every victim of sleep apnea is a sleep-deprived person. No sleep-deprived persons are nonunfortunates. At least one unfortunate is a college student. Therefore, some college students are victims of sleep apnea.

6. No humans with a sense of justice are consumers who are willing to buy clothing made in sweatshops. But only shoppers are persons looking for good deals. Hence, not all shoppers are humans with a sense of justice because some persons looking for good deals are consumers who are willing to buy clothing made in sweatshops.

* 7. Not all freethinkers are rational people. Every agnostic is a person who proportions his or her religious beliefs to the evidence. All people who proportion their religious beliefs to the evidence are rational people. Thus, not every freethinker is an agnostic.

8. Only reasons that override mere prudence are ethical reasons. Some motives for laying down one's life are ethical reasons. No motives for laying down one's life are intentions that make sense if there's no life after death. It follows that not all reasons that override mere prudence are intentions that make sense if there's no life after death.

9. There exist rapscallions that are human vermin. Only scalawags are moral freeloaders. All human vermin are moral freeloaders. Every scalawag is a nongangster. Hence, not every rapscallion is a gangster.

* 10. No badly informed voters are responsible voters. Some folks who vote in every election are citizens who do not proportion their political beliefs to the evidence. Every citizen who does not proportion his or her political beliefs to the evidence is a badly informed voter. Only responsible voters are people who ought to vote. Accordingly, not all folks who vote in every election are people who ought to vote.

6.7 Rules for Evaluating Syllogisms

Prior to the invention of Venn diagrams, categorical syllogisms were evaluated by means of a set of rules. Although the rules lack the visual intuitiveness of Venn diagrams, they are equally effective in testing syllogisms for validity. In this section, we will explore a set of rules for evaluating categorical syllogisms.

Our first rule is:

Rule 1: A valid standard-form categorical syllogism must contain exactly three terms, and each term must be used with the same meaning throughout the argument.

A *fallacy of equivocation* occurs if a term is used with more than one meaning in a categorical syllogism. For example:

54. Only man is rational. But no woman is a man. Hence, no woman is rational.

Argument (54) violates Rule 1 because in the first premise, "man" means "human beings," while in the second premise, "man" means "male human."

The next two rules depend crucially on the concept of a term's being *distributed*. So, the rather technical concept of distribution must be explained in some detail before we proceed any further. A term is **distributed** in a statement if the statement says something about every member of the class that the term denotes. A term is **undistributed** in a statement if the statement does not say something about every member of the class the term denotes. For example:

55. All ants are insects.

Statement (55) says something about all members of the class of ants—namely, that every member of the class of ants belongs to the class of insects. Hence, the term "ants" is distributed in (55). But the term "insects" is undistributed, for the statement does not say anything about every member of the class of insects. In general, the subject term of a universal affirmative (or A) statement is distributed, while the predicate term is not.

Both terms are distributed in a universal negative (or E) statement. For instance:

56. No trumpets are flutes.

This says that every trumpet is excluded from the class of flutes and that every flute is excluded from the class of trumpets. Hence, both the subject term "trumpets" and the predicate term "flutes" are distributed.

Neither term is distributed in a particular affirmative (or I) statement. For example:

57. Some precious stones are diamonds.

This statement makes no assertion about all precious stones. Furthermore, it makes no assertion about all diamonds. Both the subject term and the predicate term of a particular affirmative statement are undistributed.

The predicate term of a particular negative (or O) statement is distributed, but its subject term is undistributed. For example:

58. Some precious stones are not diamonds.

Statement (58) does not say anything about *all* precious stones. But it does refer to all members of the class of diamonds, and it says that *all* diamonds are excluded from a portion of the class of precious stones.

To recap: Universal (A and E) statements distribute their subject terms, while negative (E and O) statements distribute their predicate terms. The following list summarizes our discussion of distribution:

Letter Name	Form	Terms Distributed
A	All S are P.	S
E	No S are P.	S and P
I	Some S are P.	None
O	Some S are not P.	P

We are now in position to state Rule 2.

Rule 2: In a valid standard-form categorical syllogism, the middle term must be distributed in at least one premise.

Here is an example of a syllogism that violates Rule 2:

59. All eagles are birds. All penguins are birds. So, all penguins are eagles.

The middle term, "birds," is not distributed in either premise since the predicate terms of A statements are not distributed. Within the Aristotelian scheme, a violation of Rule 2 is called a *fallacy of the undistributed middle*.

Why does the distribution of the middle term matter? Because the middle term has to serve as a link between the other terms. And if the middle term is undistributed, then neither premise makes an assertion about *all* the members of the class denoted by the middle term. Hence, it is possible that the minor term relates to one part of the class denoted by the middle term, while the major term relates to a different part of that class, with the result that there is no guaranteed link between the minor and major term.

Rule 3 also involves the concept of distribution.

Rule 3: In a valid standard-form categorical syllogism, a term must be distributed in the premises if it is distributed in the conclusion.

This rule can be broken in two basic ways, depending on whether the major or minor term is distributed in the conclusion but not in the premises. Consider the following examples:

60. All birds are animals. No bats are birds. So, no bats are animals.

61. All squares are rectangles. All squares are figures. So, all figures are rectangles.

In argument (60), the major term, "animals," is distributed in the conclusion but not in the premises. This sort of violation of Rule 3 is called a *fallacy of the illicit major*. In argument (61), the minor term, "figures," is distributed in the conclusion but not in the second premise. This type of violation of Rule 3 is called a *fallacy of the illicit minor*.

Why is it important for a term to be distributed in the premises if it is distributed in the conclusion? Well, suppose a term is distributed in the conclusion but not in the premises. Then the conclusion contains more information than the premises warrant because the conclusion says something about *all* members of the class denoted by the term, while the premises do *not* say something about all the members of that class. Therefore, if a term is distributed in the conclusion but not in the premises, the conclusion "goes beyond" the information contained in the premises, and hence the argument is invalid.

The next rule concerns the *quality* of the statements composing a categorical syllogism.

Rule 4: In a valid standard-form categorical syllogism, the number of negative premises must be equal to the number of negative conclusions.[1]

Since a syllogism has only one conclusion, this rule tells us that any categorical syllogism with two negative premises is invalid. For instance:

62. No dogs are cats. Some cats are not cocker spaniels. So, some cocker spaniels are not dogs.

The premises of argument (62) are true, while the conclusion is false, so (62) is clearly invalid.

Rule 4 also tells us that the conclusion of a categorical syllogism must be negative if one of the premises is negative. Thus, the following syllogism also violates Rule 4:

63. No tigers are wolves. Some felines are tigers. So, some felines are wolves.

Summary of Rules for Determining the Validity of Categorical Syllogisms

Rule 1: A valid standard-form categorical syllogism must contain exactly three terms, and each term must be used with the same meaning throughout the argument.

Rule 2: In a valid standard-form categorical syllogism, the middle term must be distributed in at least one premise.

Rule 3: In a valid standard-form categorical syllogism, a term must be distributed in the premises if it is distributed in the conclusion.

Rule 4: In a valid standard-form categorical syllogism, the number of negative premises must be equal to the number of negative conclusions.

Rule 5: No valid standard-form categorical syllogism with a particular conclusion can have two universal premises.

Here again, the premises are true, while the conclusion is false, so the argument is invalid.

Finally, Rule 4 tells us that a categorical syllogism is invalid if it has a negative conclusion but no negative premises:

64. All collies are dogs. Some animals are collies. So, some dogs are not animals.

Argument (64) has obviously true premises and an obviously false conclusion; therefore, it is plainly invalid. In fact, violations of Rule 4 are not common because the invalidity tends to be quite obvious.

At this point, our list of rules is complete from the traditional Aristotelian perspective. If we add the following rule, however, we can bring the Aristotelian system into agreement with modern systems of logic.

Rule 5: No valid standard-form categorical syllogism with a particular conclusion can have two universal premises.[2]

Here is an example of a syllogism that violates Rule 5 but counts as valid in the traditional Aristotelian scheme:

65. All Americans are humans. All morally perfect Americans are Americans. So, some morally perfect Americans are humans.

The conclusion asserts the existence of at least one morally perfect American. But from the standpoint of modern logic, we can assert that "all morally perfect Americans are Americans" without asserting that there actually *are* any morally perfect Americans. We can analyze the statement "All morally perfect Americans are Americans" as involving a conditional, along the following lines: "If anything is a morally perfect American, then it is an American." And such a statement can be true even if the term "employed" in the if-clause denotes an empty class.

Check your understanding of the Aristotelian theory of the syllogism by completing the following exercises.

Exercise 6.7

Part A: Forms Apply the five rules set forth in this section to determine whether the following forms are valid.

* 1. No P are M. No M are S. So, no S are P.
2. All M are P. No S are M. So, no S are P.
3. All M are P. All M are S. So, all S are P.
* 4. All P are M. All S are M. So, all S are P.
5. No M are P. Some S are M. So, some S are not P.
6. No M are P. All M are S. So, some S are not P.
* 7. All P are M. Some S are not M. So, some S are not P.
8. Some M are P. All S are M. So, some S are not P.
9. All M are P. Some S are not M. So, some S are not P.
* 10. Some P are not M. Some S are not M. So, some S are not P.
11. All P are M. Some S are M. So, some S are P.
12. No P are M. All M are S. So, no S are P.
* 13. Some M are not P. All S are M. So, some S are not P.
14. No M are P. Some M are not S. So, some S are not P.
15. All P are M. No M are S. So, some S are not P.
* 16. All M are P. All S are M. So, some S are P.
17. All P are M. All M are S. So, all S are P.
18. No P are M. All S are M. So, some S are not P.
* 19. Some M are P. Some M are S. So, some S are P.
20. Some P are M. All S are M. So, some S are P.

Part B: Valid or Invalid? For each of the following categorical syllogisms, specify the form using "S" to stand for the minor term, "P" for the major term, and "M" for the middle term. (If the English argument itself is not in standard form, be sure your form puts the major premise first, the minor premise second, and the conclusion last.) Then apply the five rules set forth in this section to determine whether the syllogism has a valid form.

* 1. Some great scientists are famous. No TV stars are great scientists. So, some TV stars are not famous.

2. No deathly ill people are hypochondriacs. All hypochondriacs are dysfunctional people. Accordingly, some deathly ill people are dysfunctional people.

3. Some books written by Kant are not great books. For no great books are books that put their readers to sleep. But some books written by Kant are books that put their readers to sleep.

* 4. No humans are animals. All members of *homo sapiens* are animals. Therefore, no humans are members of *homo sapiens*.

5. All values that can be quantified are important values. No human emotions are values that can be quantified. Consequently, no human emotions are important values.

6. No great altruists are great thinkers. Some great thinkers are people who make life better for humanity in general. It follows that some people who make life better for humanity in general are not great altruists.

* 7. All cars are vehicles. All Ford automobiles are cars. Hence, some Ford automobiles are vehicles.

8. All banks are edges of rivers. Some banks are financial institutions. Thus, some financial institutions are edges of rivers.

9. All acts that promote the general welfare are commanded by God. For all acts commanded by God are obligatory acts. And all acts that promote the general welfare are obligatory acts.

* 10. All of the greatest human achievements are accomplishments that have come at a great price. Some accomplishments that have come at a great price are not brilliant discoveries. We may conclude that some of the greatest human achievements are not brilliant discoveries.

11. All kleptomaniacs are troubled persons. No Bodhisattvas are troubled persons. So, some kleptomaniacs are not Bodhisattvas.

12. All biologists are vivisectionists. Some vivisectionists are well-intentioned people. Therefore, some biologists are well-intentioned people.

13. Every schipperke is a small dog. Some small dogs are not black. Hence, not all black dogs are schipperkes.

14. No bagatelle is important. Some important things are pleasurable. It follows that at least one bagatelle is not pleasurable.

15. All Mennonites are Protestants. No Mennonites are Roman Catholics. Accordingly, no Protestants are Roman Catholics.

Notes

1. My formulation of Rule 4 is borrowed from Wesley C. Salmon, *Logic*, 3rd ed. (Englewood Cliffs, NJ: Prentice-Hall, 1984), p. 57.
2. My formulation of Rule 5 is borrowed from Irving Copi and Carl Cohen, *Introduction to Logic*, 9th ed. (Englewood Cliffs, NJ: Prentice-Hall, 1994), p. 266.