**Quarter 3-Classwork 2**

**Euclidean Division**

Let $a, b$ be two positive integers where $a < b$. Then, there exist two non-negative integers $q, r$ so that

$$b = q \cdot a + r \text{ and } 0 \leq r \leq a - 1.$$

- $q$ will be called the **quotient** of the division of $b$ by $a$, and
- $q$ will be called the **remainder** of the division of $b$ by $a$.

**Example 1.**

1. $a = 3, b = 16$.

   Observe that $16 = 5 \cdot 3 + 1$. Hence, the quotient of the division of 16 by 3 is the number 5 and the remainder is 1. Also, $r = 1 < 3 = a$.

2. $a = 8, b = 26$.

   Observe that $26 = 3 \cdot 8 + 2$. Hence, the quotient of the division of 26 by 8 is the number 3 and the remainder is 2. Also $r = 2 < 8 = a$.

3. $a = 3, b = 6$.

   Observe that $6 = 2 \cdot 3 + 0$. Hence, the quotient of the division of 6 by 3 is the number 2 and the remainder is 0. Also, $r = 0 < 3 = a$.

4. $a = 10, b = 15$.

   Observe that $15 = 1 \cdot 10 + 5$. Hence, the quotient of the division of 15 by 10 is the number 1 and the remainder is 5. Also, $r = 5 < 10 = a$.

**Exercise:** Find the quotient and the remainder for the following pairs. Write the fraction $\frac{b}{a}$ as a mixed number.

- $a = 11, b = 12$

- $a = 9, b = 13$
- $a = 5, b = 7$
- $a = 3, b = 12$
- $a = 20, b = 27$
- $a = 9, b = 28$
- $a = 7, b = 19$
- $a = 6, b = 21$
- $a = 1, b = 4$

- $a = 2, b = 8$

- $a = 20, b = 100$

- $a = 25, b = 120$

- $a = 30, b = 400$

- $a = 10, b = 10.000$

- $a = 100, b = 100.000$

- $a = 101, b = 103$

- $a = 101, b = 333$

- $a = 99, b = 400$
- $a = 87, b = 150$
- $a = 88, b = 1000$
- $a = 15, b = 45$
- $a = 15, b = 60$
- $a = 15, b = 60$
- $a = 12, b = 48$
- $a = 12, b = 70$

- $a = 13, b = 27$
- $a = 14, b = 32$
- $a = 16, b = 49$
- $a = 17, b = 50$
- $a = 18, b = 39$
- $a = 15, b = 60$
- $a = 21, b = 67$
- $a = 50, b = 123$

- $a = 50, b = 187$
- $a = 70, b = 125$
- $a = 72, b = 333$
- $a = 74, b = 223$
- $a = 78, b = 80$
- $a = 81, b = 1081$
- $a = 2^5, b = 2^{10}$
- $a = 2^5 \cdot 3^3, b = 3^5 \cdot 2^6$