

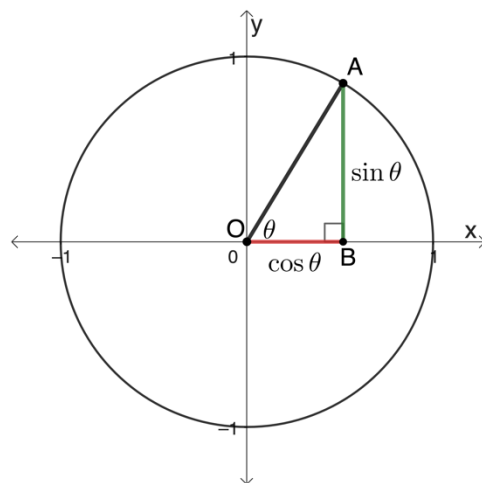


How Are the Slopes Changing?



We've spent a lot of time looking at the coordinates of the unit circle. But what about the slope made by the terminal ray?

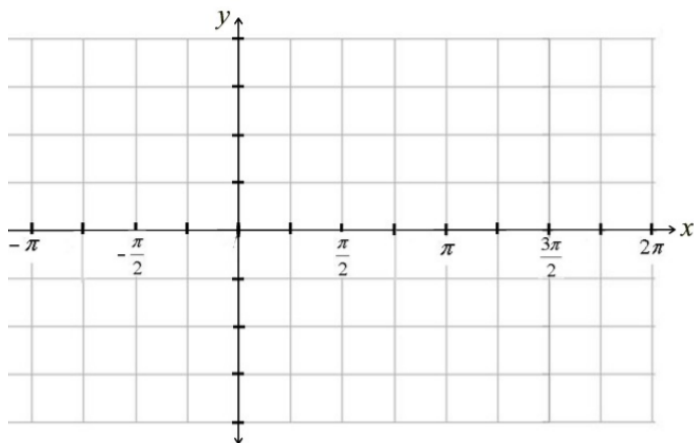
1. What is the length of segment AO?
2. Explain why the length of segment AB represents $\sin \theta$.
3. Explain why the length of segment OB represents $\cos \theta$.
4. Explain why the slope of segment OA represents $\tan \theta$.
5. Go to <https://tinyurl.com/calcmedicapc>. Use the slider to change the value of θ and watch how the values change.
 - a. In which quadrants is the slope of \overline{OA} positive?
 - b. In which quadrants is the slope of \overline{OA} negative?
 - c. For which values of θ does \overline{OA} have a slope of 0?
 - d. Set $\theta = -\frac{\pi}{2}, \frac{\pi}{2}$, and $\frac{3\pi}{2}$. What is the slope of \overline{OA} at these angles? Why does this happen?
 - e. For which values of θ is the slope of \overline{OA} equal to 1?
 - f. Do the slopes of \overline{OA} change faster between 0 radians and $\frac{\pi}{4}$ radians or between $\frac{\pi}{4}$ and $\frac{\pi}{2}$ radians? Explain.
 - g. Do the slopes of \overline{OA} change faster between $-\frac{\pi}{2}$ radians and $\frac{-\pi}{4}$ radians or between $\frac{-\pi}{4}$ and 0 radians? Explain.
6. Consider values of θ between $-\pi$ and 2π .
 - a. For how many values of θ is the slope of $\overline{OA} = 0.27$?
 - b. For how many values of θ is the slope of $\overline{OA} = -2.41$?
 - c. For $-\pi < \theta < 2\pi$, how many times do the slope values repeat? What does this tell you about the period of $y = \tan \theta$?





7. Use the Geogebra applet to complete the table of values and sketch the graph of $y = \tan \theta$.

θ	$\tan \theta$
$-\pi/2$	
$-5\pi/12$	
$-\pi/4$	
0	
$\pi/4$	
$5\pi/12$	
$\pi/2$	
$7\pi/12$	
$3\pi/4$	
π	
$5\pi/4$	
$17\pi/12$	
$3\pi/2$	



8. Explain how you can determine what the graph of $y = \tan \theta$ looks like for $\theta > \frac{3\pi}{2}$ without plotting points individually?
9. What is the domain of $y = \tan \theta$? How do you know?
10. What is the range of $y = \tan \theta$?
11. a. On which interval(s) is the graph of $y = \tan \theta$ concave up?
- b. On which interval(s) is the graph of $y = \tan \theta$ concave down?
- c. How does this relate to your answers in questions 5f and 5g?

Lesson 7.1 – The Tangent Function

QuickNotes

Check Your Understanding

1. Find an exact value for the slope of the terminal ray of an angle θ when $\theta = \frac{7\pi}{6}$ and when $\theta = \frac{5\pi}{3}$.
2. Explain why $y = \tan \theta$ is always increasing for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
3. Consider the function $f(\theta) = \tan\left(\frac{1}{2}\theta\right)$.
 - a. What transformation occurred from the parent function $y = \tan \theta$?
 - b. What is the period of f ?
 - c. For which values of θ does f have vertical asymptotes?
 - d. Sketch a graph of f .

