

## ⇒ Material Implication & Logical Arguments ∴

Material implication is involved in many logical arguments, and the five of those logical arguments shall be considered.

1. The Hypothetical Syllogism
2. *Modus Ponens* (Affirming the Antecedent)
3. The Fallacy of Affirming the Consequent
4. *Modus Tollens* (Denying the Consequent)
5. The Fallacy of Denying the Antecedent

Three of the five arguments are valid, and two of the five arguments are invalid. The two invalid arguments are the two fallacies. The other three arguments are the valid arguments. After considering the three valid arguments, the two invalid arguments shall be considered.

## (1) The Hypothetical Syllogism

The hypothetical syllogism is a logical argument involving only *conditional* statements. Both of its *premises* are conditional statements, and its *conclusion* is a conditional statement. The hypothetical syllogism has the following logical form.

- Premise 1:  $P \Rightarrow Q$
- Premise 2:  $Q \Rightarrow R$
- Conclusion:  $\therefore P \Rightarrow R$ .

For example, “If the wind blows then the lion roars, and if the lion roars then the baby cries, so if the wind blows then the baby cries”.

- Premise 1: If the wind blows then the lion roars. ( $P \Rightarrow Q$ )
- Premise 2: If the lion roars then the baby cries. ( $Q \Rightarrow R$ )
- Conclusion: So, if the wind blows then the baby cries. ( $\therefore P \Rightarrow R$ )

The hypothetical syllogism is a **valid** logical argument. Fill out the truth-table, and show how the hypothetical syllogism can be a **sound** valid argument.

			<u>Premise 1</u>	<u>Premise 2</u>	<u>Conclusion</u>
P	Q	R	$P \Rightarrow Q$	$P \Rightarrow R$	$\therefore P \Rightarrow R$
T	T	T	...		
T	T	F	...		
T	F	T	...		
F	T	T	...		
T	F	F	...		
F	T	F	...		
F	F	T	...		
F	F	F	...		

## (2) *Modus Ponens* (Affirming the Antecedent)

*Modus ponens* is a logical argument in which the conclusion follows from two premises. With *modus ponens*, the first premise puts forth a conditional statement, the second premise affirms the antecedent of the conditional statement, and the conclusion affirms the consequent of the conditional statement. So, *modus ponens* is the argument of affirming the antecedent, and it has the following logical form.

- Premise 1:  $P \Rightarrow Q$
- Premise 2:  $P$
- Conclusion:  $\therefore Q$ .

For example, “If the wind blows then the lion roars, and the wind blows, so the lion roars”.

- Premise 1: If the wind *blows* then the lion *roars*. ( $P \Rightarrow Q$ )
- Premise 2: The wind *blows*. ( $P$ )
- Conclusion: So, the lion *roars*. ( $\therefore Q$ )

*Modus ponens* is a **valid** logical argument. Fill out the truth-table, and show how it also can be a **sound** logical argument.

<u>Premise 1</u>	<u>Premise 2</u>	<u>Conclusion</u>
$P \Rightarrow Q$	$P$	$\therefore Q$
	T	T
	F	F
	T	F
	F	T

With the fallacy of affirming the consequent, the first premise puts forth a conditional statement, the second premise affirms the consequent of the conditional statement, and the conclusion affirms the antecedent of the conditional statement. Hence, the fallacy of affirming the consequent has the following logical form.

<u>Premise 1</u>	<u>Premise 2</u>	<u>Conclusion</u>
<b><math>P \Rightarrow Q</math></b>	<b><math>Q</math></b>	<b><math>\therefore P</math></b>
	T	T
	F	F
	F	T
	T	F

#### (4) *Modus Tollens* (Denying the Consequent)

*Modus tollens* is a logical argument in which the conclusion follows from two premises. With *modus tollens*, the first premise puts forth a conditional statement, the second premise denies the consequent of the conditional statement, and the conclusion denies the antecedent of the conditional statement. So, *modus tollens* is the argument of denying the consequent, and it has the following logical form.

- Premise 1:  $P \Rightarrow Q$
- Premise 2:  $\sim Q$
- Conclusion:  $\therefore \sim P$ .

Consider the following example:

“If the wind blows then the lion roars, but the lion doesn’t roar, so the wind doesn’t blow”.

- Premise 1: If the wind *blows* then the lion *roars*. ( $P \Rightarrow Q$ )
- Premise 2: The lion *doesn’t* roar ( $\sim Q$ )
- Conclusion: So, the wind *doesn’t* blow. ( $\therefore \sim P$ )

*Modus tollens* is a **valid** logical argument. Fill out the truth-table, and show how *modus tollens* also can be a **sound** logical argument.

P	Q
T	T
F	F
T	F
F	T

<u>Premise 1</u>	<u>Premise 2</u>	<u>Conclusion</u>
$P \Rightarrow Q$	$\sim Q$	$\therefore \sim P$

## (5) The Fallacy of Denying the Antecedent

With the fallacy of denying the antecedent is a logical argument in which the conclusion follows from two premises. With the fallacy of denying the antecedent, the first premise puts forth a conditional statement, the second premise denies the antecedent of the conditional statement, and the conclusion denies the consequent of the conditional statement. Hence, the fallacy of denying the antecedent has the following logical form.

- Premise 1:      $P \Rightarrow Q$
- Premise 2:      $\sim P$
- Conclusion:     $\therefore \sim Q$ .

Consider the following example:

“If the wind blows then the lion roars, and the wind doesn’t blow, so the lion doesn’t roar”.

- Premise 1:     If the wind *blows* then the lion *roars*.                      $(P \Rightarrow Q)$
- Premise 2:     The wind *doesn’t* blow.                                      $(\sim P)$
- Conclusion:                     So, the lion *doesn’t* roar.                      $(\therefore \sim Q)$

The fallacy of denying the antecedent is a fallacy, because it is an **invalid** logical argument. Fill out the truth-table, and show *why* the fallacy of denying the antecedent is an **invalid** logical argument.

P	Q
T	T
F	F
T	F
F	T

<u>Premise 1</u>	<u>Premise 2</u>	<u>Conclusion</u>
$P \Rightarrow Q$	$\sim P$	$\therefore \sim Q$