AP Precalculus - M3Y & M3Z

Polynomials - Homework 2

1. Determine whether the following functions are even, odd, or neither. Show your work.

   (i) $f(x) = x^2 + 5$

   (ii) $g(x) = x^3 - x^2 + x$

   (iii) $h(x) = x^5 + x^3 + x$

   (iv) $r(x) = 5x - 3$

   (v) $Q(x) = x^{2023} + 7x - 1$

   (vi) $q(x) = -x^3 - 3x$

2. Prove the following:

   (i) If $f(x)$ is an even function and $g(x)$ is an even function, then:

   (a) $h(x) = f(x) + g(x)$ is an even function

   (b) $k(x) = f(x)g(x)$ is an even function

   (c) $s(x) = cf(x)$ is an even function for every $c \in \mathbb{R}$

   (d) $t(x) = \dfrac{f(x)}{g(x)}$ is an even function

   (ii) If $f(x)$ is an even function and $g(x)$ is an odd function, then:

   (a) $k(x) = f(x)g(x)$ is an odd function

   (b) $t(x) = \dfrac{f(x)}{g(x)}$ is an odd function

   (iii) If $f(x)$ is an odd function and $g(x)$ is an odd function, then:

   (a) $h(x) = f(x) + g(x)$ is an odd function

   (b) $k(x) = f(x)g(x)$ is an even function

   (c) $s(x) = cf(x)$ is an odd function for every $c \in \mathbb{R}$

   (d) $t(x) = \dfrac{f(x)}{g(x)}$ is an even function

   (iv) If $f(x)$ is an even function and $g(x)$ is an even function, then:

   (a) $h(x) = f(x) + g(x)$ is an even function

   (b) $k(x) = f(x)g(x)$ is an even function

   (c) $s(x) = cf(x)$ is an even function for every $c \in \mathbb{R}$

   (d) $t(x) = \dfrac{f(x)}{g(x)}$ is an even function

3. Can a non-zero function $f(x)$ be both odd and even? Explain.

4. Find the end-behavior (that is, the limits at $+\infty$ and $-\infty$) of the following functions:

(i) $f(x) = x^5 - 4x^2 + 3x - 1$

(ii) $g(x) = -4x^{16} - 3x^2 + 7x - 9$

(iii) $h(x) = -3x^7 + 2x^6 - 2x^5 - x^3 + 2x + 1$

(iv) $r(x) = \dfrac{1}{1000}x^2 - 10000x + 5$

(v) $p(x) = 2x - 9$

(vi) $q(x) = -9x - 2$