

AUC apCalculus BC

Assignment 07

3. The zero limit test

PROBLEM 3.1. Prove that the following limits do not exist by using subsequences:

- (1) $\lim_{n \rightarrow +\infty} (-1)^n$.
- (2) $\lim_{n \rightarrow +\infty} (-1)^{n^2}$.
- (3) $\lim_{n \rightarrow +\infty} (-1)^{n^2} n^3$.
- (4) $\lim_{n \rightarrow +\infty} (-1)^{n^2} n^3 + n^2$.

PROBLEM 3.2. Prove that the following series diverge:

- (1) $\sum_{n=1}^{+\infty} (-1)^n n^3$.
- (2) $\sum_{n=1}^{+\infty} (-1)^n \frac{n^2+1}{n+1}$.
- (3) $\sum_{n=1}^{+\infty} (-1)^n \frac{n^5+1}{n^2+1}$.
- (4) $\sum_{n=1}^{+\infty} \sin(n)$.

Problem 9.1. Study the convergence of the following series:

- (3) $\sum_{n=n_0}^{+\infty} \frac{1}{10^{4n}}$.
- (2) $\sum_{n=1}^{+\infty} \frac{1}{n^5} - \frac{1}{(n+1)^5}$.
9. $\sum_{n=0}^{\infty} \left(\frac{7}{6}\right)^n$
10. $\sum_{n=0}^{\infty} 5\left(\frac{11}{10}\right)^n$
11. $\sum_{n=0}^{\infty} 1000(1.055)^n$
12. $\sum_{n=0}^{\infty} 2(-1.03)^n$

4. Geometric Series & the Zero Limit Test

PROBLEM 4.1. Prove that the following series diverge:

- (1) $\sum_{n=1}^{+\infty} (-1)^n n^3$.
- (2) $\sum_{n=1}^{+\infty} (-1)^n \frac{n^2+1}{n+1}$.
- (3) $\sum_{n=1}^{+\infty} (-1)^n \frac{n^5+1}{n^2+1}$.
- (4) $\sum_{n=1}^{+\infty} \sin(n)$.

PROBLEM 4.2. Study the convergence of the following series:

- (1) $\sum_{n=1}^{+\infty} (-1)^n n^5$.
- (2) $\sum_{n=n_0}^{+\infty} \frac{1}{10^n}$.
- (3) $\sum_{n=1}^{+\infty} \frac{5}{10^n}$.
- (4) $\sum_{n=1}^{+\infty} \frac{2^{n+2}}{10^{3n}}$.
- (5) $\sum_{n=1}^{+\infty} \frac{6^n}{7^n}$.

PROBLEM 4.3. Calculate the rational numbers with the following (unless expressed otherwise) decimal expansion:

- (1) 0.111111....
- (2) 0.888888....
- (3) 0.1515151515....
- (4) 0.1515151515... in base 6.
- (5) 0.111111... in base 2.