

# AUC apCalculus BC

## Assignment 06

PROBLEM 3.1. Prove that the following limits do not exist by using subsequences:

- (1)  $\lim_{n \rightarrow +\infty} (-1)^n$ .
- (2)  $\lim_{n \rightarrow +\infty} (-1)^{n^2}$ .
- (3)  $\lim_{n \rightarrow +\infty} (-1)^{n^2} n^3$ .
- (4)  $\lim_{n \rightarrow +\infty} (-1)^{n^2} n^3 + n^2$ .

PROBLEM 14.1. Consider a sequence  $\{a_n\}$ . Let  $\alpha \in \mathbb{R} \vee \alpha = +\infty \vee \alpha = -\infty$ . Prove that, if  $\lim_{n \rightarrow +\infty} a_{2n} = \alpha = \lim_{n \rightarrow +\infty} a_{2n+1}$ , then  $\lim_{n \rightarrow +\infty} a_n = \alpha$ , i.e., if the even and odd subsequences of a sequence have equal limits, then the limit of the sequence exists.

PROBLEM 14.2. Prove that  $\lim_{n \rightarrow +\infty} \cos(n) \neq 0$ .

PROBLEM 14.3. (1) Estimate  $\sqrt[n]{n!}$ .

(2) Estimate  $\sqrt[n^2+n+1]{n!}$ .

(3) Estimate  $\sqrt[3n]{(2n)!}$ .

PROBLEM 14.4. Prove that  $\lim_{n \rightarrow +\infty} \cos(n)$  (D.N.E.) does not exist.