

AUC apCalculus BC

Assignment 06

PROBLEM 3.1. *Prove that the following limits do not exist by using subsequences:*

- (1) $\lim_{n \rightarrow +\infty} (-1)^n.$
- (2) $\lim_{n \rightarrow +\infty} (-1)^{n^2}.$
- (3) $\lim_{n \rightarrow +\infty} (-1)^{n^2} n^3.$
- (4) $\lim_{n \rightarrow +\infty} (-1)^{n^2} n^3 + n^2.$

PROBLEM 14.1. *Consider a sequence $\{a_n\}$. Let $\alpha \in \mathbb{R} \vee \alpha = +\infty \vee \alpha = -\infty$. Prove that, if $\lim_{n \rightarrow +\infty} a_{2n} = \alpha = \lim_{n \rightarrow +\infty} a_{2n+1}$, then $\lim_{n \rightarrow +\infty} a_n = \alpha$, i.e., if the even and odd subsequences of a sequence have equal limits, then the limit of the sequence exists.*

PROBLEM 14.2. *Prove that $\lim_{n \rightarrow +\infty} \cos(n) \neq 0$.*

PROBLEM 14.3. (1) *Estimate $\sqrt[n]{n!}$.*

(2) *Estimate $\sqrt[n^2+n+1]{n!}$.*

(3) *Estimate $\sqrt[3n]{(2n)!}$.*

PROBLEM 14.4. *Prove that $\lim_{n \rightarrow +\infty} \cos(n)$ (D.N.E.) does not exist.*