
THE ARCHIMEDEAN ALGEBRA REVIEW

brachysto-Algebra

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PART I

ELEMENTARY ALGEBRA CONCEPTS

CHAPTER 1

EXPONENTS

All letters represent real numbers, and they are nonzero if they make the denominators zero.

1. Use the properties of exponents to simplify the following expressions:

In groups:

- (a) $x^7 \cdot x^6 \cdot x^{-3}$
- (b) $(-2a^3b^6)(-5b^{-1}a^{-7})$
- (c) $\left(\frac{8p^2}{q^3}\right)^2$
- (d) $\left(\frac{1}{4} - \frac{3}{8} + \frac{2}{3}\right)^2$
- (e) $\left(\frac{-16x^2y^8}{4x^6y^{-3}}\right)^{-2}$
- (f) $\left(\frac{6x^2y^{-4}z^3}{12y^3x^5z^{-4}}\right)^2 \cdot (-x^4y^5z^{-4})^{-4}$
- (g) $(3a+2)^{-7}(3a+2)^3$

Individually:

- (a) $(5x^{-4}y^{-3})(-3x^{-1}y^8)$
- (b) $\frac{x^{-3}x^6}{x^2}$
- (c) $\frac{27y^2y^9}{18x^5y^2}$
- (d) $\frac{-7p^3q^{11}}{49p^7q^3}$
- (e) $\frac{x^{-13}x^6}{x^{20}}$
- (f) $(x^{-2})^7$
- (g) $(-3y^{-4})^4$
- (h) $(-2a^5)^3$
- (i) $\left(\frac{4x^{-2}y^{-3}}{8y^4}\right)^3$
- (j) $\left(\frac{1}{4}\right)^{-2} - \left(\frac{3}{8}\right)^{-1} + \left(\frac{2}{3}\right)^0$
- (k) $\left(\frac{1}{4}\right)^{-3} - \left(\frac{3}{8}\right)^{-1} + \left(\frac{2}{3}\right)^0$

- (l) $\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right)^{-2}$
- (m) $\left(\frac{6x^4y^{-4}}{18x^{-2}y^{-3}}\right)^{-3}$
- (n) $\left(\frac{2}{6p^3q^{-3}}\right)^{-2} \left(\frac{3}{2p^{-6}q^{-7}}\right)^{-1}$
- (o) $\left(\frac{1}{4a^6b^{-1}}\right)^2 \left(\frac{2}{5a^3b^{-4}}\right)^{-1}$
- (p) $\left(\frac{14a^{-2}b^2c^5}{21c^2b^{-2}a^4}\right)^2 \cdot (-3a^{-2}c^4b^{-2})^{-3}$
- (q) $\left(\frac{(y^{-4})^{-2}y^{-3}}{y^{-5}}\right)^{-1}$
- (r) $\left(\frac{(a^{-1})^3a^{-2}}{a^{-2}}\right)^{-4}$
- (s) $(5x - 2)^{-3} (5x - 2)^4$
- (t) $(7x - 1)^4 (7x - 1)^{-3}$
- (u) $(1 - 7x)^4 (7x - 1)^{-3}$
- (v) $\left((4x + 3)^4\right)^{-3}$
- (w) $\left((-5x + 2)^{-5}\right)^2$

2. Use the properties of exponents, including rational exponents and radicals to simplify the following expressions:

In groups:

- (a) $36^{1/2}$
- (b) $\left(\frac{121}{144}\right)^{1/2}$
- (c) $-16^{3/4}$
- (d) $25^{3/2}$

Individually:

- (a) $\left(\frac{49}{81}\right)^{1/2}$
- (b) $16^{3/4}$

- (c) $(-16)^{3/4}$
- (d) $16^{-3/4}$
- (e) $-16^{-3/4}$
- (f) $(-16)^{-3/4}$
- (g) $8^{2/3}$
- (h) $-8^{2/3}$
- (i) $(-8)^{2/3}$
- (j) $8^{-2/3}$
- (k) $-8^{-2/3}$
- (l) $(-8)^{-2/3}$

3. Rewrite the following expressions using rational exponents:

In groups:

- (a) $\sqrt[4]{x^3}$
- (b) $\sqrt[6]{a^6 + b^6}$
- (c) $\sqrt{9x^5}$

Individually:

- (a) $\sqrt[8]{y^5}$
- (b) $9\sqrt{x^5}$
- (c) $\sqrt[5]{3x^2 - y^3}$
- (d) $\sqrt[4]{16p^{16} - q^{16}}$
- (e) $\sqrt{9a^8 - b^8}$

4. Simplify and rewrite the result using only positive rational exponents:

In groups:

- (a) $\frac{x^{4/3}x^{2/3}}{x^{1/3}}$
- (b) $\frac{4z^{-4/7}z^{2/7}}{12z^{-1/7}}$
- (c) $\left(\frac{a^{2/3}}{b^{3/4}}\right)^{12} \left(\frac{b^{2/5}}{b^{3/2}}\right)^{10}$
- (d) $\left(\frac{x^2}{x+y}\right)^{-1} \left(\frac{x^2}{x+y}\right)^{1/2}$

Individually:

- (a) $\frac{y^{2/5}y^{1/5}}{x^{3/5}}$
- (b) $\frac{21x^{4/3}x^{2/3}}{14x^{1/3}}$
- (c) $\frac{45a^{-4/5}a^{2/5}}{9b^{-1/3}}$
- (d) $\frac{p^{-5/7}q^{-2/7}}{(-3)^{-2}r^{-2/5}}$
- (e) $(16a^{-8}b^{1/5})^{3/4}$
- (f) $(125x^6b^{-7/5})^{1/3}$
- (g) $\left(\frac{x^2}{x-y}\right)^{-2}\left(\frac{x^2}{x-y}\right)^{3/2}$
- (h) $\left(\frac{a^{2/3}}{b^{3/4}}\right)^{-2}\left(\frac{b^{2/5}}{b^{3/2}}\right)^{10}$

5. Simplify each expression, and write the result in terms of positive exponents and radicals:

In groups:

- (a) $\sqrt{x^7}$
- (b) $\sqrt[3]{x^7}$
- (c) $\sqrt[3]{250a^2b^6c^{13}}$
- (d) $\sqrt{\frac{x^9}{36}}$
- (e) $4\sqrt[3]{\frac{p^3z^8}{8}}$
- (f) $\sqrt{10} \cdot \sqrt{14}$
- (g) $\sqrt[3]{x^2y} \cdot \sqrt[3]{xy^2}$
- (h) $\left(3\sqrt[4]{x^3}\right)\left(-5\sqrt[4]{x^3}\right)$
- (i) $\left(-\frac{1}{2}\sqrt[3]{6x^2y^2z}\right)\left(\frac{4}{3}\sqrt[3]{4x^2z^2}\right)$
- (j) $\sqrt[5]{x}\sqrt[3]{x^2}$

Individually:

- (a) $\sqrt[4]{x^7}$
- (b) $\sqrt[8]{x^7}$

- (c) $\sqrt{24}$
- (d) $\sqrt[3]{24}$
- (e) $\sqrt{54}$
- (f) $\sqrt[3]{54}$
- (g) $\sqrt[4]{40x^3y^9z^{14}}$
- (h) $\sqrt[4]{243p^{19}q^{10}}$
- (i) $\sqrt{84(a-2)^3}$
- (j) $\sqrt{12(x+42)^5}$
- (k) $\sqrt{\frac{a^{13}}{64}}$
- (l) $8\sqrt[3]{\frac{x^6y^7}{64}}$
- (m) $\sqrt{6} \cdot \sqrt{21}$
- (n) $\sqrt[4]{p^3q} \cdot \sqrt[4]{pq^3}$
- (o) $\left(7\sqrt[6]{a^5}\right)\left(-2\sqrt[6]{a^5}\right)$
- (p) $\left(\frac{1}{6}\sqrt[3]{9p^2q^5r}\right)\left(-\frac{3}{4}\sqrt[3]{6p^2qz^4}\right)$
- (q) $\sqrt[5]{x^6y^5} \cdot \sqrt[4]{x}$
- (r) $\sqrt[4]{p^3q} \cdot \sqrt[3]{pq^2}$
- (s) $\sqrt[6]{a^4\sqrt{a^3}}$
- (t) $\sqrt{x\sqrt{x\sqrt{x}}}$
- (u) $\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x}}}$

6. Add or subtract as indicated. Leave your answers as simplified as possible:

In groups:

- (a) $3\sqrt[3]{2x^2} - 9\sqrt[3]{2x^2} + \sqrt[3]{2x^2}$
- (b) $-3x\sqrt[3]{16xy^4} + xy\sqrt[3]{54xy} - 5\sqrt[3]{250x^4y^4}$

Individually:

- (a) $\frac{1}{5}\sqrt{50} - \frac{7}{3}\sqrt{18} + \frac{5}{6}\sqrt{72}$
- (b) $\frac{2}{5}\sqrt{75} - \frac{2}{3}\sqrt{27} - \frac{1}{2}\sqrt{12}$
- (c) $8\sqrt[4]{32p^5q^6} - 5q\sqrt[4]{2p^5q^2} - pq\sqrt[4]{162pq^2}$
- (d) $-\frac{1}{2}\sqrt{8x} + \frac{3}{7}\sqrt{98x}$

CHAPTER 2

FACTORING
