FULL NAME: ______________________________

Universal Gravitation

1. (20 points) Calculate the magnitude and direction of the net gravitational force on the moon due to the Earth and the Sun when the moon is in each of the positions shown in Figure 1.

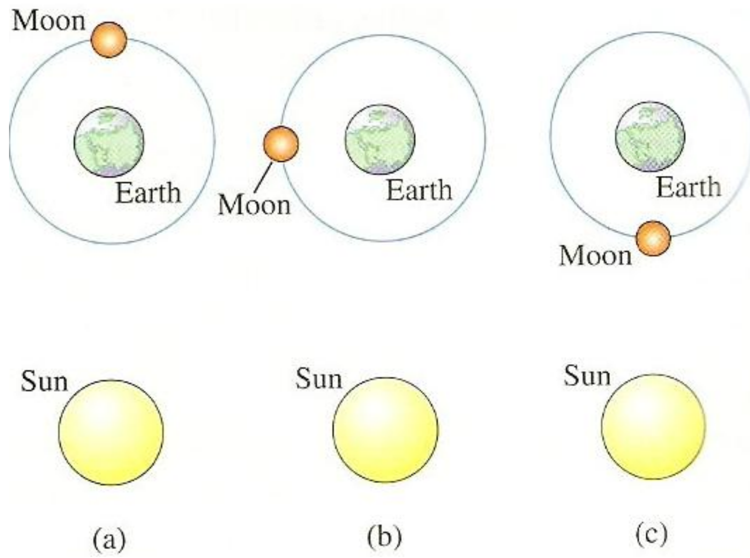

Figure 1

(Note that the figure is not drawn to scale. Assume that the sun is in the plane of the earth-moon orbit, even though this is not actually the case.) Use the data in the table below. The Sun is massive in comparison to the planets and remains at rest (the center of mass of the three objects is located on the Sun). Treat all objects as point-like particles. The orbital radius of Earth is the distance of the Earth's center from the Sun's center, and the orbital radius of the Moon is the distance of the Moon's center from the Earth's center.

Stellar object	Mass (kg)	Orbital radius (m)
Sun	$1.99 \cdot 10^{30}\, kg$	——
Earth	$5.97 \cdot 10^{24}\, kg$	$1.50 \cdot 10^{11}\, m$
Moon	$7.35 \cdot 10^{22}\, kg$	$3.84 \cdot 10^{8}\, m$

2. (20 points) The point masses m and $2m$ lie along the x-axis, with m at origin and $2m$ at $x = L$. A third point mass M is moved along the x-axis.

   (a) (10 points) At what point is the net gravitational force on M due to the other two masses equal to zero?

(b) (10 points) Sketch the x-component of the net force on M due to m and $2m$, taking quantities to the right as positive. Include the regions $x < 0$, $0 < x < L$, and $x > L$. Be especially careful to show the behavior of the graph on either side of $x = 0$ and $x = L$.

3. (30 points) (a) (10 points) Show that the sun-planet distance at perihelion is $(1 - e)a$, the sun-planet distance at aphelion is $(1 + e)a$, and therefore the sum of these two distances is $2a$.

   (b) (10 points) When the dwarf planet Pluto was at perihelion in 1989, it was 100 million km closer to the sun than Neptune. The semi-major axes of the orbits of Pluto and Neptune are $5.92 \times 10^{12}\,m$ and $4.50 \times 10^{12}\,m$, respectively, and the eccentricities are 0.248 and 0.010. Find Pluto's closest distance and Neptune's farthest distance from the sun.

   (c) (10 points) How many years after being at perihelion in 1989 will Pluto again be at perihelion?

4. (20 points) (a) (5 points) Determine the neutral point (point where the gravitational field is zero) along the imaginary axis connecting the center of the Earth with the center of the Moon. More specific find the distance of the neutral point from the Earth's center.

   (b) (5 points) Determine the gravitational potential energy of the system Earth-Moon.

   (c) (10 points) What is the minimum speed of a rocket is to be launched to reach the neutral point found in part (a)?

5. (10 points) Consider the ring-shaped body of Figure 2. A particle with mass m is placed a distance x from the center of the ring, along the line through the center of the ring and perpendicular to its plane.

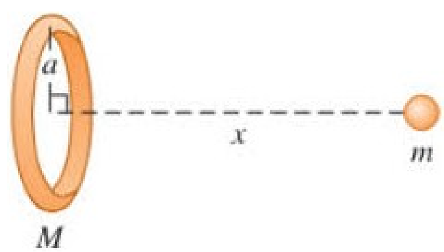

Figure 2

   (a) Calculate the gravitational potential energy U of this system. Take the potential energy to be zero when the two objects are far apart.

   (b) Show that your answer to part (a) reduces to the expected result when x is much larger than the radius a of the ring.

   (c) Use $F_x = -dU/dx$ to find the magnitude and direction of the force on the particle (see Section 7.4).

(d) Show that your answer to part (c) reduces to the expected result when x is much larger than a.

(e) What are the values of U and F_x when $x = 0$? Explain why these results make sense.