Multivariable Calculus

Assignment 12 Friday, December 9, 2016

47. (a) Suppose that $z = f(u)$ and $u = g(x, y)$. Draw a tree diagram, and use it to construct chain rules that express $\partial z/\partial x$ and $\partial z/\partial y$ in terms of dz/du, $\partial u/\partial x$, and $\partial u/\partial y$.

(b) Show that

$$\frac{\partial^2 z}{\partial x^2} = \frac{dz}{du}\frac{\partial^2 u}{\partial x^2} + \frac{d^2 z}{du^2}\left(\frac{\partial u}{\partial x}\right)^2$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{dz}{du}\frac{\partial^2 u}{\partial y^2} + \frac{d^2 z}{du^2}\left(\frac{\partial u}{\partial y}\right)^2$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{dz}{du}\frac{\partial^2 u}{\partial y \partial x} + \frac{d^2 z}{du^2}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}$$

48. (a) Let $z = f(x^2 - y^2)$. Use the result in Exercise 47(a) to show that $y\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y} = 0$

(b) Let $z = f(xy)$. Use the result in Exercise 47(a) to show that $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = 0$

(c) Confirm the result in part (a) in the case where $z = \sin(x^2 - y^2)$.

(d) Confirm the result in part (b) in the case where $z = e^{xy}$.

49. Let f be a differentiable function of one variable, and let $z = f(x + 2y)$. Show that

$$2\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

50. Let f be a differentiable function of one variable, and let $z = f(x^2 + y^2)$. Show that

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$$

51. Let f be a differentiable function of one variable, and let $w = f(u)$, where $u = x + 2y + 3z$. Show that

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 6\frac{dw}{du}$$

52. Let f be a differentiable function of one variable, and let $w = f(\rho)$, where $\rho = (x^2 + y^2 + z^2)^{1/2}$. Show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 = \left(\frac{dw}{d\rho}\right)^2$$

53. Let $z = f(x - y, y - x)$. Show that $\partial z/\partial x + \partial z/\partial y = 0$.

54. Let f be a differentiable function of three variables and suppose that $w = f(x - y, y - z, z - x)$. Show that

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$