

M9X & M9Y - Advanced Placement Statistics  
Review Set on Random Variables (handed out on April 28th)  
- SOLUTIONS

April 28, 2017

[For Question 2, review pages 424-425 of your textbook](#)

2. (a) Let  $t$  be the random variable representing total water bottles sold,  $s$  be the random variable representing the total bottles sold at the stall, and  $w$  be the random variable representing the total bottles sold via the web site. Then  $t = s + w$ . Therefore,

$$E(t) = E(s) + E(w) = 128 + 223 = 351$$

- (b) In order to calculate the standard deviation of  $t$ , we need to assume that  $s$  and  $w$  are independent. This assumption may not be reasonable as there may be confounding variables that affect both sales at the stall and via the web site, such as weather and temperature.
- (c) Since  $t = s + w$ , we know that  $\sigma^2(t) = \sigma^2(s) + \sigma^2(w)$ . Therefore,

$$\sigma(t) = \sqrt{\sigma^2(s) + \sigma^2(w)} = \sqrt{16^2 + 35^2} \approx 38.484$$

- (d) Let  $d$  be the random variable representing the difference between bottles sold via the web site and bottles sold at the stall. Then,  $d = w - s$ . Like in parts (a) and (c) we can calculate the mean(expected value) and standard deviation of  $d$  as follows:

$$E(d) = E(w) - E(s) = 223 - 128 = 95$$

and

$$\sigma(d) = \sqrt{\sigma^2(s) + \sigma^2(w)} = \sqrt{16^2 + 35^2} \approx 38.484$$

(DO notice that although this is a difference of two random variables, the variances are still added like in part (c)).

- (e) Let  $M$  be the random variable representing the total money made on a random day. Then  $M = 1.25s + 0.57w$ . Therefore,

$$E(M) = 1.25E(s) + 0.57E(w) = 1.25 \times 128 + 0.57 \times 223 = 287.11$$

and

$$\sigma(M) = \sqrt{1.25^2\sigma^2(s) + 0.57^2\sigma^2(w)} = \sqrt{1.25^216^2 + 0.57^235^2} \approx 28.249$$

For Questions 3 and 4, review pages 430-437 of your textbook

3. Let  $N$  be the random variable representing the free throws that Nick makes. Then  $N \sim \text{Binomial}(6, 0.7)$

- (a)  $P(\text{makes exactly four free throws}) = P(N = 4) = \binom{6}{4}(0.7)^4(1-0.7)^2 = \text{binompdf}(6, 0.7, 4) \approx .324$
- (b)  $P(\text{makes at least four free throws}) = P(N = 4) + P(N = 5) + P(N = 6) = \binom{6}{4}(0.7)^4(1-0.7)^2 + \binom{6}{5}(0.7)^5(1-0.7)^1 + \binom{6}{6}(0.7)^6(1-0.7)^0 = 1 - \text{binomcdf}(6, 0.7, 3) \approx .744$

Please note that the TI commands in red is needed to calculate the correct probability, but should not be written as part of your free response.

4. Let  $J$  be the random variable representing the free throws that James makes. Then  $J \sim \text{Binomial}(10, 0.35)$

- (a) i.  $P(\text{makes no free throws}) = P(J = 0) = \binom{10}{0}(0.35)^0(1-0.35)^{10} = \text{binompdf}(10, 0.35, 0) \approx .013$
- ii.  $P(\text{makes at least one free throw}) = 1 - P(\text{makes no free throws}) = 1 - .013 = .987$
- iii.  $P(\text{makes more than three free throws}) = 1 - P(\text{makes three or less free throws}) = 1 - (P(J = 0) + P(J = 1) + P(J = 2) + P(J = 3)) = 1 - \text{binomcdf}(10, 0.35, 3) \approx .486$

- (b)

$$E(J) = n \times p = 10 \times 0.35 = 3.5$$

and

$$\sigma(J) = \sqrt{n \times p \times (1 - p)} = \sqrt{10 \times 0.35 \times .65} \approx 1.508$$

For Question 5, review pages 438-439 of your textbook

5. Since we are now interested at which effort James will have his first success, the new random variable  $J \sim \text{Geometric}(0.35)$ 
  - (a)  $P(\text{first success is \#3}) = (1 - p)^2 \times p = (.65)^2 \times (0.35) \approx .148$
  - (b)  $P(\text{first success in less than three efforts}) = P(J = 1) + P(J = 2) = p + (1 - p) \times p = \text{geometcdf}(0.35, 2) = .5775$
  - (c)  $P(\text{first success in more than three efforts}) = 1 - P(\text{first success in three or less efforts}) = 1 - (.148 + .5775) = .2745$

For Questions 6 and 7, review examples 7.27 and 7.28 on pages 452-455 of your textbook

6. Let  $B$  be the the random variable representing the mass of an adult bear. Then,  $B \sim \text{Normal}(515, 88)$ 
  - (a)
    - i.  $P(480 \leq B \leq 580) = P\left(\frac{480-515}{88} \leq z \leq \frac{580-515}{88}\right) = P(-.398 \leq z \leq .739) = \text{normalcdf}(480, 580, 515, 88) \approx .425$
    - ii.  $P(B \leq 600) = P\left(z \leq \frac{600-515}{88}\right) = P(z \leq .966) = \text{normalcdf}(0, 600, 515, 88) \approx .833$
    - iii.  $P(B \geq 450) = P\left(z \geq \frac{450-515}{88}\right) = P(z \geq -.739) = \text{normalcdf}(450, 2017, 515, 88) \approx .770$
  - (b)  $P(z \geq z^*) = .20 \Rightarrow z^* \approx .842$  ( $\text{invNorm}(.80)$ )  
 $.842 = \frac{B-515}{88} \Rightarrow B = 515 + (.842 \times 88) \approx 589.06$  ( $\text{invNorm}(.80, 515, 88)$ )
7. We know that for female bears the mass is  $B \sim \text{Normal}(X, 51)$ , where  $X$  is the unknown mean value.  
Then  $P(z \leq z^*) = .22 \Rightarrow z^* \approx -.772$  ( $\text{invNorm}(.22)$ )  
Therefore,  $-.772 = \frac{240-X}{51} \Rightarrow 240-X = -.772 \times 51 \Rightarrow X = 240 + (.772 \times 51) = 279.372$