Quarter 4-Classwork 3

Euclidean Division

Definition 1. Let a and b be positive integers. We say that a **divides** b and write a/b, if there exists an integer q so that $b = q \cdot a$. In other words, a divides b if the remainder of the division of b by a is equal to 0.

Moreover, a will be called a **factor** of b.

Examples

1. 3 divides 6 because $6 = 2 \cdot 3$. Therefore, 3 is a factor of 6.
2. 2 divides 8 because $8 = 4 \cdot 2$. Therefore, 2 is a factor of 8.
3. 10 divides 100 because $100 = 10 \cdot 10$. Therefore, 10 is a factor of 100.
4. 3 does not divide 5. Why?

Exercise 1: Consider the following pairs of numbers a, b. Check if a divides b and explain your answer.

1. $a = 3, b = 24$
2. $a = 10, b = 1,000,000$
3. $a = 100, b = 15,000$
4. $a = 30, b = 120$

5. $a = 2, b = 64$

6. $a = 2, b = 70$

7. $a = 3, b = 103$

8. $a = 4, b = 64$

9. $a = 5, b = 125$

10. $a = 5, b = 95$

11. $a = 5, b = 110$

12. $a = 5, b = 37$

13. $a = 7, b = 64$

14. $a = 7, b = 56$

15. $a = 1, b = 35$

16. $a = 1, b = 32$

17. $a = 1, b = 1$

18. $a = 1, b = 9$

19. $a = 2^4, b = 2^6$

20. $a = (-2)^6, b = 2^{15}$

21. $a = 2^2 \cdot 3^2, b = 192$

22. $a = 2^5 \cdot 5, b = 5 \cdot 20$

Definition 2. A positive integer b is called **prime**, if the only positive integers that divide b is the number 1 and b (itself). In other words, if the factors/divisors of b are only 1 and b.

Examples: 2,3,5 and 7 are prime numbers.

Definition 3. A positive integer b is called **composite** if it is not prime, that is, there exists a positive integer a which divides b and a is not equal to 1 and b.

Examples

1. 10 is a composite number because 5 divides 10.
2. 6 is a composite number because 3 divides 6.

Exercise

1. Find all the prime numbers between 10 and 20.

2. Find all the composite numbers between 30 and 50.

3. Find all the prime numbers between 20 and 30.

4. Find all the composite numbers between 90 and 99.

Prime Factorization: Every positive integer can be written as a product of prime numbers.

Examples

1. The prime factorization of 10 is $2 \cdot 5$. Note that both 2 and 5 are prime numbers.
2. The prime factorization of 16 is $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$. Note that $16 = 2 \cdot 8 = 2 \cdot 2 \cdot 4 = 2 \cdot 2 \cdot 2 \cdot 2$ and that 2 is the only prime factor of 8.
3. The prime factorization of 36 is $2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$.

Exercise 2: Find the prime factorization of the following numbers.

1. 36

2. 126

3. 55

4. 527

5. 356

6. 88

7. 93

8. 100

9. 1,000,000,000

10. 99

11. 101

12. 46

13. 32

14. 654

15. 500

16. 5,000

17. 2^6

18. $3^5 \cdot 27$

19. $3^2 \cdot 56$

20. $15 \cdot 3^2 \cdot 55$

21. $25 \cdot 37 \cdot 36$

22. $9 \cdot 41 \cdot 121$

23. $37 \cdot 45$

24. $28 \cdot 33$

25. $100 \cdot 99$

26. $34 \cdot 1024$

Exercise: Find the greatest common factor of the following expressions and apply the distributive property to find their equivalent form.

1. 5+15=

2. 5+120=

3. 1,500+525=

4. 5+85=

5. $10+5=$

6. $255+5=$

7. $255+10=$

8. $255+25=$

9. $22+2=$

10. $2+24=$

11. $8+36=$

12. 18+30=

13. 18+30=

14. 36+34=

15. 36+104=

16. 50+60=

17. 270+95=

18. 2,222+888,800=

19. 125+525=

20. 27+54=

21. 28+54=

22. 36+54=

23. 56+77=

24. 75-35=

25. 80-55=

26. 5-30=

27. 16-124=

28. $2^3 - 2^2 =$

29. $2^{19} - 2^5 =$

30. $2^{19} + 2^6 =$

31. $4^{28} + 2^{55} =$

32. $1^{15} + 1^{20} =$

33. $1^{25} - 1^{20} =$

34. $3^7 - 3^6 =$

35. $20 \cdot 30 + 3 \cdot 15 =$

36. $22 \cdot 121 + 33 \cdot 15 =$

37. $33 \cdot 27 + 44 \cdot 63 =$

38. $100 \cdot 24 + 2^5 \cdot 77 =$

39. 35+15+120=

40. $33+99+132=$

41. $10+1{,}000+1{,}000{,}000=$

42. $2 + 16 + 24 =$

43. $2 + 4 + 8 =$

44. $3 + 81 + 27 =$

45. $2^5 + 2^6 + 2^7 =$

46. $2 \cdot 7 + 3 \cdot 49 + 28 \cdot 11 =$

47. $2 \cdot 3 \cdot 5 \cdot 6 + 33 \cdot 56$

48. $2 \cdot 5 \cdot 11 + 25 \cdot 11 \cdot 56 + 128 \cdot 121 \cdot 55 =$

49. $3^5 \cdot 5^{10} + 5^2 \cdot 3^3 =$

50. $5 \cdot 2^6 + 7 \cdot 2^3 =$

51. $(2^3)^5 \cdot (3^5)^7 + 256 \cdot 81 =$