CALCULUS BC

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $7 = xy - e^{xy}$, then $\frac{dy}{dx} =$

(A) $x - e^{y}$ (B) $y - e^{x}$ (C) $\frac{ye^{xy} + y}{x - xe^{xy}}$ (D) $\frac{-y}{x}$ (E) $\frac{ye^{xy} + y}{x + xe^{xy}}$

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2. The volume of the solid that results when the area between the curve $y = e^{x}$ and the line $y = 0$, from $x = 1$ to $x = 2$, is revolved around the x-axis is

(A) $2\pi(e^{4} - e^{2})$ (B) $\frac{\pi}{2}(e^{4} - e^{2})$ (C) $\frac{\pi}{2}(e^{2} - e)$ (D) $2\pi(e^{2} - e)$ (E) $2\pi e^{2}$

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3. $\int \frac{x-18}{(x+3)(x-4)}dx =$

(A) $\int \frac{5dx}{(x+3)(x-4)}$

(B) $\int \frac{dx}{(x+3)(x-4)}$

(C) $\int \frac{3dx}{x+3} + \int \frac{2dx}{x-4}$

(D) $\int \frac{15dx}{x+3} - \int \frac{14dx}{x-4}$

(E) $\int \frac{3dx}{x+3} - \int \frac{2dx}{x-4}$

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4. If $y = 5x^2 + 4x$ and $x = \ln t$ then $\frac{dy}{dt} =$

(A) $\frac{10}{t} + 4$ (B) $10t\ln t + 4t$ (C) $\frac{10\ln t + 4}{t}$ (D) $\frac{5}{t^2} + \frac{4}{t}$ (E) $10\ln t + \frac{4}{t}$

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5. $\int_0^{\frac{\pi}{2}} \sin^5 x \cos x dx =$

(A) $\frac{1}{6}$ (B) $-\frac{1}{6}$ (C) 0 (D) –6 (E) 6

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6. The tangent line to the curve $y = x^3 - 4x + 8$ at the point (2, 8) has an x-intercept at

(A) (–1, 0) (B) (1, 0) (C) (0, –8) (D) (0, 8) (E) (8, 0)

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7. The graph in the xy–plane represented by $x = 3\sin(t)$ and $y = 2\cos(t)$ is

(A) a circle (B) an ellipse (C) a hyperbola (D) a parabola (E) a line

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8. $\int \frac{dx}{\sqrt{4-9x^2}} =$

(A) $\frac{1}{6}\sin^{-1}\left(\frac{3x}{2}\right)+C$

(B) $\frac{1}{2}\sin^{-1}\left(\frac{3x}{2}\right)+C$

(C) $6\sin^{-1}\left(\frac{3x}{2}\right)+C$

(D) $3\sin^{-1}\left(\frac{3x}{2}\right)+C$

(E) $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+C$

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9. $\lim_{x\to\infty} 4x\sin\left(\frac{1}{x}\right)$ is

(A) 0 (B) 2 (C) 4 (D) 4π (E) nonexistent

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10. The position of a particle moving along the x-axis at time t is given by $x(t) = e^{\cos(2t)}$, $0 \le t \le \pi$. For which of the following values of t will $x'(t) = 0$?

I. $t = 0$

II. $t = \frac{\pi}{2}$

III. $t = \pi$

(A) I only (B) II only (C) I and III only (D) I and II only (E) I, II, and III

11. $\lim_{h \to 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$

(A) -1 (B) 0 (C) $\frac{1}{\sqrt{2}}$ (D) 1 (E) $\sqrt{2}$

12. Use differentials to approximate the change in the volume of a cube when the side is decreased from 8 to 7.99 cm. (in cm^3)

(A) -19.2
(B) -15.36
(C) -1.92
(D) -0.01
(E) -0.0001

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13. The radius of convergence of $\sum_{n=1}^{\infty}\frac{a^n}{(x+2)^n}$; $a > 0$ is

(A) $(a-2) \le x \le (a+2)$
(B) $(a-2) < x < (a+2)$
(C) $(-a-2) > x > (a-2)$
(D) $(a-2) > x > (-a-2)$
(E) $(a-2) \le x \le (-a-2)$

14. $\int_0^1 \sin^{-1}(x)dx =$

(A) 0
(B) $\frac{\pi+2}{2}$
(C) $\frac{\pi-2}{2}$
(D) $\frac{\pi}{2}$
(E) $\frac{-\pi}{2}$

15. The equation of the line *normal* to $y = \sqrt{\frac{5-x^2}{5+x^2}}$ at $x = 2$ is

(A) $81x - 60y = 142$
(B) $81x + 60y = 182$
(C) $20x + 27y = 49$
(D) $20x + 27y = 31$
(E) $81x - 60y = 182$

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16. If c satisfies the conclusion of the Mean Value Theorem for derivatives for $f(x) = 2\sin x$ on the interval $[0, \pi]$, then c could be

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

(E) There is no value of c on $[0, \pi]$

17. The average value of $f(x) = x \ln x$ on the interval $[1, e]$ is

(A) $\frac{e^2+1}{4}$ (B) $\frac{e^2+1}{4(e+1)}$ (C) $\frac{e+1}{4}$ (D) $\frac{e^2+1}{4(e-1)}$ (E) $\frac{3e^2+1}{4(e-1)}$

18. A 17-foot ladder is sliding down a wall at a rate of –5 feet/sec. When the top of the ladder is 8 feet from the ground, how fast is the foot of the ladder sliding away from the wall (in feet/sec)?

(A) $\frac{75}{8}$ (B) $\frac{8}{3}$ (C) $\frac{3}{8}$ (D) -16 (E) $\frac{-75}{3}$

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19. If $\frac{dy}{dx} = 3y\cos x$, and $y = 8$ when $x = 0$, then $y =$

(A) $8e^{3\sin x}$ (B) $8e^{3\cos x}$ (C) $8e^{3\sin x} + 3$ (D) $3\frac{y^2}{2}\cos x + 8$ (E) $3\frac{y^2}{2}\sin x + 8$

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20. The length of the curve determined by $x = 3t$ and $y = 2t^2$ from $t = 0$ to $t = 9$ is

(A) $\int_0^9 \sqrt{9t^2 + 4t^4}\,dt$

(B) $\int_0^{162} \sqrt{9 - 16t^2}\,dt$

(C) $\int_0^{162} \sqrt{9 + 16t^2}\,dt$

(D) $\int_0^3 \sqrt{9 - 16t^2}\,dt$

(E) $\int_0^9 \sqrt{9 + 16t^2}\,dt$

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21. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left(e^{t^2}, e^{-t^3}\right)$, then its velocity vector at time $t = 3$ is

(A) $(\ln 6, \ln(-27))$

(B) $(\ln 9, \ln(-27))$

(C) (e^9, e^{-27})

(D) $(6e^9, -27e^{-27})$

(E) $(9e^9, -27e^{-27})$

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22. The graph of $f(x) = \sqrt{11 + x^2}$ has a point of inflection at

(A) $\left(0, \sqrt{11}\right)$

(B) $\left(-\sqrt{11}, 0\right)$

(C) $\left(0, -\sqrt{11}\right)$

(D) $\left(\sqrt{\frac{11}{2}}, \sqrt{\frac{33}{2}}\right)$

(E) There is no point of inflection.

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23. What is the volume of the solid generated by rotating about the y-axis the region enclosed by $y = \sin x$ and the x-axis, from $x = 0$ to $x = \pi$?

(A) π^2 (B) $2\pi^2$ (C) $4\pi^2$ (D) 2 (E) 4

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24. $\displaystyle\int_{\frac{2}{\pi}}^{\infty} \frac{\sin\left(\frac{1}{t}\right)}{t^2}\,dt =$

(A) 1 (B) 0 (C) -1 (D) 2 (E) Undefined

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25. A rectangle is to be inscribed between the parabola $y = 4 - x^2$ and the x-axis, with its base on the x-axis. A value of x that maximizes the area of the rectangle is

(A) 0 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{\sqrt{3}}{2}$

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26. $\int \frac{dx}{\sqrt{9-x^2}} =$

(A) $\sin^{-1} 3x + C$

(B) $1\text{n}\left|x + \sqrt{9-x^2}\right| + C$

(C) $\frac{1}{3}\sin^{-1} x + C$

(D) $\sin^{-1} \frac{x}{3} + C$

(E) $\frac{1}{3}1\text{n}\left|x + \sqrt{9-x^2}\right| + C$

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27. Find $\lim_{x \to \infty} x^{\frac{1}{x}}$

(A) 0 (B) 1 (C) ∞ (D) -1 (E) $-\infty$

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28. What is the sum of the Maclaurin series $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \ldots + (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} + \ldots$?

(A) 1 (B) 0 (C) −1 (D) e (E) There is no sum.

STOP

END OF PART A SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The first three non-zero terms in the Taylor series about $x = 0$ for $f(x) = \cos x$

(A) $x+\frac{x^3}{3!}+\frac{x^5}{5!}$

(B) $x-\frac{x^3}{3!}+\frac{x^5}{5!}$

(C) $1-\frac{x^2}{2!}+\frac{x^4}{4!}$

(D) $1-\frac{x^2}{2!}-\frac{x^4}{4!}$

(E) $1+\frac{x^2}{2!}+\frac{x^4}{4!}$

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30. $\int \cos^3 x \, dx =$

(A) $\frac{\cos^4 x}{4} + C$

(B) $\frac{\sin^4 x}{4} + C$

(C) $\sin x - \frac{\sin^3 x}{3} + C$

(D) $\sin x + \frac{\sin^3 x}{3} + C$

(E) $\sin^3 x + C$

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31. If $f(x) = (3x)^{(3x)}$ then $f'(x) =$

(A) $(3x)^{(3x)}(3\ln(3x) + 3)$

(B) $(3x)^{(3x)}(3\ln(3x) + 3x)$

(C) $(9x)^{(3x)}(\ln(3x) + 1)$

(D) $(3x)^{(3x-1)}(3x)$

(E) $(3x)^{(3x-1)}(9x)$

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32. To what limit does the sequence $S_n = \frac{3+n}{3^n}$ converge as n approaches infinity?

(A) 1 (B) $\frac{1}{3}$ (C) 0 (D) ∞ (E) 3

33. $\int \frac{18x-17}{(2x-3)(x+1)} dx =$

(A) $8\ln|2x-3| + 7\ln|x+1| + C$

(B) $2\ln|2x-3| + 7\ln|x+1| + C$

(C) $4\ln|2x-3| + 7\ln|x+1| + C$

(D) $7\ln|2x-3| + 2\ln|x+1| + C$

(E) $\frac{7}{2}\ln|2x-3| + 4\ln|x+1| + C$

34. A particle moves along a path described by $x = \cos^3 t$ and $y = \sin^3 t$. The distance that the particle travels along the path from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 0.75 (B) 1.50 (C) 0 (D) −3.50 (E) −0.75

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35. The sale price of an item is 800 – $35x$ dollars and the total manufacturing cost is $2x^3 - 140x^2 + 2{,}600x + 10{,}000$ dollars, where x is the number of items. What number of items should be manufactured in order to optimize the manufacturer's total profit?

(A) 35 (B) 25 (C) 10 (D) 15 (E) 20

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36. The area enclosed by the polar equation $r = 4 + \cos\theta$, for $0 \le \theta \le 2\pi$, is

(A) 0 (B) $\frac{9\pi}{2}$ (C) 18π (D) $\frac{33\pi}{2}$ (E) $\frac{33\pi}{4}$

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37. Use the trapezoid rule with $n = 4$ to approximate the area between the curve $y = x^3 - x^2$ and the x-axis from $x = 3$ to $x = 4$.

(A) 35.266 (B) 27.766 (C) 63.031 (D) 31.516 (E) 25.125

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38. If $f(x)=\sum_{k=0}^{\infty}(\cos^2 x)^k$, then $f\left(\frac{\pi}{4}\right)$ is

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

39. The volume of the solid that results when the area between the graph of $y = x^2 + 2$ and the graph of $y = 10 - x^2$ from $x = 0$ to $x = 2$ is rotated around the x-axis is

(A) $2\pi\int_0^2 y\left(\sqrt{y-2}\right)dy + 2\pi\int_0^2 y\left(\sqrt{10-y}\right)dy$

(B) $2\pi\int_2^6 y\left(\sqrt{y-2}\right)dy + 2\pi\int_6^{10} y\left(\sqrt{10-y}\right)dy$

(C) $2\pi\int_2^6 y\left(\sqrt{y-2}\right)dy - 2\pi\int_6^{10} y\left(\sqrt{10-y}\right)dy$

(D) $2\pi\int_0^2 y\left(\sqrt{y-2}\right)dy - 2\pi\int_0^2 y\left(\sqrt{10-y}\right)dy$

(E) $2\pi\int_0^2 y\left(\sqrt{10-y}\right)dy - 2\pi\int_0^2 y\left(\sqrt{y-2}\right)dy$

40. $\int_0^4 \frac{dx}{\sqrt{9+x^2}} =$

(A) ln3 (B) ln4 (C) −ln2 (D) −ln4 (E) Undefined

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41. The rate that an object cools is directly proportional to the difference between its temperature (in Kelvins) at that time and the surrounding temperature (in Kelvins). If an object is initially at 35*K*, and the surrounding temperature remains constant at 10*K*, it takes 5 minutes for the object to cool to 25*K*. How long will it take for the object to cool to 20*K*?

(A) 6.66 min. (B) 7.50 min. (C) 7.52 min. (D) 8.97 min. (E) 10.00 min.

42. $\int e^x \cos x \, dx =$

(A) $\frac{e^x}{2}(\sin x + \cos x) + C$

(B) $\frac{e^x}{2}(\sin x - \cos x) + C$

(C) $\frac{e^x}{2}(\cos x - \sin x) + C$

(D) $2e^x(\sin x + \cos x) + C$

(E) $e^x(\sin x + \cos x) + C$

43. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4 \sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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44. The minimum value of the function $y = x^3 - 7x + 11,\ x \geq 0$, is approximately

(A) 18.128 (B) 9.283 (C) 6.698 (D) 5.513 (E) 3.872

45. Use Euler's Method with $h = 0.2$ to estimate $y(1)$, if $y' = y$ and $y(0) = 1$.

(A) 1.200 (B) 2.075 (C) 2.488 (D) 4.838 (E) 9.677

STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.