

Άσκηση: Να αποδείξετε
τα παρακάτω.

$$① (a+b) \cdot (\gamma+\delta) = (\gamma+\delta) \cdot (a+b)$$

$$② (a+b) \cdot \delta = \delta \cdot (a+b)$$

$$③ \gamma \cdot (a+b) \cdot \delta = (\beta+a) \cdot \delta \cdot \gamma$$

Λύση:

$$① (a+b) \cdot (x+d) = (x+d) \cdot (a+b)$$

$(\square \cdot \Delta = \Delta \cdot \square)$ Αντιμεταθετική
↓

$$\square = (a+b), \Delta = (x+d)$$

$$② (a+b) \cdot d = d \cdot (a+b)$$

↓
Αντιμεταθετική

$$\square \cdot \Delta = \Delta \cdot \square \quad \square = (a+b)$$

$$\Delta = d$$

$$(3) \gamma \cdot (\alpha + \beta) \cdot \delta = \gamma \cdot (\beta + \alpha) \cdot \delta$$

Αντιμετάθεση

$$\square + \Delta = \Delta + \square$$

$$\square = \alpha, \Delta = \beta$$

$$= (\beta + \alpha) \cdot \gamma \cdot \delta$$

$$= (\beta + \alpha) \cdot \delta \cdot \gamma$$

Αντιμετάθεση

$$\square \cdot \Delta = \Delta \cdot \square$$

$$\square = \beta + \alpha, \Delta = \gamma$$

Αντιμετάθεση

$$\square \cdot \Delta = \Delta \cdot \square$$

$$\square = \gamma, \Delta = \delta$$

Άσκηση: Να αποδείξετε τα παρακάτω.

$$(1) \gamma \cdot \delta \cdot \gamma = \gamma^2 \cdot \delta$$

$$(2) \gamma + \delta + \gamma = 2\gamma \cdot \delta$$

$$(3) \gamma \cdot \delta \cdot \gamma \cdot \delta \cdot \gamma = \delta^2 \cdot \gamma^3$$

$$(4) ((a + b - \gamma) + (\delta - \epsilon)) + \zeta = (a + b - \gamma) + (\delta + (\zeta - \epsilon))$$

Λύση

$$\begin{aligned} \textcircled{1} \quad \gamma \cdot \delta \cdot \gamma &= \gamma \cdot \gamma \cdot \delta = 1 \cdot 1 \gamma^{1+1} \cdot \delta \\ &= 1\gamma^2 \cdot \delta \\ &= \gamma^2 \cdot \delta \end{aligned}$$

Αντιμεταθετική

$$\square \cdot \Delta = \Delta \cdot \square$$

$$\square = \delta, \Delta = \gamma$$

$$\begin{aligned} \textcircled{2} \quad \gamma + \delta + \gamma &= \gamma + \gamma + \delta = (1+1)\gamma + \delta \\ &= 2\gamma + \delta \end{aligned}$$

Αντιμεταθετική

$$\square + \Delta = \Delta + \square$$

$$\square = \delta, \Delta = \gamma$$

$$(3) \gamma \cdot \delta \cdot \gamma \cdot \delta \cdot \gamma = \delta \cdot \gamma \cdot \gamma \cdot \delta \cdot \gamma$$

Αντιμεταθετική

$$\square \cdot \Delta = \Delta \cdot \square$$

$$\square = \delta, \Delta = \gamma$$

$$= \delta \cdot \gamma^2 \cdot \delta \cdot \gamma$$

$$= \delta \cdot \delta \cdot \gamma^2 \cdot \gamma$$

$$= \delta^2 \cdot \gamma^3$$

Αντιμεταθετική

$$\square \cdot \Delta = \Delta \cdot \square$$

$$\square = \gamma^2, \Delta = \delta$$

4

$$\textcircled{\otimes} + (-\square) = -\square$$

$$((a+b-\gamma)+(\delta-\epsilon))+\zeta = (a+b-\gamma)+((\delta-\epsilon)+\zeta)$$

Προσεταιριστική

$$\square = (a+b-\gamma)$$

$$\Delta = (\delta-\epsilon)$$

$$0 = \zeta$$

Προσεταιριστική

$$\square = \delta$$

$$\Delta = (-\epsilon)$$

$$0 = \zeta$$

$$\textcircled{\otimes} = (a+b-\gamma) + ((\delta+(-\epsilon))+\zeta)$$

$$= (a+b-\gamma) + (\delta + (\epsilon) + \zeta)$$

$$= (a+b-\gamma) + (\delta + (\zeta + (-\epsilon)))$$

$$\textcircled{\otimes} = (a+b-\gamma) + (\delta + (\zeta - \epsilon))$$

Αντιμεταθετική

$$\square = (-\epsilon)$$

$$\Delta = \zeta$$