

Να αποδείξετε πως ισχύουν  
οι παρακάτω ισότητες.

$$1) ((a+b)+\gamma)+\delta = a+(\beta+(\gamma+\delta))$$

$$2) ((a+b)+\gamma)-\delta = a+(\beta+(\gamma-\delta))$$

$$3) ((a-b)+\gamma)+\delta = a+(-\beta+(\gamma+\delta))$$

$$4) ((a-b)-\gamma)+\delta = a+(-\beta+(-\gamma+\delta))$$

$$5) ((a+b)+\gamma)+\delta = (a+b)+(\gamma+\delta)$$

$$6) ((a-b-\gamma)+\delta)+\epsilon = (a-b-\gamma)+(\delta+\epsilon)$$

# Λύσεις

$$\begin{aligned}\square &= a \\ \Delta &= \beta \\ \diamond &= \gamma\end{aligned}$$

$$1) ((a+\beta)+\gamma)+\delta = (a+(\beta+\gamma))+\delta$$

$$\begin{aligned}\square &= a \\ \Delta &= (\beta+\gamma) \\ \diamond &= \delta\end{aligned} \quad \begin{aligned}\square &= a \\ \Delta &= (\beta+\gamma) \\ \diamond &= \delta\end{aligned} = a + ((\beta+\gamma)+\delta)$$

$$\begin{aligned}\square &= \beta \\ \Delta &= \gamma \\ \diamond &= \delta\end{aligned} \quad \begin{aligned}\square &= \beta \\ \Delta &= \gamma \\ \diamond &= \delta\end{aligned} = a + (\beta + (\gamma+\delta))$$

$$2) ((a+\beta)+\gamma)-\delta = ((a+\beta)+\gamma)+(-\delta)$$

$$\begin{aligned}\square &= a \\ \Delta &= \beta \\ \diamond &= \gamma\end{aligned} \quad \begin{aligned}\square &= a \\ \Delta &= \beta \\ \diamond &= \gamma\end{aligned} = ((a+(\beta+\gamma))+(-\delta))$$

$$\begin{aligned}\square &= a \\ \Delta &= (\beta+\gamma) \\ \diamond &= (-\delta)\end{aligned} \quad \begin{aligned}\square &= a \\ \Delta &= (\beta+\gamma) \\ \diamond &= (-\delta)\end{aligned} = a + ((\beta+\gamma)+(-\delta))$$

$$\begin{aligned}\square &= \beta \\ \Delta &= \gamma \\ \diamond &= (-\delta)\end{aligned} \quad \begin{aligned}\square &= \beta \\ \Delta &= \gamma \\ \diamond &= (-\delta)\end{aligned} = a + (\beta + (\gamma+(-\delta)))$$
$$= a + (\beta + (\gamma-\delta))$$

Reminder: Προσεταιριστική ιδιότητα

$$(\square + \Delta) + \diamond = \square + (\Delta + \diamond)$$

$$3) ((a-b)+x)+d = ((a+(-b))+x)+d$$

$$\square = a$$

$$\Delta = (-b)$$

$$\diamond = x$$

$$\stackrel{\otimes}{=} (a + ((-b) + x)) + d$$

$$\square = a$$

$$\Delta = ((-b) + x)$$

$$\diamond = d$$

$$\stackrel{\otimes}{=} a + (((-b) + x) + d)$$

$$\stackrel{\otimes}{=} a + ((-b) + (x + d))$$

$$\square = (-b)$$

$$\Delta = x$$

$$\diamond = d$$

$$= a + (-b + (x + d))$$

$$4) ((a-b)-x)+d = ((a+(-b))+(-x))+d$$

$$\square = a$$

$$\Delta = (-b)$$

$$\diamond = (-x)$$

$$\stackrel{\otimes}{=} (a + ((-b) + (-x))) + d$$

$$\square = a$$

$$\Delta = ((-b) + (-x))$$

$$\diamond = d$$

$$\stackrel{\otimes}{=} a + (((-b) + (-x)) + d)$$



$$\otimes = a + \left( (-\beta) + (1-\gamma) + \delta \right)$$

$$= a + \left( -\beta + (-\gamma + \delta) \right)$$


---

$$5) \left( (a + \beta) + \gamma \right) + \delta \otimes = (a + \beta) + (\gamma + \delta)$$

$$\square = (a + \beta)$$

$$\Delta = \gamma$$

$$\diamond = \delta$$


---

Reminder:  $(\square + \Delta) + \diamond = \square + (\Delta + \diamond)$

6)

$$((a - b - \gamma) + \delta) + \varepsilon = (a - b - \gamma) + (\delta + \varepsilon)$$

⊗



Προσεταιριστική  
ιδιότητα.

$$\square = (a - b - \gamma)$$

$$\Delta = \delta$$

$$\diamond = \varepsilon$$