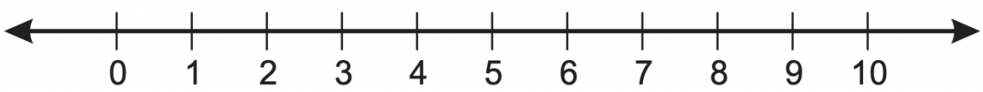
Let's Be Rational!

When you first started learning about numbers, you learned about the natural numbers, or counting numbers: 1, 2, 3, 4, 5, …Later on you learned that there were numbers that lived between the whole numbers called rational numbers or fractions. You learned that some numbers can be written as improper fractions and as mixed numbers. Today we'll see how these concepts extend to polynomial and rational functions.

1. Consider the number $\frac{22}{7}$.
   a. Write the improper fraction as a mixed number.
   b. How many times does 7 go into 22?
   c. How many times greater is 22 than 7?
   d. Is 7 a factor of 22? Explain.
   e. Graph the number on the number line below.

2. Now think about the mixed number $8\frac{5}{6}$.
   a. Write the number as an improper fraction.
   b. How did you do this?
   c. Graph the number on the number line below.



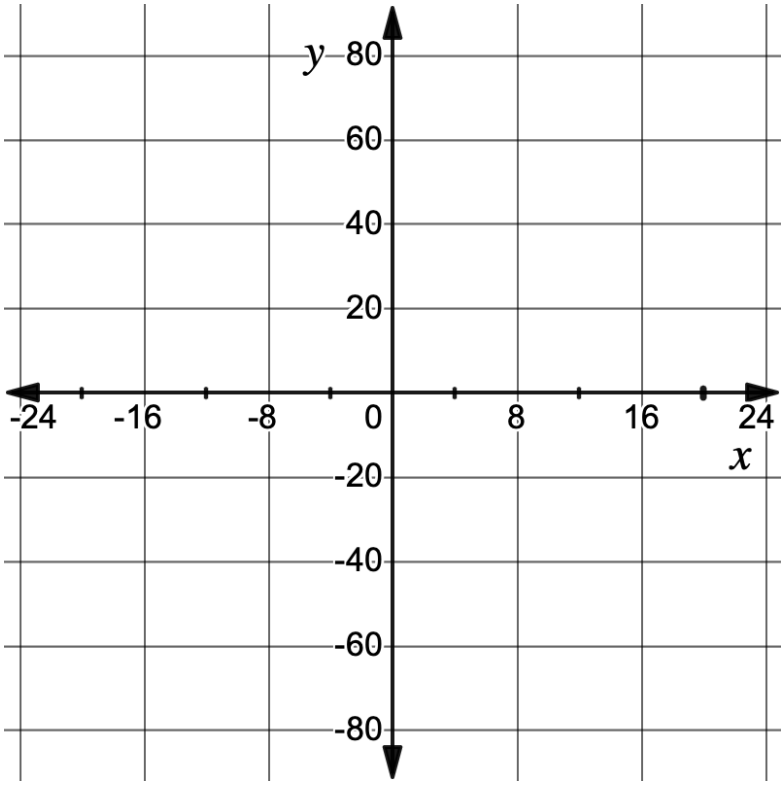
3. How does rewriting a number in different ways reveal different properties about the number?

4. Now consider the rational function $f(x) = \frac{2x^2+5x+10}{x+4}$.
   a. A rational function, like a rational number, can be thought of as a division problem. Use the area model to divide the two polynomials.

x	$2x^2$	
4		

   b. Does $x + 4$ go into $2x^2 + 5x + 10$ evenly? If not, what is the remainder?
   c. Rewrite the expression for $f(x)$ as a "mixed" function.

5. Use desmos.com to graph $y = f(x)$. Sketch it below.



6. Identify any holes or vertical asymptotes.

7. Explain why f looks linear for very large values of x and very small (negative) values of x.

8. How does rewriting a function in a different way reveal different properties about the function?

Lesson 2.8 – Equivalent Representations of Rational Functions

QuickNotes

Check Your Understanding

1. Divide.
   $(x^3 + 2x^2 - 5x + 4) \div (x - 3)$

2. Find the slant asymptote of $y = \frac{x^2+3x+2}{x-2}$.

3. Let $f(x) = \frac{g(x)}{x+4}$ for some polynomial g. If $x + 4$ goes into $g(x)$ evenly, describe what is happening on the graph of f at $x = -4$.

4. a. Find $(x^2 + 4)(x - 6)$.

   b. Is $x^2 + 4$ a factor of $x^3 - 6x^2 + 4x - 20$? How do you know? If not, what is the remainder?