Mutual-Exclusivity

In the English language, the word “or” is ambiguous. In other words, it has more than one meaning. Either it expresses mutual *inclusivity* or it expresses mutual *exclusivity*. On the one hand, the *inclusive*-“or” means “one, the other, or both”. On the other hand, the *exclusive*-“or” means “one or the other, but *not* both”. To see the difference, consider the truth table below.

The 3rd column exhibits mutual-*inclusivity*. In the 3rd column, the disjunction is true in three ways: if only P is true, if only Q is true, or if both P and Q are true. In contrast, the 6th column exhibits mutual-*exclusivity*. In the 6th column, the conjunction is true in only two ways: if only P is true or if only Q is true. if both P and Q are true.

1st column	2nd column	3rd column	4th column	5th column	6th column
P	**Q**	**P∨Q**	**P∧Q**	**~(P∧Q)**	**(P∨Q)∧~(P∧Q)**
TRUE	TRUE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	FALSE	FALSE	TRUE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE

The difference between the two meanings of “or” can be explained with an illustrative example. Let “P” stand for the proposition “the dog is chasing the cat”, and let “Q” stand for the proposition “the cat is chasing the rat”. Accordingly, proposition “P∨Q” would stand for the disjunction “the dog is chasing the cat, the cat is chasing the rat, or *both*”. In other words, the disjunction “P∨Q” expresses the idea that “P” and “Q” are mutually *inclusive*: each one of the two can be true *individually*, and *both* can be true together. In stark contrast, “(P∨Q)∧~(P∧Q)” would stand for “either the dog is chasing the cat or the cat is chasing the rat, but *not* both”. In other words, “(P∨Q)∧~(P∧Q)” expresses the idea that “P” and “Q” are mutually *exclusive*: each one of the two can be true *individually*, but both *cannot* be true together.

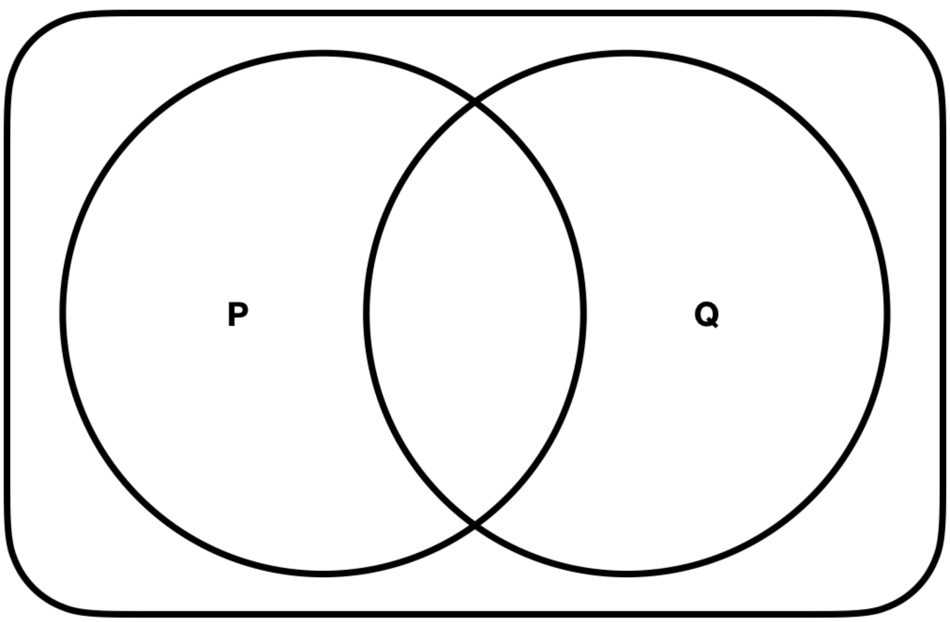
Sometimes, it is helpful to illustrate these ideas with the help of Venn diagrams. On the following pages, consider the following seven Venn diagrams.

Venn Diagrams

1. If "~(P∧Q)" is *true*, then "P∧Q" is *false*.

P	**Q**	**P∧Q**	**~(P∧Q)**
T	T	T	F
F	F	F	T
T	F	F	T
F	T	F	T

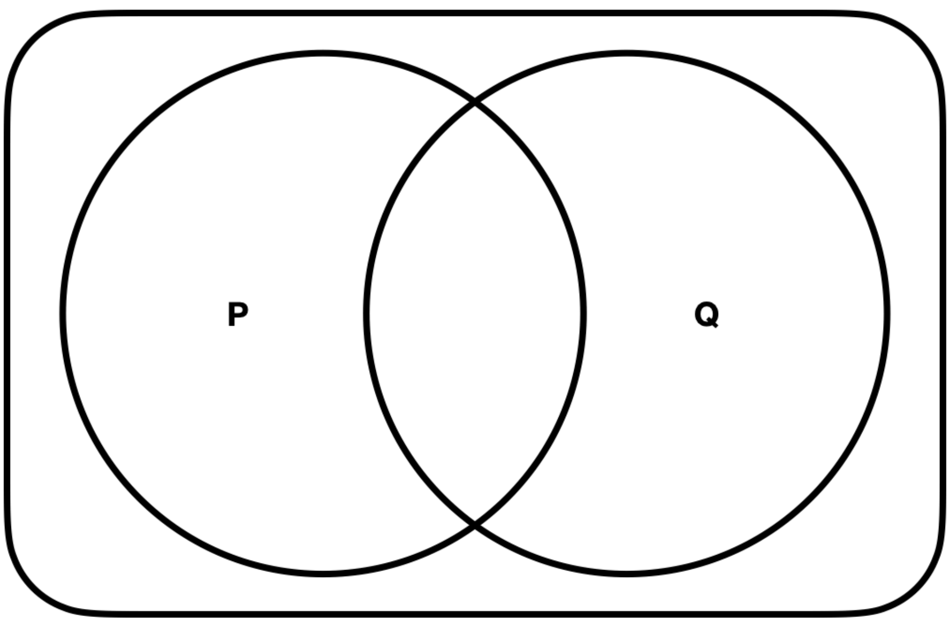
Assuming that "~(P∧Q)" is *true*, shade in each part of the diagram that represents a *false* proposition.



2. If “P∧Q” is *true*, then “P∧~Q” is *false*, “~P∧Q” is *false*, and “~P∧~Q” is *false*.

P	**Q**	**P∧Q**	**~P**	**~Q**	**P∧~Q**	**~P∧Q**	**~P∧~Q**
T	T	T	F	F	F	F	F
F	F	F	T	T	F	F	T
T	F	F	F	T	T	F	F
F	T	F	T	F	F	T	F

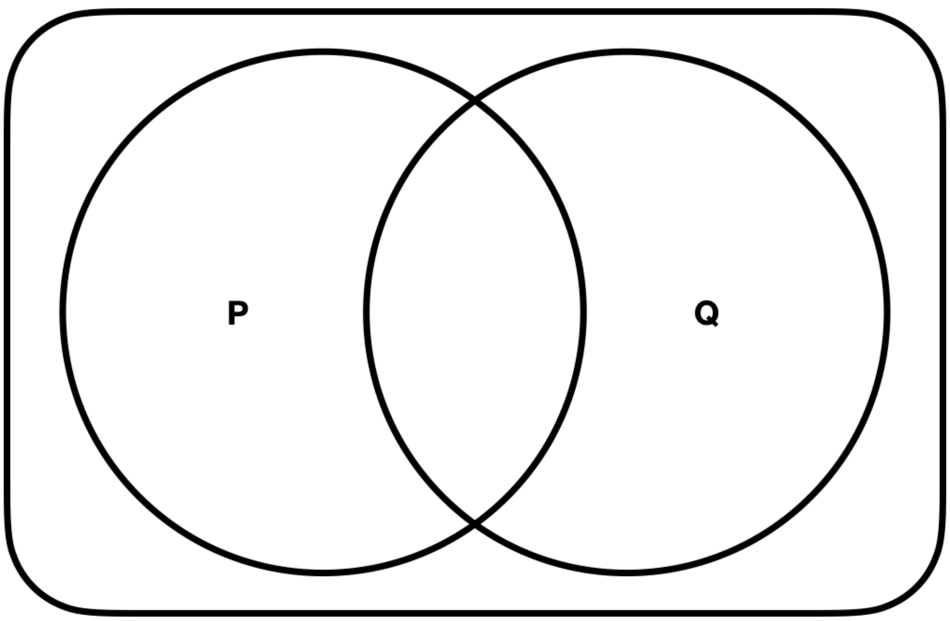
Assuming that “P∧Q” is *true*, shade in each part of the diagram that represents a *false* proposition.



3. If “P∧~Q” is *true*, then “P∧Q” is *false*, “~P∧Q” is *false*, and “~P∧~Q” is *false*.

P	**Q**	**P∧Q**	**~P**	**~Q**	**P∧~Q**	**~P∧Q**	**~P∧~Q**
T	T	T	F	F	F	F	F
F	F	F	T	T	F	F	T
T	F	F	F	T	T	F	F
F	T	F	T	F	F	T	F

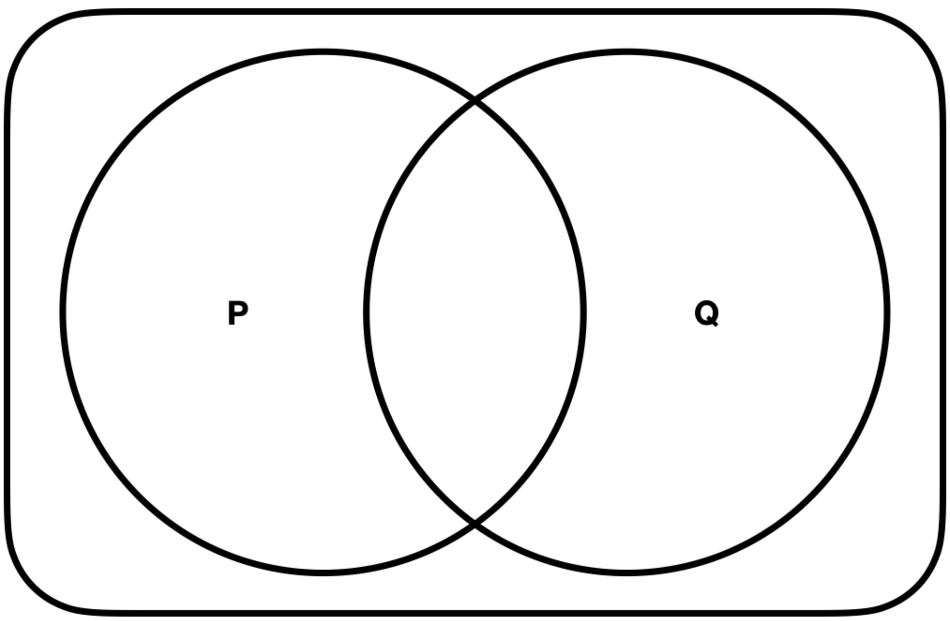
Accordingly, shade in each part of the diagram that represents a *false* proposition.



4. If “~P∧Q” is *true*, then “P∧Q”, is *false*, “P∧~Q” is *false*, and “~P∧~Q” is *false*.

P	**Q**	**P∧Q**	**~P**	**~Q**	**P∧~Q**	**~P∧Q**	**~P∧~Q**
T	T	T	F	F	F	F	F
F	F	F	T	T	F	F	T
T	F	F	F	T	T	F	F
F	T	F	T	F	F	T	F

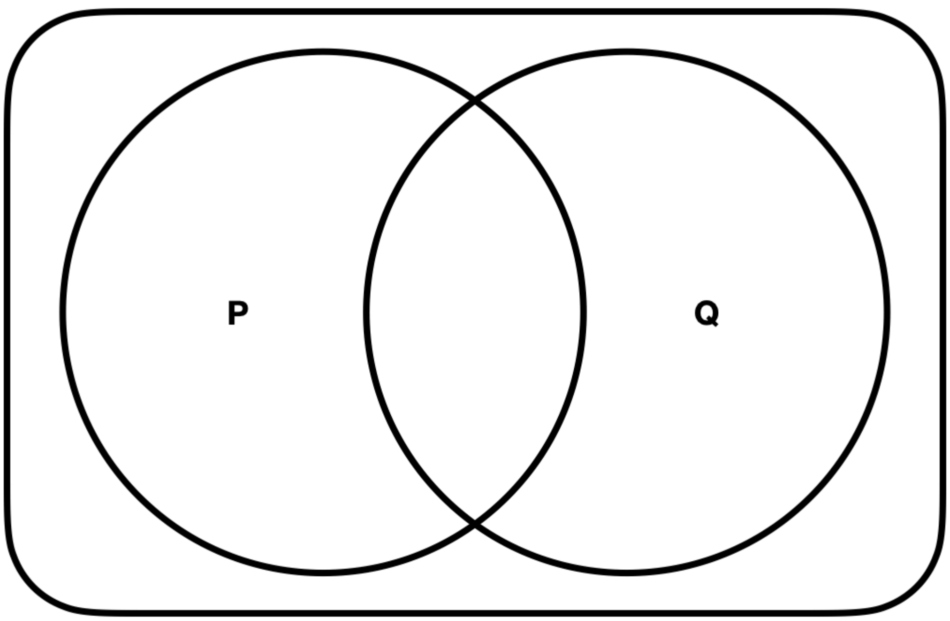
Accordingly, shade in each part of the diagram that represents a *false* proposition.



5. If “P∨Q” is *true*, then “~P∧~Q” is *false*.

P	**Q**	**P∧Q**	**~P**	**~Q**	**~(P∧Q)**	**P∧~Q**	**~P∧Q**	**~P∧~Q**	**P∨Q**
T	T	T	F	F	F	F	F	F	T
F	F	F	T	T	T	F	F	T	F
T	F	F	F	T	T	T	F	F	T
F	T	F	T	F	T	F	T	F	T

Accordingly, shade in each part of the diagram that represents a *false* proposition.

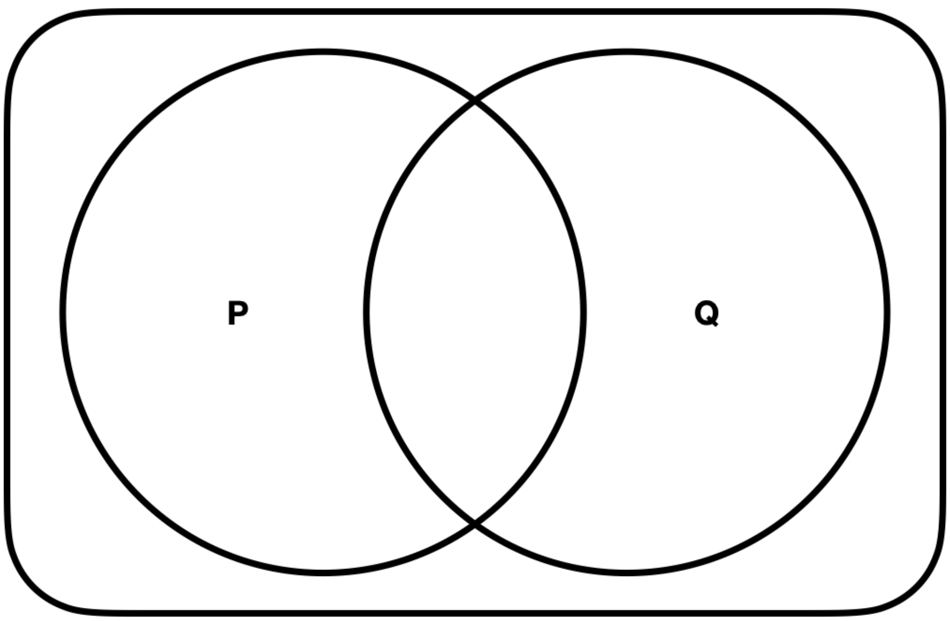


This is mutual-*inclusivity*! This is the *inclusive*-‘or’. The diagram shows that propositions “P” and “Q” describe mutually-*inclusive* possibilities.

6. If “(P∨Q)∧~(P∧Q)” is *true*, then “~P∧~Q” and “P∧Q” are *false*.

P	**Q**	**P∧Q**	**~P**	**~Q**	**~(P∧Q)**	**P∧~Q**	**~P∧Q**	**~P∧~Q**	**P∨Q**	**(P∨Q)∧~(P∧Q)**
T	T	T	F	F	F	F	F	F	T	F
F	F	F	T	T	T	F	F	T	F	F
T	F	F	F	T	T	T	F	F	T	T
F	T	F	T	F	T	F	T	F	T	T

Accordingly, shade in each part of the diagram that represents a *false* proposition.



This is mutual-exclusivity! This is the *exclusive*-‘or’. The diagram shows that propositions “P” and “Q” describe mutually-*exclusive* possibilities.

7. If “~P∧~Q” is *true*, then “P∨Q” is *false*.

P	**Q**	**P∧Q**	**~P**	**~Q**	**~(P∧Q)**	**P∧~Q**	**~P∧Q**	**~P∧~Q**	**P∨Q**	**(P∨Q)∧~(P∧Q)**
T	T	T	F	F	F	F	F	F	T	F
F	F	F	T	T	T	F	F	T	F	F
T	F	F	F	T	T	T	F	F	T	T
F	T	F	T	F	T	F	T	F	T	T

Accordingly, shade in each part of the diagram that represents a *false* proposition.