

Study Guide

Chapter 1

1-2: Points, Lines, and Planes

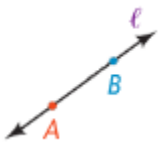
A **point** is a mark of position or location. It has no size or magnitude. Points can be named using capital letters.

Example:



A **straight line** is a collection or set of infinite points all in a straight path. It extends **indefinitely** in two opposite directions. A line could be named using any two points on it or using a small letter. A line **has no thickness**.

Example:



A **plane** is a collection of points and lines on a **flat surface** which extends indefinitely in each direction. A plane has no thickness.

Example:



A plane can be named using at least 3 non-collinear points on it. Points on the same plane are coplanar. If not, non-coplanar.

A **line segment** is a part of a line that has two **endpoints** and contain all of the points between them.

Example:



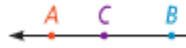
A **ray** is a part of a line that consists of one endpoint and all the points of the line on one side of the endpoint. It extends indefinitely in one direction.

Example:



Opposite rays are two rays that share the same endpoint and go in opposite directions to form a straight line.

Example:



Postulate/Axiom is an accepted statement of fact. Postulates, like undefined terms, are basic building blocks of the logical system in geometry.

Postulate 1-1:

take note

Postulate 1-1

Through any two points there is exactly one line.

Line t passes through points A and B . Line t is the only line that passes through both points.

When you have two or more geometric figures, their **intersection** is the set of points the figures have in common.

Postulate 1-2:

take note

Postulate 1-2

If two distinct lines intersect, then they intersect in exactly one point.

\overleftrightarrow{AE} and \overleftrightarrow{DB} intersect in point C .

There is a similar postulate about the intersection of planes.

Postulate 1-3:

take note

Postulate 1-3

If two distinct planes intersect, then they intersect in exactly one line.

Plane RST and plane WST intersect in \overleftrightarrow{ST} .

Postulate 1-4:

take note

Postulate 1-4

Through any three noncollinear points there is exactly one plane.

Points Q , R , and S are noncollinear. Plane P is the only plane that contains them.

1-3: Measuring Segments

Postulate 1-5:

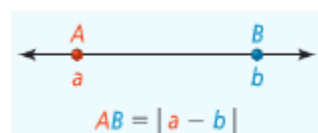
take note **Postulate 1-5 Ruler Postulate**

Every point on a line can be paired with a real number. This makes a one-to-one correspondence between the points on the line and the real numbers. The real number that corresponds to a point is called the **coordinate** of the point.

coordinate of A coordinate of B

The Ruler Postulate allows you to measure lengths of segments using a given unit and to find distances between points on a number line. Consider line AB below.

The distance between points A and B is the absolute value of the difference of their coordinates, or



$|a - b|$. This value is also AB, or the length of line segment AB.

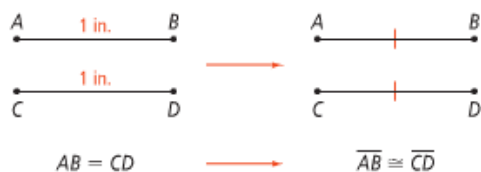
Postulate 1-6:

take note **Postulate 1-6 Segment Addition Postulate**

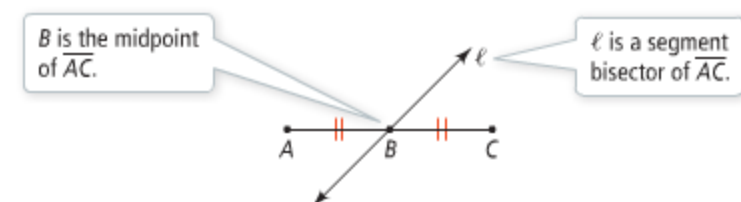
If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

When numerical expressions have the same value, you say that they are equal. Similarly, if two segments have the same length, then the segments are **congruent**.

This means that if $AB = CD$, then $\overline{AB} \cong \overline{CD}$. You can also say that if $\overline{AB} \cong \overline{CD}$, then $AB = CD$.



The **midpoint** of a segment is a point that divides the segment into two congruent segments. A point, line, ray, or other segment that intersects a segment at its midpoint is said to bisect the segment. That is called the **segment bisector**.



1-4: Measuring Angles

An **angle** is formed by two rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.



The sides of the angle are \overrightarrow{AB} and \overrightarrow{AC} .

The vertex is A.

You can name an angle by:

- Its vertex
- A point on each ray and the vertex.
- A number

One way to measure an angle is in degrees. To indicate the measure of an angle, write a lowercase m in front of the angle symbol.

A circle has 360 degrees, so 1 degree is $1/360$ of a circle.

Postulate 1-7:

take note

Postulate 1-7 Protractor Postulate

Consider \overrightarrow{OB} and a point A on one side of \overrightarrow{OB} . Every ray of the form \overrightarrow{OA} can be paired one to one with a real number from 0 to 180.



Types of Angles

take note

Key Concept Types of Angles

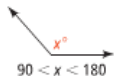
acute angle



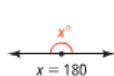
right angle



obtuse angle



straight angle



Postulate 1-8:

take note

Postulate 1-8 Angle Addition Postulate

If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.



1-5: Explaining Angle Pairs

Types of Angle Pairs:

Take note

Key Concept Types of Angle Pairs

Definition	Example
Adjacent angles are two coplanar angles with a common side, a common vertex, and no common interior points.	$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$
Vertical angles are two angles whose sides are opposite rays.	$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$
Complementary angles are two angles whose measures have a sum of 90. Each angle is called the <i>complement</i> of the other.	$\angle 1$ and $\angle 2$, $\angle A$ and $\angle B$
Supplementary angles are two angles whose measures have a sum of 180. Each angle is called the <i>supplement</i> of the other.	$\angle 3$ and $\angle 4$, $\angle B$ and $\angle C$

A **linear pair** is a pair of adjacent angles whose noncommon sides are opposite rays. The angles of a linear pair form a straight angle.

An angle bisector is a ray that divides an angle into two congruent angles. Its endpoint is at the

Take note

Postulate 1-9 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.


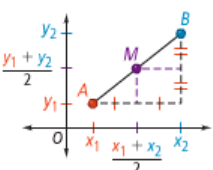
angle vertex. Within the ray, a segment with the same endpoint is also an angle bisector. The ray or segment bisects the angle.

1-7: Midpoint and Distance in the Coordinate Plane

Midpoint Formulas

Take note

Key Concept Midpoint Formulas

Description	Formula	Diagram
On a Number Line The coordinate of the midpoint is the <i>average</i> or <i>mean</i> of the coordinates of the endpoints.	The coordinate of the midpoint M of \overline{AB} is $\frac{a+b}{2}$.	
In the Coordinate Plane The coordinates of the midpoint are the average of the x-coordinates and the average of the y-coordinates of the endpoints.	Given \overline{AB} where $A(x_1, y_1)$ and $B(x_2, y_2)$, the coordinates of the midpoint of \overline{AB} are $M(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.	

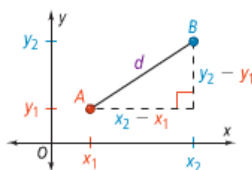
Distance Formula

Take note

Key Concept Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



1-8: Perimeter, Circumference, and Area

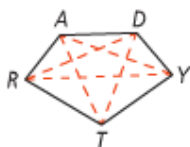
A **polygon** is a shape with at least 3 sides.

The **perimeter P** of a polygon is the sum of the lengths of its sides. The **area A** of a polygon is the number of square units it encloses.

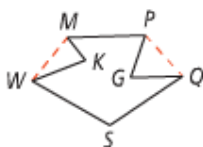
A polygon of equal size (each side is the same length) is a **regular** polygon, such as an equilateral triangle or a quadrilateral square.

A polygon of unequal size (each side is a different length) is a **complex** polygon.

You can also classify a polygon as **concave or convex**, using the diagonals of the polygon. A **diagonal** is a segment that connects two nonconsecutive vertices in a polygon.



A **convex polygon** has no diagonal with points outside the polygon.



A **concave polygon** has at least one diagonal with points outside the polygon.

Perimeter and Area for Common Shapes

Take note

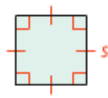
Key Concept Perimeter, Circumference, and Area

Square

side length s

$$P = 4s$$

$$A = s^2$$

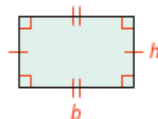


Rectangle

base b and height h

$$P = 2b + 2h, \text{ or } 2(b + h)$$

$$A = bh$$

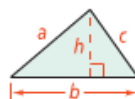


Triangle

side lengths a , b , and c ,
base b , and height h

$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

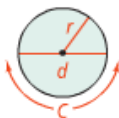


Circle

radius r and diameter d

$$C = \pi d, \text{ or } C = 2\pi r$$

$$A = \pi r^2$$

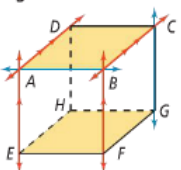


Note: When describing the area of a polygon, you must use square units.

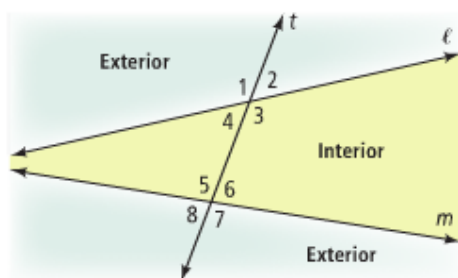
Chapter 3

3-1: Lines and Angles

Parallel and Skew

Key Concept Parallel and Skew		
Definition Parallel lines are coplanar lines that do not intersect. The symbol \parallel means "is parallel to."	Symbols $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$	Diagram  Use arrows to show $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.
Skew lines are noncoplanar; they are not parallel and do not intersect.	\overleftrightarrow{AB} and \overleftrightarrow{CG} are skew.	
Parallel planes are planes that do not intersect.	plane $ABCD \parallel$ plane $EFGH$	

A **transversal** is a line that intersects two or more coplanar lines at distinct points.



Angle Pairs formed by Transversals

Key Concept Angle Pairs Formed by Transversals	
Definition Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.	Example $\angle 4$ and $\angle 6$ $\angle 3$ and $\angle 5$
Same-side interior angles are interior angles that lie on the same side of the transversal.	$\angle 4$ and $\angle 5$ $\angle 3$ and $\angle 6$
Corresponding angles lie on the same side of the transversal t and in corresponding positions.	$\angle 1$ and $\angle 5$ $\angle 4$ and $\angle 8$ $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$
Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of the transversal.	$\angle 1$ and $\angle 7$ $\angle 2$ and $\angle 8$

3-2: Properties of Parallel Lines

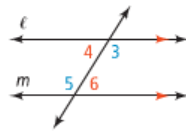
Take note

Postulate 3-1 Same-Side Interior Angles Postulate

Postulate

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

If ...
 $\ell \parallel m$



Then ...

$$m\angle 4 + m\angle 5 = 180$$

$$m\angle 3 + m\angle 6 = 180$$

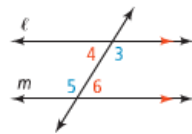
Take note

Theorem 3-1 Alternate Interior Angles Theorem

Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

If ...
 $\ell \parallel m$



Then ...

$$\angle 4 \cong \angle 6$$

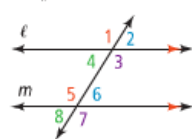
$$\angle 3 \cong \angle 5$$

Theorem 3-2 Corresponding Angles Theorem

Theorem

If a transversal intersects two parallel lines, then corresponding angles are congruent.

If ...
 $\ell \parallel m$



Then ...

$$\angle 1 \cong \angle 5$$

$$\angle 2 \cong \angle 6$$

$$\angle 3 \cong \angle 7$$

$$\angle 4 \cong \angle 8$$

You will prove Theorem 3-2 in Exercise 25.

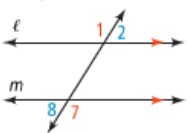
Take note

Theorem 3-3 Alternate Exterior Angles Theorem

Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

If ...
 $\ell \parallel m$



Then ...

$$\angle 1 \cong \angle 8$$

$$\angle 2 \cong \angle 7$$

3-3: Proving Lines Parallel

Theorems 3-4, 3-5, 3-6, and 3-7.

Take note

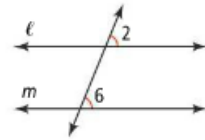
Theorem 3-4 Converse of the Corresponding Angles Theorem

Theorem

If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.

If ...

$$\angle 2 \cong \angle 6$$



Then ...

$$\ell \parallel m$$

You will prove Theorem 3-4 in Lesson 5-5.

Take note

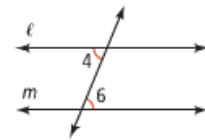
Theorem 3-5 Converse of the Alternate Interior Angles Theorem

Theorem

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

If ...

$$\angle 4 \cong \angle 6$$



Then ...

$$\ell \parallel m$$

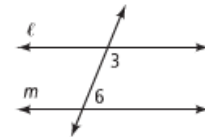
Theorem 3-6 Converse of the Same-Side Interior Angles Postulate

Theorem

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

If ...

$$m\angle 3 + m\angle 6 = 180$$



Then ...

$$\ell \parallel m$$

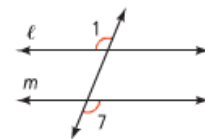
Theorem 3-7 Converse of the Alternate Exterior Angles Theorem

Theorem

If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.

If ...

$$\angle 1 \cong \angle 7$$



Then ...

$$\ell \parallel m$$

3-4: Parallel & Perpendicular Lines

take note

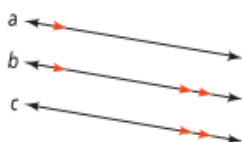
Theorem 3-8

Theorem

If two lines are parallel to the same line, then they are parallel to each other.

If ...

$$a \parallel b \text{ and } b \parallel c$$



Then ...

$$a \parallel c$$

You will prove Theorem 3-8 in Exercise 7.

take note

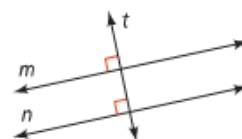
Theorem 3-9

Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

If ...

$$m \perp t \text{ and } n \perp t$$



Then ...

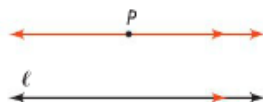
$$m \parallel n$$

3-5: Parallel Lines and Triangles

take note

Postulate 3-2 Parallel Postulate

Through a point not on a line, there is one and only one line parallel to the given line.

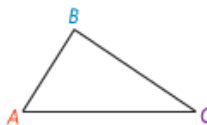


There is exactly one line through P parallel to ℓ .

take note

Theorem 3-11 Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180.



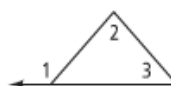
$$m\angle A + m\angle B + m\angle C = 180$$

take note

Theorem 3-12 Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

$$m\angle 1 = m\angle 2 + m\angle 3$$



You will prove Theorem 3-12 in Exercise 33.

3-7: Equations of Line in the Coordinate Plane

Slope

take note

Key Concept Slope

Definition
The **slope** m of a line is the ratio of the vertical change (**rise**) to the horizontal change (**run**) between any two points.

Symbols
A line contains the points (x_1, y_1) and (x_2, y_2) .
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Diagram

Forms of Linear Equations

take note

Key Concept Forms of Linear Equations

Definition
The **slope-intercept form** of an equation of a nonvertical line is $y = mx + b$, where m is the slope and b is the y -intercept.

Symbols
$$y = \underset{\substack{\uparrow \\ \text{slope}}}{m}x + \underset{\substack{\uparrow \\ \text{y-intercept}}}{b}$$

The **point-slope form** of an equation of a nonvertical line is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

$$\underset{\substack{\uparrow \\ \text{y-coordinate}}}{y} - \underset{\substack{\uparrow \\ \text{slope}}}{y_1} = \underset{\substack{\uparrow \\ \text{slope}}}{m}(\underset{\substack{\uparrow \\ \text{x-coordinate}}}{x} - \underset{\substack{\uparrow \\ \text{slope}}}{x_1})$$

Chapter 4

4-1: Congruent Figures

take note

Key Concept Congruent Figures

Definition
Congruent polygons have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.

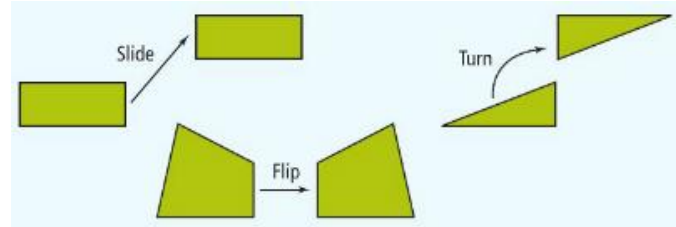
Example

$ABCD \cong EFGH$

$\overline{AB} \cong \overline{EF}$ $\overline{BC} \cong \overline{FG}$
 $\overline{CD} \cong \overline{GH}$ $\overline{DA} \cong \overline{HE}$

$\angle A \cong \angle E$ $\angle B \cong \angle F$
 $\angle C \cong \angle G$ $\angle D \cong \angle H$

Congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one.



Theorem 4-1: Third Angles Theorem

Take note

Theorem 4-1 Third Angles Theorem

Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

If ...

$$\angle A \cong \angle D \text{ and } \angle B \cong \angle E$$

Then ...

$$\angle C \cong \angle F$$



4-2: Triangle Congruence by SSS and SAS

Take note

Postulate 4-1 Side-Side-Side (SSS) Postulate

Postulate

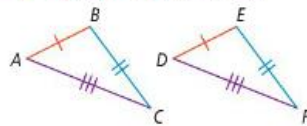
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

If ...

$$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$$

Then ...

$$\triangle ABC \cong \triangle DEF$$



Take note

Postulate 4-2 Side-Angle-Side (SAS) Postulate

Postulate

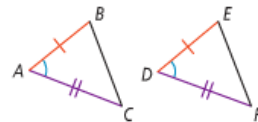
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

If ...

$$\overline{AB} \cong \overline{DE}, \angle A \cong \angle D, \overline{AC} \cong \overline{DF}$$

Then ...

$$\triangle ABC \cong \triangle DEF$$



4-3: Triangle Congruence by ASA & AAS

Take note

Postulate 4-3 Angle-Side-Angle (ASA) Postulate

Postulate

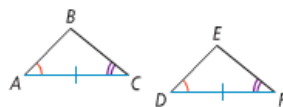
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

If ...

$$\angle A \cong \angle D, \overline{AC} \cong \overline{DF}, \angle C \cong \angle F$$

Then ...

$$\triangle ABC \cong \triangle DEF$$



Take note

Theorem 4-2 Angle-Angle-Side (AAS) Theorem

Theorem

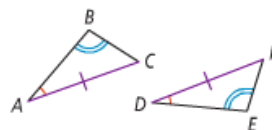
If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

If ...

$$\angle A \cong \angle D, \angle B \cong \angle E, \overline{AC} \cong \overline{DF}$$

Then ...

$$\triangle ABC \cong \triangle DEF$$



4-5: Isosceles and Equilateral Triangles

take note

Theorem 4-3 Isosceles Triangle Theorem

Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

If ...
 $\overline{AC} \cong \overline{BC}$



Then ...

$\angle A \cong \angle B$



take note

Theorem 4-4 Converse of the Isosceles Triangle Theorem

Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

If ...
 $\angle A \cong \angle B$



Then ...

$\overline{AC} \cong \overline{BC}$



You will prove Theorem 4-4 in Exercise 23.

take note

Theorem 4-5

Theorem

If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

If ...
 $\overline{AC} \cong \overline{BC}$ and
 $\angle ACD \cong \angle BCD$



Then ...
 $\overline{CD} \perp \overline{AB}$ and
 $\overline{AD} \cong \overline{BD}$



You will prove Theorem 4-5 in Exercise 26.

take note

Corollary to Theorem 4-3

Corollary

If a triangle is equilateral, then the triangle is equiangular.

If ...
 $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



Then ...

$\angle X \cong \angle Y \cong \angle Z$



Corollary to Theorem 4-4

Corollary

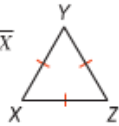
If a triangle is equiangular, then the triangle is equilateral.

If ...
 $\angle X \cong \angle Y \cong \angle Z$



Then ...

$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



4-6: Congruence in Right Triangles

HL Theorem

Take note

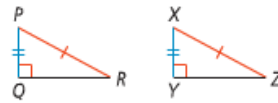
Theorem 4-6 Hypotenuse-Leg (HL) Theorem

Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

If ...

$\triangle PQR$ and $\triangle XYZ$ are right \triangle ,
 $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$



Then ...

$\triangle PQR \cong \triangle XYZ$

Chapter 5

5-1: Midsegments of Triangles

Take note

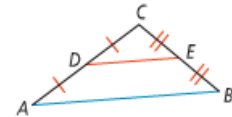
Theorem 5-1 Triangle Midsegment Theorem

Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

If ...

D is the midpoint of \overline{CA} and
 E is the midpoint of \overline{CB}



Then ...

$\overline{DE} \parallel \overline{AB}$ and
 $DE = \frac{1}{2}AB$

You will prove Theorem 5-1 in Lesson 6-9.

5-2: Perpendicular and Angle Bisectors

Take note

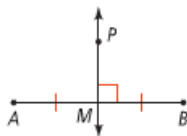
Theorem 5-2 Perpendicular Bisector Theorem

Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

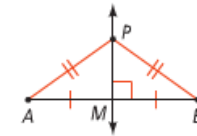
If ...

$\overline{PM} \perp \overline{AB}$ and $MA = MB$



Then ...

$PA = PB$



You will prove Theorem 5-2 in Exercise 32.

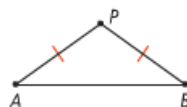
Theorem 5-3 Converse of the Perpendicular Bisector Theorem

Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

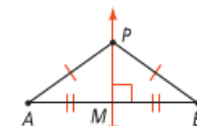
If ...

$PA = PB$



Then ...

$\overline{PM} \perp \overline{AB}$ and $MA = MB$



You will prove Theorem 5-3 in Exercise 33.

Take note

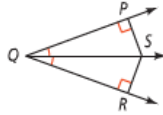
Theorem 5-4 Angle Bisector Theorem

Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

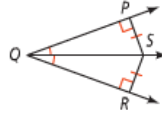
If . . .

\overline{QS} bisects $\angle PQR$, $\overline{SP} \perp \overline{QP}$,
and $\overline{SR} \perp \overline{QR}$



Then . . .

$SP = SR$



You will prove Theorem 5-4 in Exercise 34.

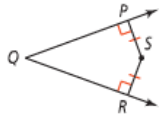
Theorem 5-5 Converse of the Angle Bisector Theorem

Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.

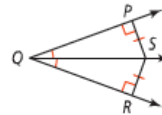
If . . .

$\overline{SP} \perp \overline{QP}$, $\overline{SR} \perp \overline{QR}$,
and $SP = SR$



Then . . .

\overline{QS} bisects $\angle PQR$



You will prove Theorem 5-5 in Exercise 35.

5-3: Bisectors in Triangles

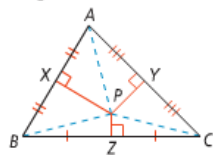
Take note

Theorem 5-6 Concurrence of Perpendicular Bisectors Theorem

Theorem

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

Diagram



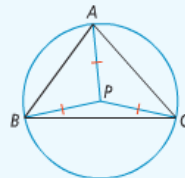
Symbols

Perpendicular bisectors
 \overline{PX} , \overline{PY} , and \overline{PZ} are
concurrent at P.

$PA = PB = PC$

The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle**.

Since the circumcenter is equidistant from the vertices, you can use the circumcenter as the center of the circle that contains each vertex of the triangle. You say the circle is **circumscribed about** the triangle.



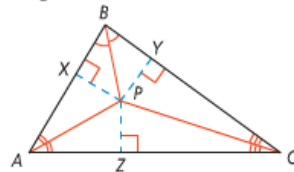
Take note

Theorem 5-7 Concurrence of Angle Bisectors Theorem

Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Diagram



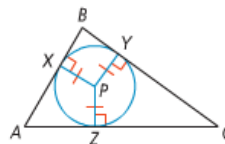
Symbols

Angle bisectors
 \overline{AP} , \overline{BP} , and \overline{CP} are
concurrent at P.

$PX = PY = PZ$

You will prove Theorem 5-7 in Exercise 24.

The point of concurrency of the angle bisectors of a triangle is called the **incenter of the triangle**. For any triangle, the incenter is always inside the triangle. In the diagram, points X, Y, and Z are equidistant from P, the incenter of $\triangle ABC$. P is the center of the circle that is **inscribed in** the triangle.



5-4: Medians and Altitudes



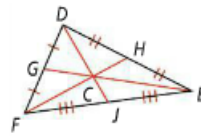
Theorem 5-8 Concurrency of Medians Theorem

The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.

$$DC = \frac{2}{3}DJ$$

$$EC = \frac{2}{3}EG$$

$$FC = \frac{2}{3}FH$$



You will prove Theorem 5-8 in Lesson 6-9.

In a triangle, the point of concurrency of the medians is the **centroid of the triangle**. The point is also called the *center of gravity* of a triangle because it is the point where a triangular shape will balance. For any triangle, the centroid is always inside the triangle.

An **altitude** of a triangle is the perpendicular segment from a vertex of the triangle to the line containing the opposite side.

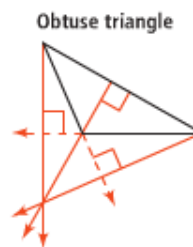
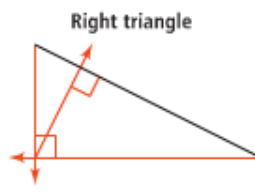
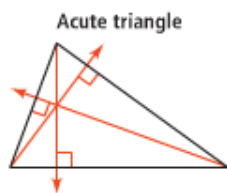


Theorem 5-9 Concurrency of Altitudes Theorem

The lines that contain the altitudes of a triangle are concurrent.

You will prove Theorem 5-9 in Lesson 6-9.

The lines that contain the altitudes of a triangle are concurrent at the orthocenter of the triangle.



5-7: Inequalities in Two Triangles



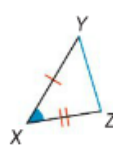
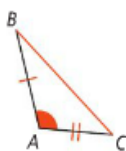
Theorem 5-13 The Hinge Theorem (SAS Inequality Theorem)

Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

If ...

$$m\angle A > m\angle X$$



Then ...

$$BC > YZ$$

You will prove Theorem 5-13 in Exercise 25.

Chapter 6

6-1: The Polygon Angle-Sum Theorems

The sum of the interior measures of a polygon depends on the number of sides the polygon has.

By dividing a polygon with n sides into $(n-2)$ triangles, you can show that the sum of the interior angle measures of any polygon is a multiple of 180.

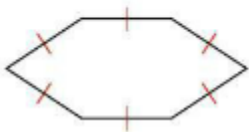
Theorem 6-1

take note

Theorem 6-1 Polygon Angle-Sum Theorem

The sum of the measures of the interior angles of an n -gon is $(n - 2)180$.

An **equilateral polygon** is a polygon with all sides congruent.



An **equiangular polygon** is a polygon with all angles congruent.



A **regular polygon** is a polygon that is both equilateral and equiangular.



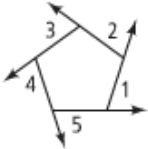
Theorem 6-2

take note

Theorem 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

For the pentagon, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$.



You will prove Theorem 6-2 in Exercise 39.

6-2: Properties of Parallelograms

A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

Parallelograms have special properties regarding their sides, angles, and diagonals.

In a quadrilateral, **opposite sides** do not share a vertex and **opposite angles** do not share a side.



Theorem 6-3

take note

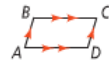
Theorem 6-3

Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

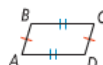
If ...

$ABCD$ is a \square



Then ...

$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



Angles of a polygon that share a side are **consecutive angles**.

Theorem 6-4

take note

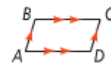
Theorem 6-4

Theorem

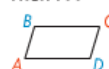
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If ...

$ABCD$ is a \square



Then ...



$$m\angle A + m\angle B = 180$$

$$m\angle B + m\angle C = 180$$

$$m\angle C + m\angle D = 180$$

$$m\angle D + m\angle A = 180$$

You will prove Theorem 6-4 in Exercise 32.

Theorem 6-5

take note

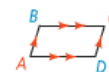
Theorem 6-5

Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

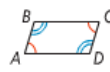
If ...

$ABCD$ is a \square .



Then ...

$\angle A \cong \angle C$ and $\angle B \cong \angle D$



Theorem 6-6

take note

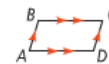
Theorem 6-6

Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

If ...

$ABCD$ is a \square



Then ...

$\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$



You will prove Theorem 6-6 in Exercise 13.

Theorem 6-7

take note

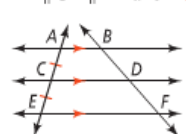
Theorem 6-7

Theorem

If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

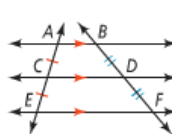
If ...

$\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ and $\overline{AC} \cong \overline{CE}$



Then ...

$\overline{BD} \cong \overline{DF}$



You will prove Theorem 6-7 in Exercise 43.

6-3: Proving that a Quadrilateral is a Parallelogram

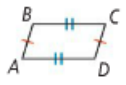
Theorem 6-8 (Converse of Theorem 6-3)

take note

Theorem 6-8

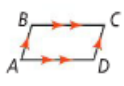
Theorem
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If ...



$\overline{AB} \cong \overline{CD}$
 $\overline{BC} \cong \overline{DA}$

Then ...
ABCD is a \square



You will prove Theorem 6-8 in Exercise 20.

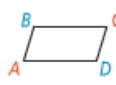
Theorem 6-9 (Converse of Theorem 6-4) & Theorem 6-10 (Converse of Theorem 6-5)

take note

Theorem 6-9

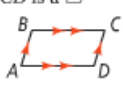
Theorem
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.

If ...



$m\angle A + m\angle B = 180$
 $m\angle A + m\angle D = 180$

Then ...
ABCD is a \square

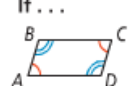


You will prove Theorem 6-9 in Exercise 21.

Theorem 6-10

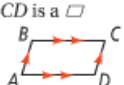
Theorem
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If ...



$\angle A \cong \angle C$
 $\angle B \cong \angle D$

Then ...
ABCD is a \square



You will prove Theorem 6-10 in Exercise 18.

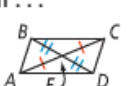
Theorem 6-11 (Converse of Theorem 6-6)

take note

Theorem 6-11

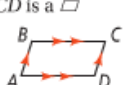
Theorem
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If ...



$\overline{AE} \cong \overline{CE}$
 $\overline{BE} \cong \overline{DE}$

Then ...
ABCD is a \square



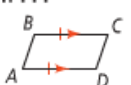
Theorem 6-12

take note

Theorem 6-12


Theorem
If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

If ...



$\overline{BC} \cong \overline{DA}$
 $\overline{BC} \parallel \overline{DA}$

Then ...
ABCD is a \square



You will prove Theorem 6-12 in Exercise 19.

Theorem Summary

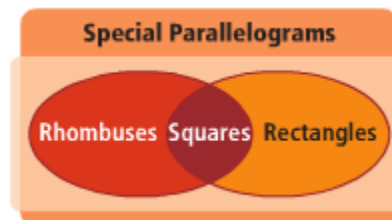
<div> <div>take note</div> <div> Concept Summary Proving That a Quadrilateral Is a Parallelogram </div> </div>		
Method	Source	Diagram
Prove that both pairs of opposite sides are parallel.	Definition of parallelogram	
Prove that both pairs of opposite sides are congruent.	Theorem 6-8	
Prove that an angle is supplementary to both of its consecutive angles.	Theorem 6-9	
Prove that both pairs of opposite angles are congruent.	Theorem 6-10	
Prove that the diagonals bisect each other.	Theorem 6-11	
Prove that one pair of opposite sides is congruent and parallel.	Theorem 6-12	

6-4: Properties of Rhombuses, Rectangles, and Squares

Special Parallelograms

<div> <div>take note</div> <div> Key Concept Special Parallelograms </div> </div>	
Definition	Diagram
A rhombus is a parallelogram with four congruent sides.	
A rectangle is a parallelogram with four right angles.	
A square is a parallelogram with four congruent sides and four right angles.	

The Venn Diagram at the right shows the relationships among special parallelograms.



Theorem 6-13 and Theorem 6-14

Take note

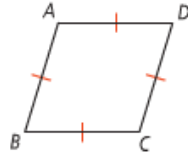
Theorem 6-13

Theorem

If a parallelogram is a rhombus, then its diagonals are perpendicular.

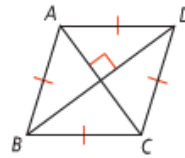
If ...

$ABCD$ is a rhombus



Then ...

$\overline{AC} \perp \overline{BD}$



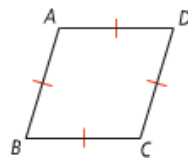
Theorem 6-14

Theorem

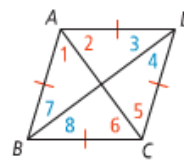
If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

If ...

$ABCD$ is a rhombus



Then ...



$$\angle 1 \cong \angle 2$$

$$\angle 3 \cong \angle 4$$

$$\angle 5 \cong \angle 6$$

$$\angle 7 \cong \angle 8$$

You will prove Theorem 6-14 in Exercise 45.

Theorem 6-15

Take note

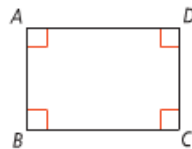
Theorem 6-15

Theorem

If a parallelogram is a rectangle, then its diagonals are congruent.

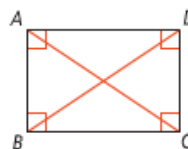
If ...

$ABCD$ is a rectangle



Then ...

$\overline{AC} \cong \overline{BD}$



You will prove Theorem 6-15 in Exercise 41.

6-5: Conditions for Rhombuses, Rectangles, and Squares

You can determine whether a parallelogram is a rhombus or a rectangle based on the properties of its diagonals.

Theorem 6-16

Take note

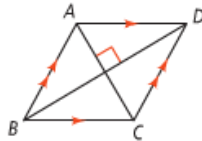
Theorem 6-16

Theorem

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

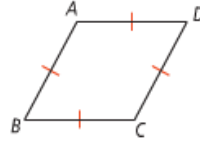
If ...

$ABCD$ is a \square and $\overline{AC} \perp \overline{BD}$



Then ...

$ABCD$ is a rhombus



Theorem 6-17 and 6-18

Take note

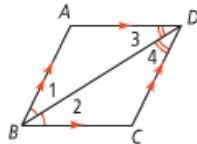
Theorem 6-17

Theorem

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

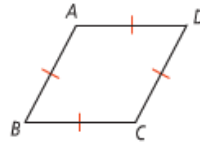
If ...

$ABCD$ is a \square , $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$



Then ...

$ABCD$ is a rhombus



You will prove Theorem 6-17 in Exercise 23.

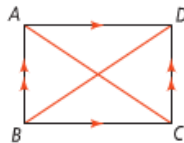
Theorem 6-18

Theorem

If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

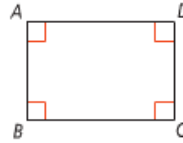
If ...

$ABCD$ is a \square , and $\overline{AC} \cong \overline{BD}$



Then ...

$ABCD$ is a rectangle

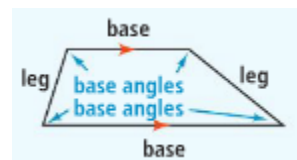


You will prove Theorem 6-18 in Exercise 24.

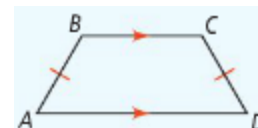
6-6: Trapezoids and Kites

The diagonals, angles, and sides of a trapezoid have certain properties.

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called **bases**. The nonparallel sides are called **legs**. The two angles that share a base of a trapezoid are called base angles. A trapezoid has two pairs of **base angles**.



An isosceles trapezoid is a trapezoid with legs that are congruent. ABCD below is an isosceles trapezoid. The angles of an isosceles trapezoid have some unique properties.



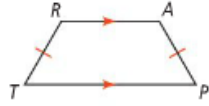
Theorem 6-19

take note

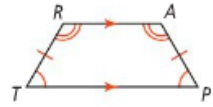
Theorem 6-19

Theorem
If a quadrilateral is an isosceles trapezoid, then each pair of base angles is congruent.

If ...
 $TRAP$ is an isosceles trapezoid with bases \overline{RA} and \overline{TP}



Then ...
 $\angle T \cong \angle P, \angle R \cong \angle A$



You will prove Theorem 6-19 in Exercise 45.

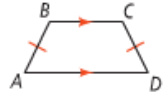
Theorem 6-20

take note

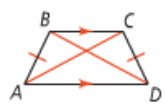
Theorem 6-20

Theorem
If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent.

If ...
 $ABCD$ is an isosceles trapezoid



Then ...
 $\overline{AC} \cong \overline{BD}$



You will prove Theorem 6-20 in Exercise 54.

In Lesson 5-1, you learned about midsegments of triangle. Trapezoid also have midsegments. The **midsegment of a trapezoid** is the segment that joins the midpoint of its legs. The midsegment has two unique properties.

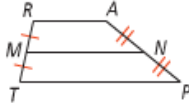
Theorem 6-21

take note

Theorem 6-21 Trapezoid Midsegment Theorem

Theorem
If a quadrilateral is a trapezoid, then
(1) the midsegment is parallel to the bases, and
(2) the length of the midsegment is half the sum of the lengths of the bases.

If ...
 $TRAP$ is a trapezoid with midsegment \overline{MN}



Then ...
(1) $\overline{MN} \parallel \overline{TP}, \overline{MN} \parallel \overline{RA}$, and
(2) $MN = \frac{1}{2}(TP + RA)$

You will prove Theorem 6-21 in Lesson 6-9.

Theorem 6-22

take note

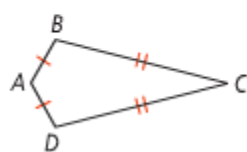
Theorem 6-22

Theorem

If a quadrilateral is a kite,
then its diagonals are
perpendicular.

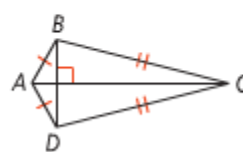
If ...

$ABCD$ is a kite



Then ...

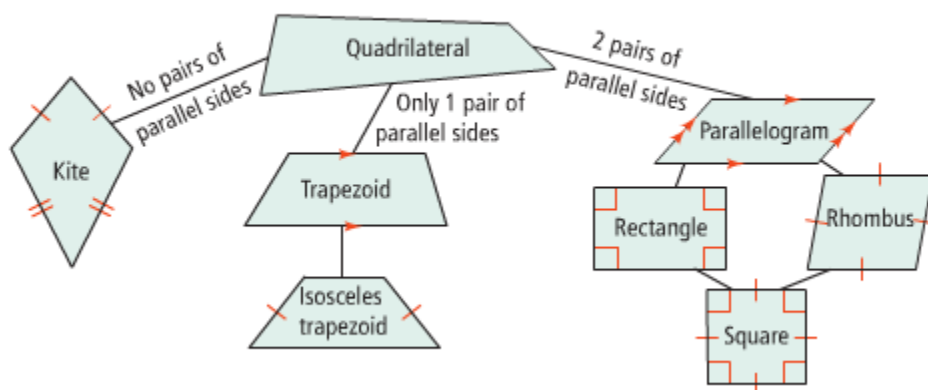
$\overline{AC} \perp \overline{BD}$



Relationships Among Quadrilaterals Summary

take note

Concept Summary Relationships Among Quadrilaterals



6-7: Polygons in the Coordinate Plane

Take note

Key Concept Formulas and the Coordinate Plane

Formula

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

When to Use It

To determine whether

- sides are congruent
- diagonals are congruent

To determine

- the coordinates of the midpoint of a side
- whether diagonals bisect each other

To determine whether

- opposite sides are parallel
- diagonals are perpendicular
- sides are perpendicular

6-8: Applying Coordinate Geometry

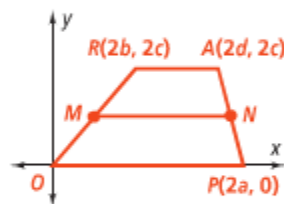
You can use coordinate geometry and algebra to prove theorems in geometry. This kind of proof is called a **coordinate proof**.

Plan a coordinate proof of the Trapezoid Midsegment Theorem (Theorem 6-21).

- (1) The midsegment of a trapezoid is parallel to the bases.
- (2) The length of the midsegment of a trapezoid is half the sum of the lengths of the bases.

Step 1 Draw and label a figure.

Midpoints will be involved, so use multiples of 2 to name coordinates.



Step 2 Write the *Given* and *Prove* statements.

Use the information on the diagram to write the statements.

Given: \overline{MN} is the midsegment of trapezoid $ORAP$.

Prove: $\overline{MN} \parallel \overline{OP}$, $\overline{MN} \parallel \overline{RA}$,
 $MN = \frac{1}{2}(OP + RA)$

Step 3 Determine the formulas you will need. Then write the plan.

- First, use the Midpoint Formula to find the coordinates of M and N .
- Then, use the Slope Formula to determine whether the slopes of \overline{MN} , \overline{OP} , and \overline{RA} are equal. If they are, \overline{MN} , \overline{OP} , and \overline{RA} are parallel.
- Finally, use the Distance Formula to find and compare the lengths of \overline{MN} , \overline{OP} , and \overline{RA} .

7-1: Ratios and Proportions

A **ratio** is a comparison of two quantities by division. You can write the ratio of two numbers a and b , where $b \neq 0$, in three ways: $\frac{a}{b}$, $a : b$, and a to b . You usually express a and b in the same unit and write the ratio in simplest form.

An equation that states that two ratios are equal is called a **proportion**. The first and last numbers in a proportion are the **extremes**. The middle two numbers are the **means**.



Take note

Key Concept Cross Products Property

Words

In a proportion, the product of the **extremes** equals the product of the **means**.

Symbols

If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

Example

$$\begin{aligned}\frac{2}{3} &= \frac{4}{6} \\ 2 \cdot 6 &= 3 \cdot 4 \\ 12 &= 12\end{aligned}$$

Take note

Key Concept Properties of Proportions

a , b , c , and d do not equal zero.

Property

(1) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{b}{a} = \frac{d}{c}$.

(2) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{c} = \frac{b}{d}$.

(3) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a+b}{b} = \frac{c+d}{d}$.

How to Apply It

Write the reciprocal of each ratio.

$\left(\frac{2}{3} = \frac{4}{6}\right)$ becomes $\frac{3}{2} = \frac{6}{4}$.

Switch the means.

$\frac{2}{3} \nearrow \frac{4}{6}$ becomes $\frac{2}{4} = \frac{3}{6}$.

In each ratio, add the denominator to the numerator.

$\frac{2}{3} = \frac{4}{6}$ becomes $\frac{2+3}{3} = \frac{4+6}{6}$.

7-3: Proving Triangles Similar

take note

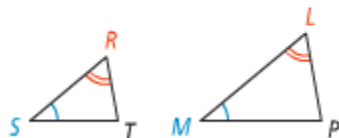
Postulate 7-1 Angle-Angle Similarity (AA ~) Postulate

Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

If ...

$$\angle S \cong \angle M \text{ and } \angle R \cong \angle L$$



Then ...

$$\triangle SRT \sim \triangle MLP$$

take note

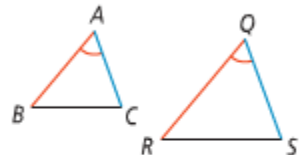
Theorem 7-1 Side-Angle-Side Similarity (SAS ~) Theorem

Theorem

If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar.

If ...

$$\frac{AB}{QR} = \frac{AC}{QS} \text{ and } \angle A \cong \angle Q$$



Then ...

$$\triangle ABC \sim \triangle QRS$$

You will prove Theorem 7-1 in Exercise 35.

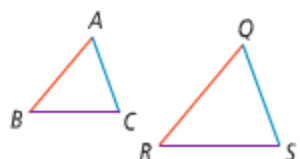
Theorem 7-2 Side-Side-Side Similarity (SSS ~) Theorem

Theorem

If the corresponding sides of two triangles are proportional, then the triangles are similar.

If ...

$$\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$$



Then ...

$$\triangle ABC \sim \triangle QRS$$

You will prove Theorem 7-2 in Exercise 36.

7-4: Similarity in Right Triangles

Take note

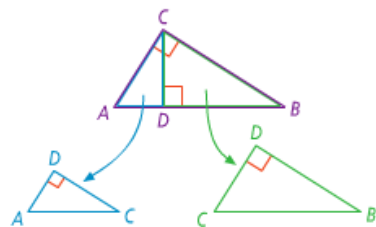
Theorem 7-3

Theorem

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

If ...

$\triangle ABC$ is a right triangle with right $\angle ACB$, and \overline{CD} is the altitude to the hypotenuse



Then ...

$$\triangle ABC \sim \triangle ACD$$

$$\triangle ABC \sim \triangle CBD$$

$$\triangle ACD \sim \triangle CBD$$

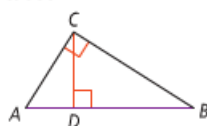
Take note

Corollary 1 to Theorem 7-3

Corollary

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

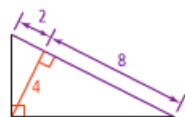
If ...



Then ...

$$\frac{AD}{CD} = \frac{CD}{DB}$$

Example



Segments of hypotenuse

$$\frac{2}{4} = \frac{4}{8}$$

Altitude to hypotenuse

You will prove Corollary 1 in Exercise 42.

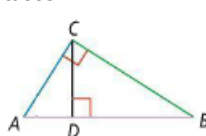
Take note

Corollary 2 to Theorem 7-3

Corollary

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to the leg.

If ...

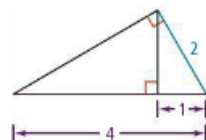


Then ...

$$\frac{AB}{AC} = \frac{AC}{AD}$$

$$\frac{AB}{CB} = \frac{CB}{DB}$$

Example



Hypotenuse

Leg

$$\frac{4}{2} = \frac{2}{1}$$

Segment of hypotenuse adjacent to leg

You will prove Corollary 2 in Exercise 43.

7-5: Proportions in Triangles

take note

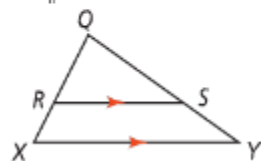
Theorem 7-4 Side-Splitter Theorem

Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

If ...

$$\overrightarrow{RS} \parallel \overrightarrow{XY}$$



Then ...

$$\frac{XR}{RQ} = \frac{YS}{SQ}$$

take note

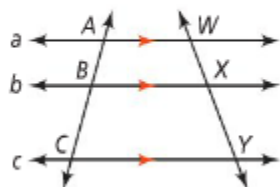
Corollary Corollary to the Side-Splitter Theorem

Corollary

If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

If ...

$$a \parallel b \parallel c$$



Then ...

$$\frac{AB}{BC} = \frac{WX}{XY}$$

You will prove the Corollary to Theorem 7-4 in Exercise 46.

take note

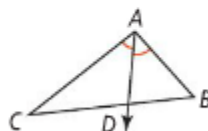
Theorem 7-5 Triangle-Angle-Bisector Theorem

Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

If ...

$$\overrightarrow{AD} \text{ bisects } \angle CAB$$



Then ...

$$\frac{CD}{DB} = \frac{CA}{BA}$$

You will prove the Triangle-Angle-Bisector Theorem in Exercise 47.

Chapter 8

8-1: The Pythagorean Theorem and Its Converse

take note

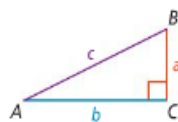
Theorem 8-1 Pythagorean Theorem

Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

If ...

$\triangle ABC$ is a right triangle



Then ...

$$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$$

$$a^2 + b^2 = c^2$$

You will prove Theorem 8-1 in Exercise 49.

A **Pythagorean triple** is a set of nonzero whole numbers a , b , and c that satisfy the equation $a^2 + b^2 = c^2$. Below are some common Pythagorean triples.

3, 4, 5

5, 12, 13

8, 15, 17

7, 24, 25

take note

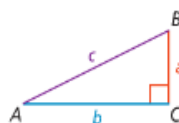
Theorem 8-2 Converse of the Pythagorean Theorem

Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

If ...

$$a^2 + b^2 = c^2$$



Then ...

$\triangle ABC$ is a right triangle

You will prove Theorem 8-2 in Exercise 52.

take note

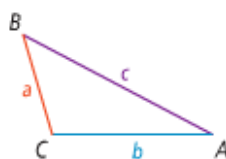
Theorem 8-3

Theorem

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If ...

$$c^2 > a^2 + b^2$$



You will prove Theorem 8-3 in Exercise 53.

Then ...

$\triangle ABC$ is obtuse

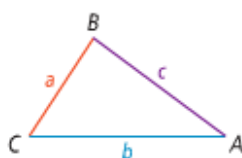
Theorem 8-4

Theorem

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

If ...

$$c^2 < a^2 + b^2$$



You will prove Theorem 8-4 in Exercise 54.

Then ...

$\triangle ABC$ is acute

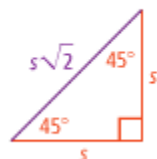
8-2: Special Right Triangles

take note

Theorem 8-5 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

$$\text{hypotenuse} = \sqrt{2} \cdot \text{leg}$$



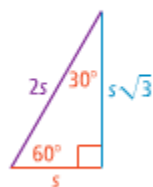
take note

Theorem 8-6 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

$$\text{hypotenuse} = 2 \cdot \text{shorter leg}$$

$$\text{longer leg} = \sqrt{3} \cdot \text{shorter leg}$$



8-3: Trigonometry

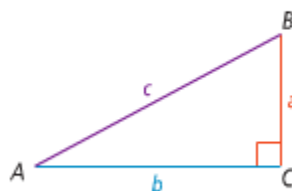
take note

Key Concept Trigonometric Ratios

$$\text{sine of } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{a}{c}$$

$$\text{cosine of } \angle A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{b}{c}$$

$$\text{tangent of } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{a}{b}$$



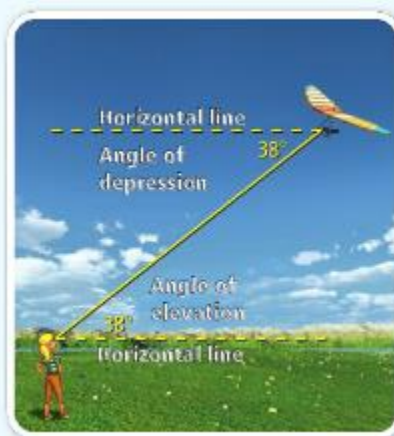
8-4: Angles of Elevation and Depression

Suppose a person on the ground sees a hang glider at a 38° angle above a horizontal line. This angle is the **angle of elevation**.

At the same time, a person in the hang glider sees the person on the ground at a 38° angle below a horizontal line. This angle is the **angle of depression**.

Notice that the angle of elevation is congruent to the angle of depression because they are alternate interior angles.

Essential Understanding You can use the angles of elevation and depression as the acute angles of right triangles formed by a horizontal distance and a vertical height.



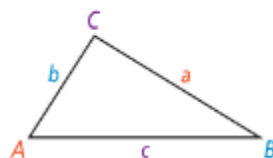
8-5: Law of Sines



Key Concept Law of Sines

For any $\triangle ABC$, let the lengths of the sides opposite angles A , B , and C be a , b , and c , respectively. Then the **Law of Sines** relates the sine of each angle to the length of the opposite side.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



8-6: Law of Cosines



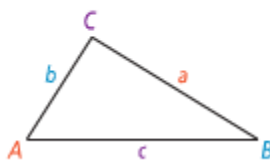
Key Concept Law of Cosines

For any $\triangle ABC$, the **Law of Cosines** relates the cosine of each angle to the side lengths of the triangle.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Chapter 9

9-1: Translations

A transformation of a geometric figure is a function, or mapping that results in a change in the position, shape, or size of the figure.

In a transformation, the original figure is the preimage. The resulting figure is the image. A transform that preserves distance and angle measures is called a rigid motion.



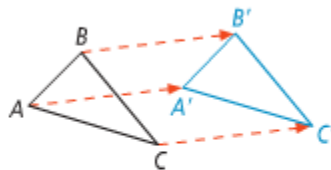
Key Concept Translation

A **translation** is a transformation that maps all points of a figure the same distance in the same direction.

You write the translation that maps $\triangle ABC$ onto $\triangle A'B'C'$ as $T(\triangle ABC) = \triangle A'B'C'$. A translation is a rigid motion with the following properties.

If $T(\triangle ABC) = \triangle A'B'C'$, then

- $AA' = BB' = CC'$
- $AB = A'B'$, $BC = B'C'$, $AC = A'C'$
- $m\angle A = m\angle A'$, $m\angle B = m\angle B'$, $m\angle C = m\angle C'$



9-2: Reflections

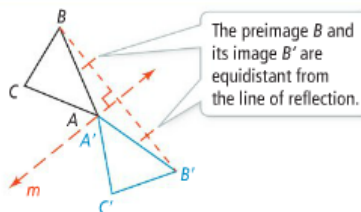
Take note

Key Concept Reflection Across a Line

A **reflection** across a line m , called the **line of reflection**, is a transformation with the following properties:

- If a point A is on line m , then the image of A is itself (that is, $A' = A$).
- If a point B is not on line m , then m is the perpendicular bisector of $\overline{BB'}$.

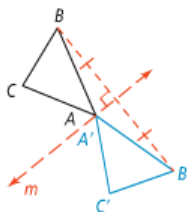
You write the reflection across m that takes P to P' as $R_m(P) = P'$.



Take note

Property Properties of Reflections

- Reflections preserve distance.
If $R_m(A) = A'$ and $R_m(B) = B'$, then $AB = A'B'$.
- Reflections preserve angle measure.
If $R_m(\angle ABC) = \angle A'B'C'$, then $m\angle ABC = m\angle A'B'C'$.
- Reflections map each point of the preimage to one and only one corresponding point of its image.
 $R_m(A) = A'$ if and only if $R_m(A') = A$.



9-3: Rotations

Take note

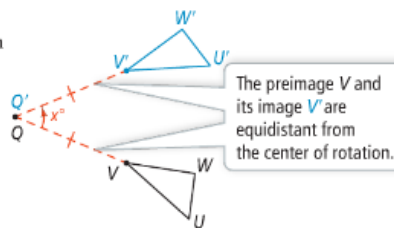
Key Concept Rotation About a Point

A **rotation** of x° about a point Q , called the **center of rotation**, is a transformation with these two properties:

- The image of Q is itself (that is, $Q' = Q$).
- For any other point V , $QV' = QV$ and $m\angle VQV' = x$.

The number of degrees a figure rotates is the **angle of rotation**.

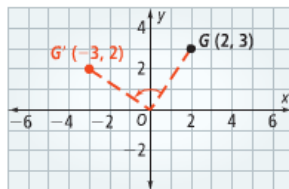
A rotation about a point is a rigid motion.
You write the x° rotation of $\triangle UVW$ about point Q as $r_{(x^\circ, Q)}(\triangle UVW) = \triangle U'V'W'$.



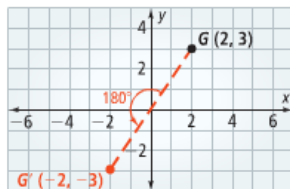
Take note

Key Concept Rotation in the Coordinate Plane

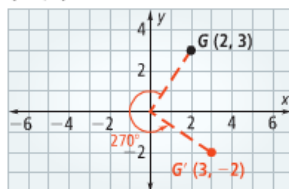
$$r_{(90^\circ, O)}(x, y) = (-y, x)$$



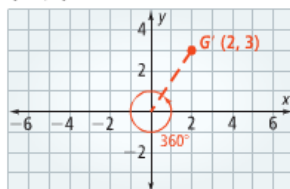
$$r_{(180^\circ, O)}(x, y) = (-x, -y)$$



$$r_{(270^\circ, O)}(x, y) = (y, -x)$$



$$r_{(360^\circ, O)}(x, y) = (x, y)$$



9-4: Compositions of Isometries

The term isometry means same distance. An isometry is a transformation that preserves distance, or length.

take note

Theorem 9-1

The composition of two or more isometries is an isometry.

There are only four kinds of isometries.



take note

Theorem 9-2 Reflections Across Parallel Lines

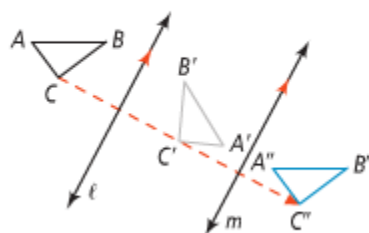
A composition of reflections across two parallel lines is a translation.

You can write this composition as

$$(R_m \circ R_\ell)(\triangle ABC) = \triangle A''B''C''$$

$$\text{or } R_m(R_\ell(\triangle ABC)) = \triangle A''B''C''.$$

$\overline{AA''}$, $\overline{BB''}$, and $\overline{CC''}$ are all perpendicular to lines ℓ and m .



take note

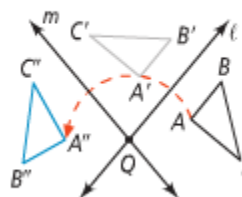
Theorem 9-3 Reflections Across Intersecting Lines

A composition of reflections across two intersecting lines is a rotation.

$$(R_m \circ R_\ell)(\triangle ABC) = \triangle A''B''C''$$

$$\text{or } R_m(R_\ell(\triangle ABC)) = \triangle A''B''C''.$$

The figure is rotated about the point where the two lines intersect. In this case, point Q.



9-5: Congruence Transformations

take note

Key Concept Congruent Figures

Two figures are **congruent** if and only if there is a sequence of one or more rigid motions that maps one figure onto the other.

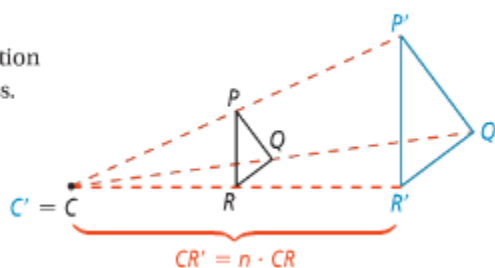
9-6: Dilations

take note

Key Concept Dilation

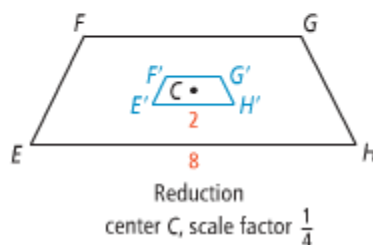
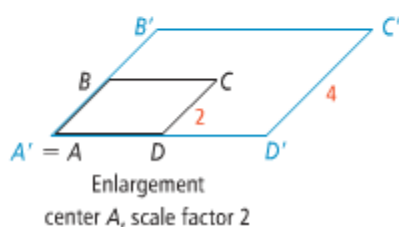
A **dilation** with **center of dilation** C and **scale factor** n , $n > 0$, can be written as $D_{(n, C)}$. A dilation is a transformation with the following properties.

- The image of C is itself (that is, $C' = C$).
- For any other point R , R' is on \overrightarrow{CR} and $CR' = n \cdot CR$, or $n = \frac{CR'}{CR}$.
- Dilations preserve angle measure.



The scale factor n of a dilation is the ratio of a length of the image to the corresponding length in the preimage, with the image length always in the numerator. For the figure shown on page 587, $n = \frac{CR'}{CR} = \frac{R'P'}{RP} = \frac{P'Q'}{PQ} = \frac{Q'R'}{QR}$.

A dilation is an **enlargement** if the scale factor n is greater than 1. The dilation is a **reduction** if the scale factor n is between 0 and 1.



9-7: Similarity Transformations

take note

Key Concept Similar Figures

Two figures are **similar** if and only if there is a similarity transformation that maps one figure onto the other.

Chapter 10

10-1: Areas of Parallelograms and Triangles

take note

Theorem 10-1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

$$A = bh$$



Theorem 10-2 Area of a Parallelogram

The area of a parallelogram is the product of a base and the corresponding height.

$$A = bh$$



A **base of a parallelogram** can be any one of its sides. The corresponding **altitude** is a segment perpendicular to the line containing that base, drawn from the side opposite the base. The **height** is the length of an altitude.

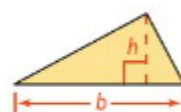


take note

Theorem 10-3 Area of a Triangle

The area of a **triangle** is half the product of a base and the corresponding height.

$$A = \frac{1}{2}bh$$



A **base of a triangle** can be any of its sides. The corresponding **height** is the length of the altitude to the line containing that base.



10-2: Areas of Trapezoids, Rhombuses, and Kites

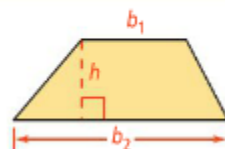
The height of the trapezoid is the perpendicular distance between the bases.

take note

Theorem 10-4 Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$

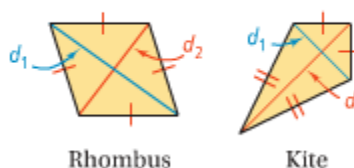


take note

Theorem 10-5 Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the lengths of its diagonals.

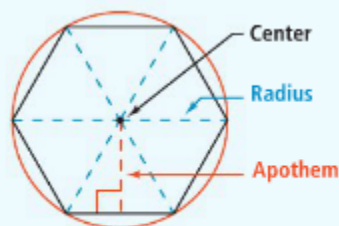
$$A = \frac{1}{2}d_1d_2$$



10-3: Areas of Regular Polygons

Essential Understanding The area of a regular polygon is related to the distance from the center to a side.

You can circumscribe a circle about any regular polygon. The center of a regular polygon is the center of the circumscribed circle. The **radius of a regular polygon** is the distance from the center to a vertex. The **apothem** is the perpendicular distance from the center to a side.



take note

Postulate 10-1

If two figures are congruent, then their areas are equal.

take note

Theorem 10-6 Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.

$$A = \frac{1}{2}ap$$



10-4: Perimeters and Areas of Similar Figures

Take note

Theorem 10-7 Perimeters and Areas of Similar Figures

If the scale factor of two similar figures is $\frac{a}{b}$, then

- (1) the ratio of their perimeters is $\frac{a}{b}$ and
- (2) the ratio of their areas is $\frac{a^2}{b^2}$.

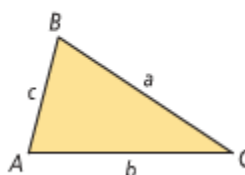
10-5: Trigonometry and Area

Take note

Theorem 10-8 Area of a Triangle Given SAS

The area of a triangle is half the product of the lengths of two sides and the sine of the included angle.

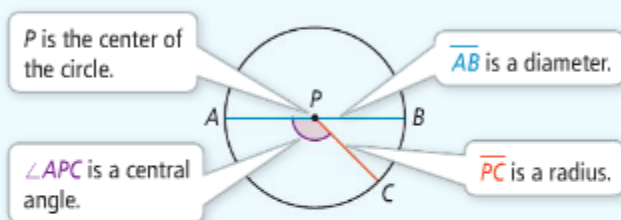
$$\text{Area of } \triangle ABC = \frac{1}{2}bc(\sin A)$$



10-6: Circles and Arcs

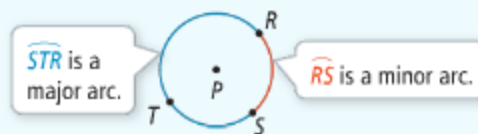
In a plane, a **circle** is the set of all points equidistant from a given point called the **center**. You name a circle by its center. Circle P ($\odot P$) is shown below.

A **diameter** is a segment that contains the center of a circle and has both endpoints on the circle. A **radius** is a segment that has one endpoint at the center and the other endpoint on the circle. **Congruent circles** have congruent radii. A **central angle** is an angle whose vertex is the center of the circle.



Essential Understanding You can find the length of part of a circle's circumference by relating it to an angle in the circle.

An arc is a part of a circle. One type of arc, a **semicircle**, is half of a circle. A **minor arc** is smaller than a semicircle. A **major arc** is larger than a semicircle. You name a minor arc by its endpoints and a major arc or a semicircle by its endpoints and another point on the arc.



take note

Key Concept Arc Measure

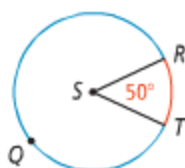
Arc Measure

The measure of a minor arc is equal to the measure of its corresponding central angle.

The measure of a major arc is the measure of the related minor arc subtracted from 360.

The measure of a semicircle is 180.

Example



$$\begin{aligned} m\widehat{RT} &= m\angle RST = 50 \\ m\widehat{TQR} &= 360 - m\widehat{RT} \\ &= 310 \end{aligned}$$

Adjacent arcs are arcs of the same circle that have exactly one point in common. You can add the measures of adjacent arcs just as you can add the measures of adjacent angles.

take note

Postulate 10-2 Arc Addition Postulate

The **measure of the arc** formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$$



The **circumference** of a circle is the distance around the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter.

take note

Theorem 10-9 Circumference of a Circle

The circumference of a circle is π times the diameter.

$$C = \pi d \text{ or } C = 2\pi r$$

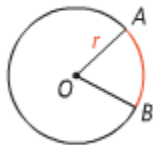


take note

Theorem 10-10 Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

$$\begin{aligned} \text{length of } \widehat{AB} &= \frac{m\widehat{AB}}{360} \cdot 2\pi r \\ &= \frac{m\widehat{AB}}{360} \cdot \pi d \end{aligned}$$



10-7: Areas of Circles and Sectors

Take note

Theorem 10-11 Area of a Circle

The area of a circle is the product of π and the square of the radius.

$$A = \pi r^2$$



A **sector of a circle** is a region bounded by an arc of the circle and the two radii to the arc's endpoints. You name a sector using one arc endpoint, the center of the circle, and the other arc endpoint.

The area of a sector is a fractional part of the area of a circle. The area of a sector formed by a 60° arc is $\frac{60}{360}$, or $\frac{1}{6}$, of the area of the circle.



Sector *RPS*

Take note

Theorem 10-12 Area of a Sector of a Circle

The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.

$$\text{Area of sector } AOB = \frac{m\widehat{AB}}{360} \cdot \pi r^2$$



A part of a circle bounded by an arc and the segment joining its endpoints is a **segment of a circle**.

To find the area of a segment for a minor arc, draw radii to form a sector. The area of the segment equals the area of the sector minus the area of the triangle formed.



Segment of a circle

Take note

Key Concept Area of a Segment



Area of sector

—



Area of triangle

=

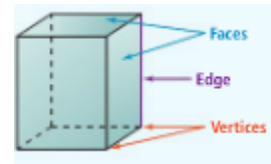


Area of segment

Chapter 11

11-1: Space Figures and Cross Sections

A **polyhedron** is a space figure, or three-dimensional figure, whose surfaces are polygons. Each polygon is a **face** of the polyhedron. An **edge** is a segment that is formed by the intersection of two faces. A **vertex** is a point where three or more edges intersect.



Euler's Formula:

For polyhedrons

The sum of the number of faces (F) and vertices (V) of a polyhedron is two more than the number of its edges (E).

$$F + V = E + 2$$

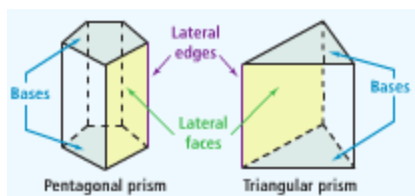
For nets of polyhedrons

In two dimensions, Euler's formula reduces to $F + V = E + 1$, where F is the number of regions formed by V vertices linked by E segments.

$$F + V = E + 1$$

11-2: Surface Areas of Prisms and Cylinders

A **prism** is a polyhedron with two congruent, parallel faces, called **bases**. The other faces are **lateral faces**. You can make a prism using the shape of its bases.



An **altitude** of a prism is a perpendicular segment that joins the planes of the bases. The **height** h of a prism is the length of the altitude. A prism may either be right or oblique.



In a **right prism**, the lateral faces are rectangles and a lateral edge is an altitude. In an **oblique prism**, some or all of the lateral faces are nonregular. You may assume that a prism is a right prism unless stated or picture otherwise.

The **lateral area** (L.A.) of a prism is the sum of the areas of the lateral faces. The **surface area** (S.A.) is the sum of the lateral area and the area of the two bases.

Theorem 11-1: Lateral and Surface Areas of a Prism

take note

Theorem 11-1 Lateral and Surface Areas of a Prism

The lateral area of a right prism is the product of the perimeter of the base and the height of the prism.

$$L.A. = ph$$

The surface area of a right prism is the sum of the lateral area and the areas of the two bases.

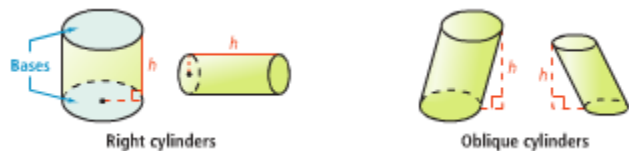
$$S.A. = L.A. + 2B$$

p is the perimeter of a base.

B is the area of a base.

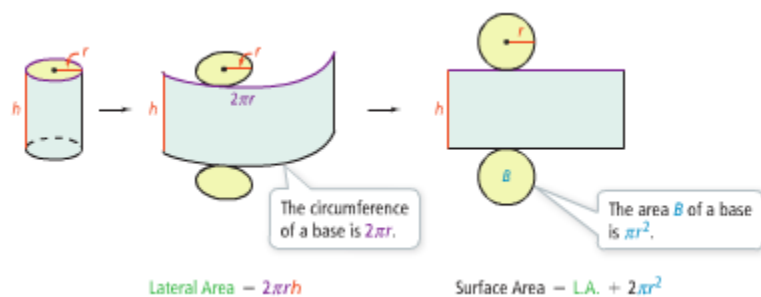
Cylinders

A **cylinder** is a solid that has two congruent parallel **bases** that are circles. An **altitude** of a cylinder is a perpendicular segment that joins the planes of the bases. The **height** h of a cylinder is the length of an altitude.



In a **right cylinder**, the segment joining the centers of the bases is an altitude. In an **oblique cylinder**, the segment joining the centers is not perpendicular to the planes containing the bases. You may assume that a cylinder is a right cylinder unless stated or pictured otherwise.

To find the area of the curved surface of a cylinder, visualize “unrolling it”. The area of the resulting rectangle is the **lateral area** of the cylinder. The **surface area** of a cylinder is the sum of the lateral area and the areas of the two circular bases. You can find formulas for these areas by looking at a net for a cylinder.



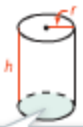
Theorem 11-2: Lateral and Surface Areas of a Cylinder

Take note

Theorem 11-2 Lateral and Surface Areas of a Cylinder

The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.
 $L.A. = 2\pi r \cdot h$, or $L.A. = \pi dh$

The surface area of a right cylinder is the sum of the lateral area and the areas of the two bases.
 $S.A. = L.A. + 2B$, or $S.A. = 2\pi rh + 2\pi r^2$



B is the area of a base.

11-3: Surface Areas of Pyramids and Cones

A **pyramid** is a polyhedron in which one face (the **base**) can be any polygon and the other faces (the **lateral faces**) are triangles that meet at a common vertex (called the **vertex** of the pyramid.)

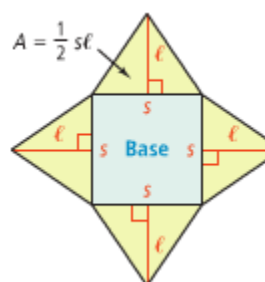
You name a pyramid by the shape of its base. The **altitude** of a pyramid is the perpendicular segment from the vertex to the plane of the base. The length of the altitude is the **height** h of the pyramid.

A **regular pyramid** is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles. The **slant height** l is the length of the altitude of a lateral face of the pyramid.

You can assume that a pyramid is regular unless stated otherwise.

The **lateral area** of a pyramid is the sum of the areas of the congruent lateral faces. You can find a formula for the lateral area of a pyramid by looking at its net.

$$\begin{aligned} \text{L.A.} &= 4\left(\frac{1}{2}s\ell\right) && \text{The area of each lateral face is } \frac{1}{2}s\ell. \\ &= \frac{1}{2}(4s)\ell && \text{Commutative and Associative} \\ &= \frac{1}{2}p\ell && \text{Properties of Multiplication} \\ &&& \text{The perimeter } p \text{ of the base is } 4s. \end{aligned}$$



To find the **surface area** of a pyramid, add the area of its base to its lateral area.

Theorem 11-3: Lateral and Surface Areas of a Pyramid

take note

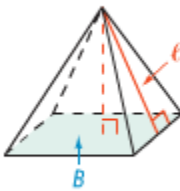
Theorem 11-3 Lateral and Surface Areas of a Pyramid

The lateral area of a regular pyramid is half the product of the perimeter p of the base and the slant height ℓ of the pyramid.

$$\text{L.A.} = \frac{1}{2}p\ell$$

The surface area of a regular pyramid is the sum of the lateral area and the area B of the base.

$$\text{S.A.} = \text{L.A.} + B$$



Like a pyramid, a **cone** is a solid that has one base and a vertex that is not in the same plane as the base. However, the **base** of a cone is circle. In a **right cone**, the **altitude** is a perpendicular segment from the **vertex** to the center of the base. The **height** h is the length of the altitude. The **slant height** l is the distance from the vertex to a point on the edge of the base. You may assume that a cone is a right cone unless stated or pictured otherwise.



The **lateral area** is half the circumference of the base times the slant height. The formulas for the lateral area and **surface area** of a cone are similar to those for a pyramid.

Theorem 11-4: Lateral and Surface Areas of a Cone

take note

Theorem 11-4 Lateral and Surface Areas of a Cone

The lateral area of a right cone is half the product of the circumference of the base and the slant height of the cone.

$$\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot \ell, \text{ or } \text{L.A.} = \pi r \ell$$

The surface area of a cone is the sum of the lateral area and the area of the base.

$$\text{S.A.} = \text{L.A.} + B$$



11-4: Volumes of Prisms and Cylinders

Volume is the space that a figure occupies. It is measured in cubic units such as cubic inches, cubic feet, or cubic centimeters. The volume V of a cube is the cube of the length of its edge e , or $V = e^3$

Theorem 11-5: Cavalieri's Principle

take note

Theorem 11-5 Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.

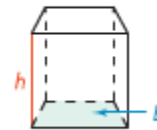
Theorem 11-6: Volume of a Prism

take note

Theorem 11-6 Volume of a Prism

The volume of a prism is the product of the area of the base and the height of the prism.

$$V = Bh$$



To find the volume of a cylinder, you use the same formula $V = Bh$ that you use to find the volume of a prism. Now, however, B is the area of the circle.

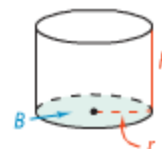
Theorem 11-7: Volume of a Cylinder

take note

Theorem 11-7 Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.

$$V = Bh, \text{ or } V = \pi r^2 h$$



11-5: Volumes of Pyramids and Cones

The volume of a pyramid is related to the volume of a prism with the same base and height.



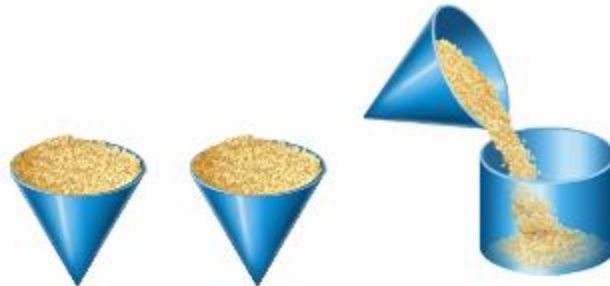
Theorem 11-8 Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

$$V = \frac{1}{3}Bh$$



The volume of a cone is related to the volume of a cylinder with the same base and height.



The cones and the cylinder have the same base and height.
It takes three cones full of rice to fill the cylinder.



Theorem 11-9 Volume of a Cone

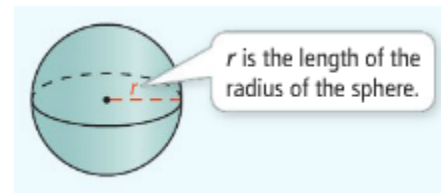
The volume of a cone is one third the product of the area of the base and the height of the cone.

$$V = \frac{1}{3}Bh, \text{ or } V = \frac{1}{3}\pi r^2h$$



11-6: Surfaces Areas and Volumes of Spheres

A **sphere** is the set of all points in space equidistant from a given point called the **center**. A **radius** is a segment that has one endpoint at the center and the other endpoint on the sphere. A **diameter** is a segment passing through the center with endpoints on the sphere.



Theorem 11-10: Surface Area of a Sphere



Theorem 11-10 Surface Area of a Sphere

The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

$$S.A. = 4\pi r^2$$



Theorem 11-11: Volume of a Sphere



Theorem 11-11 Volume of a Sphere

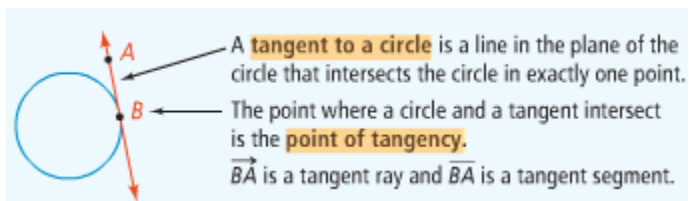
The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$



Chapter 12

12-1: Tangent Lines



A radius of a circle and the tangent that intersects the endpoint of the radius on the circle have a special relationship.

Theorem 12-1

take note

Theorem 12-1

Theorem If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.	If . . . \overleftrightarrow{AB} is tangent to $\odot O$ at P	Then . . . $\overleftrightarrow{AB} \perp \overline{OP}$

Theorem 12-2 is the converse of Theorem 12-1. You can use it to prove that a line or segment is tangent to a circle.

take note

Theorem 12-2

Theorem If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.	If . . . $\overleftrightarrow{AB} \perp \overline{OP}$ at P	Then . . . \overleftrightarrow{AB} is tangent to $\odot O$

You will prove Theorem 12-2 in Exercise 30.

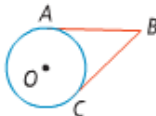
Theorem 12-3

take note

Theorem 12-3

Theorem
If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.

If ...
 \overline{BA} and \overline{BC} are tangent to $\odot O$



Then ...
 $\overline{BA} \cong \overline{BC}$

You will prove Theorem 12-3 in Exercise 23.

12-2: Chords and Arcs

The following theorems and their converses confirm that if you know that chords, arcs, or central angles in a circle are congruent, then you know the other two parts are congruent.

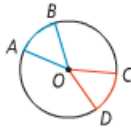
Theorem 12-4 and Its Converse

take note

Theorem 12-4 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have **congruent arcs**.

Converse
Within a circle or in congruent circles, congruent arcs have congruent central angles.



If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.
If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.

You will prove Theorem 12-4 and its converse in Exercises 19 and 35.


Theorem 12-5 and Theorem 12-6 & Their Converses

take note

Theorem 12-5 and Its Converse

Theorem
Within a circle or in congruent circles, congruent central angles have congruent chords.

Converse
Within a circle or in congruent circles, congruent chords have congruent central angles.




If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.
If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.

You will prove Theorem 12-5 and its converse in Exercises 20 and 36.

Theorem 12-6 and Its Converse

Theorem
Within a circle or in congruent circles, congruent chords have congruent arcs.

Converse
Within a circle or in congruent circles, congruent arcs have congruent chords.



If $\overline{AB} \cong \overline{CD}$, then $\overline{AB} \cong \overline{CD}$.
If $\overline{AB} \cong \overline{CD}$, then $\overline{AB} \cong \overline{CD}$.

You will prove Theorem 12-6 and its converse in Exercises 21 and 37.

Theorem 12-7 and Its Converse



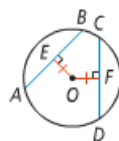
Theorem 12-7 and Its Converse

Theorem

Within a circle or in congruent circles, chords equidistant from the center or centers are congruent.

Converse

Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers).



If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.

You will prove the converse of Theorem 12-7 in Exercise 38.

The Converse of the Perpendicular Bisector Theorem from Lesson 5-2 has special applications to a circle and its diameter, chords, and arcs.

Theorems 12-8, 12-9, and 12-10



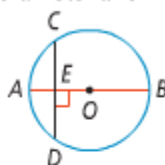
Theorem 12-8

Theorem

In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

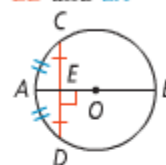
If ...

\overline{AB} is a diameter and $\overline{AB} \perp \overline{CD}$



Then ...

$\overline{CE} \cong \overline{ED}$ and $\widehat{CA} \cong \widehat{AD}$



You will prove Theorem 12-8 in Exercise 22.

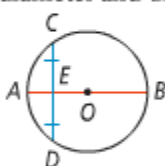
Theorem 12-9

Theorem

In a circle, if a diameter bisects a chord (that is not a diameter), then it is perpendicular to the chord.

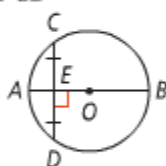
If ...

\overline{AB} is a diameter and $\overline{CE} \cong \overline{ED}$



Then ...

$\overline{AB} \perp \overline{CD}$



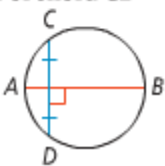
Theorem 12-10

Theorem

In a circle, the perpendicular bisector of a chord contains the center of the circle.

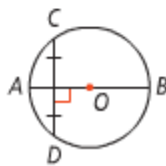
If ...

\overline{AB} is the perpendicular bisector of chord \overline{CD}



Then ...

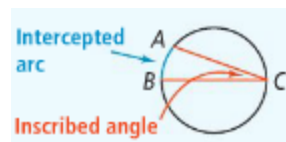
\overline{AB} contains the center of $\odot O$



You will prove Theorem 12-10 in Exercise 33.

12-3: Inscribed Angles

An angle whose vertex is on the circle and whose sides are chords of the circle is an inscribed angle. An arc with endpoints on the sides of an inscribed angle, and its other points in the interior angle is an intercepted arc. In the diagram, inscribed angle C intercepts arc AB.



Theorem 12-11: Inscribed Angle Theorem

take note

Theorem 12-11 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle B = \frac{1}{2} m\widehat{AC}$$

You will use three corollaries to the Inscribed Angle Theorem to find measures of angles in circles.

Corollaries to Theorem 12-11: The Inscribed Angle Theorem

take note

Corollary 1
Two inscribed angles that intercept the same arc are congruent.

Corollary 2
An angle inscribed in a semicircle is a right angle.

Corollary 3
The opposite angles of a quadrilateral inscribed in a circle are supplementary.

You will prove these corollaries in Exercises 31–33.

Theorem 12-12

take note

Theorem 12-12

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

$$m\angle C = \frac{1}{2} m\widehat{BDC}$$

You will prove Theorem 12-12 in Exercise 34.

12-4: Angle Measures and Segment Lengths

Angles formed by intersecting lines have a special relationship to the related arcs formed when the lines intersect a circle. In this lesson, you will study angles and arcs formed by lines intersecting either within a circle or outside a circle.

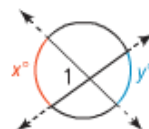
Theorems 12-13 and 12-14

take note

Theorem 12-13

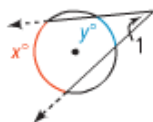
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}(x + y)$$



Theorem 12-14

The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.



$$m\angle 1 = \frac{1}{2}(x - y)$$

You will prove Theorem 12-14 in Exercises 35 and 36.

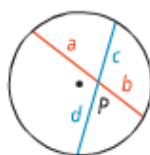
Theorem 12-15

take note

Theorem 12-15

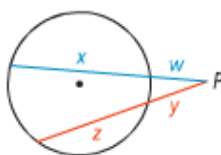
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.

I.



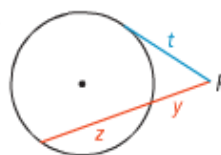
$$a \cdot b = c \cdot d$$

II.



$$(w + x)w = (y + z)y$$

III.



$$(y + z)y = t^2$$

As you use Theorem 12-15, remember the following.

- Case 1: The products of the chord segments are equal.
- Case 2: The products of the secants and their outer segments are equal.
- Case 3: The product of a secant and its outer segments equals the square of the tangent.

12-5: Circles in the Coordinate Plane

The information in the equation of a circle allows you to graph the circle. Also, you can write the equation of a circle if you know its center and radius.

Theorem 12-16: Equation of a Circle

take note

Theorem 12-16 Equation of a Circle

An equation of a circle with center (h, k) and radius r is
$$(x - h)^2 + (y - k)^2 = r^2.$$

