

Διαγώνισμα 2

Όνομα:

Ημερομηνία:

Να υπολογίσετε τα παρακάτω. Να δείξετε την δουλειά σας.

1. (10 βαθμοί) $-3\chi^5 + 7\chi^5 = (-3 + 7)\chi^5 = 4\chi^5$

2. (10 βαθμοί) $-3\beta^5 \cdot 4\beta^7 = (-3 \cdot 4)\beta^{5+7} = -12\beta^{12}$

3. (10 βαθμοί)

$$(-3\theta^7 \cdot 2\theta^4)^3 = ((-3 \cdot 2)\theta^{7+4})^3 = (-6\theta^{11})^3 = (-6)^3\theta^{11 \cdot 3} = -6^3\theta^{33}$$

$$6^3 = 6 \cdot 6 \cdot 6 = 216$$

4. (10 βαθμοί)

$$\begin{aligned} 3\beta - 3\beta^3 + 4\beta^6 - 8\beta + 6\beta^3 &= 3\beta - 8\beta - 3\beta^3 + 6\beta^3 + 4\beta^6 \\ &= (3 - 8)\beta + (-3 + 6)\beta^3 + 4\beta^6 \\ &= -5\beta + 3\beta^3 + 4\beta^6 \end{aligned}$$

5. (10 βαθμοί)

$$\begin{aligned} (-\chi^3)^6 + (3\chi^9)^2 &= (-1\chi^3)^6 + (3\chi^9)^2 \\ &= (-1)^6\chi^{3 \cdot 6} + 3^2\chi^{9 \cdot 2} \\ &= 1\chi^{18} + 9\chi^{18} \\ &= (1 + 9)\chi^{18} \\ &= 10\chi^{18} \end{aligned}$$

6. (10 βαθμοί)

$$\begin{aligned}
\frac{1}{3}\zeta^2 \cdot 8\theta \cdot (-1)^3 \zeta^2 \cdot 3\theta^7 \cdot \zeta^{12} \cdot (-7)\theta &= \frac{1}{3}\zeta^2 \cdot (-1)^3 \zeta^2 \cdot \zeta^{12} \cdot 8\theta \cdot 3\theta^7 \cdot (-7)\theta \\
&= \left(\frac{1}{3} \cdot (-1)^3 \cdot 1\right) \zeta^{2+2+12} \cdot (8 \cdot 3 \cdot (-7)) \theta^{1+7+1} \\
&=^{(*)} \left(-\frac{1}{3}\right) \zeta^{16} \cdot (-168) \theta^9 \\
&= \left(-\frac{1}{3} \cdot -168\right) \zeta^{16} \theta^9 \\
&= \frac{168}{3} \zeta^{16} \theta^9 \\
&= 56 \zeta^{16} \theta^9
\end{aligned}$$

$$(*) \frac{1}{3} \cdot (-1)^3 \cdot 1 = \frac{1}{3} \cdot (-1) \cdot 1 = \frac{1 \cdot (-1) \cdot 1}{3} = \frac{-1}{3} = -\frac{1}{3}$$

και

$$8 \cdot 3 \cdot (-7) = 24 \cdot (-7) = -24 \cdot 7 = -168$$

7. (10 βαθμοί)

$$\begin{aligned}
(3\chi^5 \cdot (-2)\chi^6) \cdot (-3)\chi^{20} &= (3\chi^5 \cdot -2\chi^6) \cdot (-3)\chi^{20} \\
&= ((3 \cdot -2)\chi^{5+6}) \cdot (-3)\chi^{20} \\
&= (-6\chi^{11}) \cdot (-3)\chi^{20} \\
&= -6\chi^{11} \cdot (-3)\chi^{20} \\
&= -6\chi^{11} \cdot -3\chi^{20} \\
&= (-6 \cdot -3)\chi^{11+20} \\
&= 18\chi^{31}
\end{aligned}$$

8. (10 βαθμοί)

$$\begin{aligned}
(-\theta^2 + 8\theta) \cdot (-5\theta^6 - \theta^4) &=^* (-\theta^2 \cdot -5\theta^6) - (-\theta^2 \cdot \theta^4) + (8\theta \cdot -5\theta^6) - (8\theta \cdot \theta^4) \\
&= ((-1 \cdot -5)\theta^{2+6}) - ((-1 \cdot 1)\theta^{2+4}) + ((8 \cdot -5)\theta^{1+6}) - ((8 \cdot 1)\theta^{1+4}) \\
&= (5\theta^8) - (-1\theta^6) + (-40\theta^7) - (8\theta^5) \\
&= 5\theta^8 + 1\theta^6 - 40\theta^7 - 8\theta^5
\end{aligned}$$

$$^*(a + b) \cdot (c - d) = a \cdot c - a \cdot d + b \cdot c - b \cdot d$$

$$a = -\theta^2, b = 8\theta, c = -5\theta^6, d = \theta^4$$

9. (5 βαθμοί)

$$\begin{aligned}
&((\chi^2 - 2\chi + 1) \cdot \chi^3) \cdot \chi - \chi^4 \cdot (\chi^2 + 2\chi + 1) \\
&= (\chi^2 \cdot \chi^3 - 2\chi \cdot \chi^3 + 1 \cdot \chi^3) \cdot \chi - \chi^4 \cdot (\chi^2 + 2\chi + 1) \\
&= (\chi^5 - 2\chi^4 + \chi^3) \cdot \chi - \chi^4 \cdot (\chi^2 + 2\chi + 1) \\
&= \chi^5 \cdot \chi - 2\chi^4 \cdot \chi + \chi^3 \cdot \chi - \chi^4 \cdot \chi^2 - \chi^4 \cdot 2\chi - \chi^4 \cdot 1 \\
&= \chi^6 - 2\chi^5 + \chi^4 - \chi^6 - 2\chi^5 - \chi^4 \\
&= \chi^6 - \chi^6 - 2\chi^5 - 2\chi^5 + \chi^4 - \chi^4 \\
&= (1 - 1)\chi^6 + (-2 - 2)\chi^5 + (1 - 1)\chi^4 \\
&= 0\chi^6 + (-4)\chi^5 + 0\chi^4 \\
&= -4\chi^5
\end{aligned}$$

10. (5 βαθμοί)

$$\begin{aligned}
(4\beta^9 - 3\beta^5) \cdot (\beta^3)^6 + 3\beta^{23} - 8\beta^{27} &= (4\beta^9 - 3\beta^5) \cdot \beta^{3 \cdot 6} + 3\beta^{23} - 8\beta^{27} \\
&= (4\beta^9 - 3\beta^5) \cdot \beta^{18} + 3\beta^{23} - 8\beta^{27}
\end{aligned}$$

$$\begin{aligned}
&=^* (4\beta^9 \cdot \beta^{18} - 3\beta^5 \cdot \beta^{18}) + 3\beta^{23} - 8\beta^{27} \\
&= (4\beta^9 \cdot 1\beta^{18} - 3\beta^5 \cdot 1\beta^{18}) + 3\beta^{23} - 8\beta^{27} \\
&= ((4 \cdot 1)\beta^{9+18} - (3 \cdot 1)\beta^{5+18}) + 3\beta^{23} - 8\beta^{27} \\
&= (4\beta^{27} - 3\beta^{23}) + 3\beta^{23} - 8\beta^{27} \\
&= 4\beta^{27} - 3\beta^{23} + 3\beta^{23} - 8\beta^{27} \\
&= (4 - 8)\beta^{27} + (-3 + 3)\beta^{23} \\
&= -4\beta^{27} + 0\beta^{23} \\
&= -4\beta^{27} + 0 \\
&= -4\beta^{27}
\end{aligned}$$

$$^*(a - b) \cdot c = a \cdot c - b \cdot c$$

$$a = 4\beta^9, b = 4\beta^5, c = \beta^{18}$$

11. (5 βαῖμοι)

$$\begin{aligned}
-2^3\zeta^3 \cdot ((-1)^3\zeta^4 + (-1)^4\zeta^5) &= -8\zeta^3 \cdot (-1\zeta^4 + 1\zeta^5) \\
&= (-8\zeta^3 \cdot -1\zeta^4) + (-8\zeta^3 \cdot 1\zeta^5) \\
&= (-8 \cdot -1)\zeta^{3+4} + (-8 \cdot 1)\zeta^{3+5} \\
&= 8\zeta^7 + (-8)\zeta^8 \\
&= 8\zeta^7 - 8\zeta^8
\end{aligned}$$

12. (5 βαῖμοι)

$$\alpha \cdot \left(\alpha + \alpha \cdot \left(((\alpha \cdot \alpha + \alpha) \cdot \alpha + \alpha) \cdot \alpha + \alpha \right) \right) \cdot \alpha =$$

$$\begin{aligned}
& \alpha \cdot \left(\alpha + \alpha \cdot \left(((\alpha^2 + \alpha) \cdot \alpha + \alpha) \cdot \alpha + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha \cdot \left(((\alpha^2 \cdot \alpha + \alpha \cdot \alpha) + \alpha) \cdot \alpha + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha \cdot \left(((\alpha^3 + \alpha^2) + \alpha) \cdot \alpha + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha \cdot \left((\alpha^3 + \alpha^2 + \alpha) \cdot \alpha + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha \cdot \left((\alpha^3 \cdot \alpha + \alpha^2 \cdot \alpha + \alpha \cdot \alpha) + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha \cdot \left((\alpha^4 + \alpha^3 + \alpha^2) + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha \cdot \left(\alpha^4 + \alpha^3 + \alpha^2 + \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \left(\alpha \cdot \alpha^4 + \alpha \cdot \alpha^3 + \alpha \cdot \alpha^2 + \alpha \cdot \alpha \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \left(\alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 \right) \right) \cdot \alpha = \\
& \alpha \cdot \left(\alpha + \alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 \right) \cdot \alpha = \\
& \left(\alpha \cdot \alpha + \alpha \cdot \alpha^5 + \alpha \cdot \alpha^4 + \alpha \cdot \alpha^3 + \alpha \cdot \alpha^2 \right) \cdot \alpha = \\
& \left(\alpha^2 + \alpha^6 + \alpha^5 + \alpha^4 + \alpha^3 \right) \cdot \alpha = \\
& \alpha^2 \cdot \alpha + \alpha^6 \cdot \alpha + \alpha^5 \cdot \alpha + \alpha^4 \cdot \alpha + \alpha^3 \cdot \alpha = \\
& \alpha^3 + \alpha^7 + \alpha^6 + \alpha^5 + \alpha^4
\end{aligned}$$