

Algebra Lab Module 1

Chapter 1: Exponents and Radicals

All letters represent real numbers, and they are nonzero if they make the denominators zero.

1. Use the properties of exponents to simplify the following expressions:

In groups:

- (a) $x^7 \cdot x^6 \cdot x^{-3}$
- (b) $(-2a^3b^6)(-5b^{-1}a^{-7})$
- (c) $\left(\frac{8p^2}{q^3}\right)^2$
- (d) $\left(\frac{1}{4} - \frac{3}{8} + \frac{2}{3}\right)^2$
- (e) $\left(\frac{-16x^2y^8}{4x^6y^{-3}}\right)^{-2}$
- (f) $\left(\frac{6x^2y^{-4}z^3}{12y^3x^5z^{-4}}\right)^2 \cdot (-x^4y^5z^{-4})^{-4}$
- (g) $(3a+2)^{-7}(3a+2)^3$

Individually:

- (a) $(5x^{-4}y^{-3})(-3x^{-1}y^8)$
- (b) $\frac{x^{-3}x^6}{x^2}$
- (c) $\frac{27y^2y^9}{18x^5y^2}$
- (d) $\frac{-7p^3q^{11}}{49p^7q^3}$
- (e) $\frac{x^{-13}x^6}{x^{20}}$
- (f) $(x^{-2})^7$
- (g) $(-3y^{-4})^4$
- (h) $(-2a^5)^3$
- (i) $\left(\frac{4x^{-2}y^{-3}}{8y^4}\right)^3$
- (j) $\left(\frac{1}{4}\right)^{-2} - \left(\frac{3}{8}\right)^{-1} + \left(\frac{2}{3}\right)^0$

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- (k) $\left(\frac{1}{4}\right)^{-3} - \left(\frac{3}{8}\right)^{-1} + \left(\frac{2}{3}\right)^0$
- (l) $\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right)^{-2}$
- (m) $\left(\frac{6x^4y^{-4}}{18x^{-2}y^{-3}}\right)^{-3}$
- (n) $\left(\frac{2}{6p^3q^{-3}}\right)^{-2} \left(\frac{3}{2p^{-6}q^{-7}}\right)^{-1}$
- (o) $\left(\frac{1}{4a^6b^{-1}}\right)^2 \left(\frac{2}{5a^3b^{-4}}\right)^{-1}$
- (p) $\left(\frac{14a^{-2}b^2c^5}{21c^2b^{-2}a^4}\right)^2 \cdot (-3a^{-2}c^4b^{-2})^{-3}$
- (q) $\left(\frac{(y^{-4})^{-2}y^{-3}}{y^{-5}}\right)^{-1}$
- (r) $\left(\frac{(a^{-1})^3a^{-2}}{a^{-2}}\right)^{-4}$
- (s) $(5x - 2)^{-3}(5x - 2)^4$
- (t) $(7x - 1)^4(7x - 1)^{-3}$
- (u) $(1 - 7x)^4(7x - 1)^{-3}$
- (v) $\left((4x + 3)^4\right)^{-3}$
- (w) $\left((-5x + 2)^{-5}\right)^2$

2. Use the properties of exponents, including rational exponents and radicals to simplify the following expressions:

In groups:

- (a) $36^{1/2}$
- (b) $\left(\frac{121}{144}\right)^{1/2}$
- (c) $-16^{3/4}$
- (d) $25^{3/2}$

Individually:

- (a) $\left(\frac{49}{81}\right)^{1/2}$

- (b) $16^{3/4}$
- (c) $(-16)^{3/4}$
- (d) $16^{-3/4}$
- (e) $-16^{-3/4}$
- (f) $(-16)^{-3/4}$
- (g) $8^{2/3}$
- (h) $-8^{2/3}$
- (i) $(-8)^{2/3}$
- (j) $8^{-2/3}$
- (k) $-8^{-2/3}$
- (l) $(-8)^{-2/3}$

3. Rewrite the following expressions using rational exponents:

In groups:

- (a) $\sqrt[4]{x^3}$
- (b) $\sqrt[6]{a^6 + b^6}$
- (c) $\sqrt{9x^5}$

Individually:

- (a) $\sqrt[8]{y^5}$
- (b) $9\sqrt{x^5}$
- (c) $\sqrt[5]{3x^2 - y^3}$
- (d) $\sqrt[4]{16p^{16} - q^{16}}$
- (e) $\sqrt{9a^8 - b^8}$

4. Simplify and rewrite the result using only positive rational exponents:

In groups:

- (a) $\frac{x^{4/3}x^{2/3}}{x^{1/3}}$
- (b) $\frac{4z^{-4/7}z^{2/7}}{12z^{-1/7}}$
- (c) $\left(\frac{a^{2/3}}{b^{3/4}}\right)^{12} \left(\frac{b^{2/5}}{b^{3/2}}\right)^{10}$
- (d) $\left(\frac{x^2}{x+y}\right)^{-1} \left(\frac{x^2}{x+y}\right)^{1/2}$

Individually:

(a) $\frac{y^{2/5}y^{1/5}}{x^{3/5}}$

(b) $\frac{21x^{4/3}x^{2/3}}{14x^{1/3}}$

(c) $\frac{45a^{-4/5}a^{2/5}}{9b^{-1/3}}$

(d) $\frac{p^{-5/7}q^{-2/7}}{(-3)^{-2}r^{-2/5}}$

(e) $(16a^{-8}b^{1/5})^{3/4}$

(f) $(125x^6b^{-7/5})^{1/3}$

(g) $\left(\frac{x^2}{x-y}\right)^{-2}\left(\frac{x^2}{x-y}\right)^{3/2}$

(h) $\left(\frac{a^{2/3}}{b^{3/4}}\right)^{-2}\left(\frac{b^{2/5}}{b^{3/2}}\right)^{10}$

Algebra Lab Module 2

From the previous chapter on exponents:

All letters represent real numbers, and they are nonzero if they make the denominators zero.

5. Simplify each expression, and write the result in terms of positive exponents and radicals:

In groups:

- (a) $\sqrt{x^7}$
- (b) $\sqrt[3]{x^7}$
- (c) $\sqrt[3]{250a^2b^6c^{13}}$
- (d) $\sqrt{\frac{x^9}{36}}$
- (e) $4\sqrt[3]{\frac{p^3z^8}{8}}$
- (f) $\sqrt{10} \cdot \sqrt{14}$
- (g) $\sqrt[3]{x^2y} \cdot \sqrt[3]{xy^2}$
- (h) $\left(3\sqrt[4]{x^3}\right)\left(-5\sqrt[4]{x^3}\right)$
- (i) $\left(-\frac{1}{2}\sqrt[3]{6x^2y^2z}\right)\left(\frac{4}{3}\sqrt[3]{4x^2z^2}\right)$
- (j) $\sqrt[5]{x\sqrt[3]{x^2}}$

Individually:

- (a) $\sqrt[4]{x^7}$
- (b) $\sqrt[8]{x^7}$
- (c) $\sqrt{24}$
- (d) $\sqrt[3]{24}$
- (e) $\sqrt{54}$
- (f) $\sqrt[3]{54}$
- (g) $\sqrt[4]{40x^3y^9z^{14}}$
- (h) $\sqrt[4]{243p^{19}q^{10}}$
- (i) $\sqrt{84(a-2)^3}$
- (j) $\sqrt{12(x+42)^5}$

$$(k) \sqrt{\frac{a^{13}}{64}}$$

$$(l) 8\sqrt[3]{\frac{x^6y^7}{64}}$$

$$(m) \sqrt{6} \cdot \sqrt{21}$$

$$(n) \sqrt[4]{p^3q} \cdot \sqrt[4]{pq^3}$$

$$(o) \left(7\sqrt[6]{a^5}\right) \left(-2\sqrt[6]{a^5}\right)$$

$$(p) \left(\frac{1}{6}\sqrt[3]{9p^2q^5r}\right) \left(-\frac{3}{4}\sqrt[3]{6p^2qz^4}\right)$$

$$(q) \sqrt[5]{x^6y^5} \cdot \sqrt[4]{x}$$

$$(r) \sqrt[4]{p^3q} \cdot \sqrt[3]{pq^2}$$

$$(s) \sqrt[6]{a\sqrt[4]{a^3}}$$

$$(t) \sqrt{x\sqrt{x\sqrt{x}}}$$

$$(u) \sqrt[3]{x\sqrt[3]{x\sqrt[3]{x}}}$$

6. Add or subtract as indicated. Leave your answers as simplified as possible:

In groups:

$$(a) 3\sqrt[3]{2x^2} - 9\sqrt[3]{2x^2} + \sqrt[3]{2x^2}$$

$$(b) -3x\sqrt[3]{16xy^4} + xy\sqrt[3]{54xy} - 5\sqrt[3]{250x^4y^4}$$

Individually:

$$(a) \frac{1}{5}\sqrt{50} - \frac{7}{3}\sqrt{18} + \frac{5}{6}\sqrt{72}$$

$$(b) \frac{2}{5}\sqrt{75} - \frac{2}{3}\sqrt{27} - \frac{1}{2}\sqrt{12}$$

$$(c) 8\sqrt[4]{32p^5q^6} - 5q\sqrt[4]{2p^5q^2} - pq\sqrt[4]{162pq^2}$$

$$(d) -\frac{1}{2}\sqrt{8x} + \frac{3}{7}\sqrt{98x}$$

Chapter 2: Factoring

1. Factor the Greatest Common Factor (GCF).

Groups:

- (a) $10x^7 - 40x^6 + 15x^5$
- (b) $5a(b - 6c) + 7(b - 6c)$

Individually:

- (c) $21x^3y^6 - 14x^4y^5 + 35x^5y^2$
- (d) $8p^3(4k + 9) + 4p^2(4k + 9)$
- (e) $36a^5b^8 + 18a^4b^6 - 27a^3b^7$
- (f) $-7q^3p^2 + 49p^4q^3$
- (g) $-8y^4 + 24y^3 - 16y^2$

2. Factor out the indicated common factor.

Groups:

- (a) Factor out a -1 from $a - b$
- (b) Factor out a -1 from $-5k^4 - 15c^3 + 20$
- (c) Factor out a -5 from $-15a^2 - 10y + 25$
- (d) Factor out a 3 from $-6y^2 + 12y + 9$
- (e) Factor out $-7x^3y$ from $-14x^4y^3 + 21x^3y^2 - 7x^3y$

Individually:

- (f) Factor out a -1 from $-3y - 2x$
- (g) Factor out a -1 from $3y^2 - 5x^3 + 17$
- (h) Factor out a 5 from $-15a^2 - 10y + 25$
- (i) Factor out a -3 from $-6y^2 + 12y + 9$
- (j) Factor out $-4a^2b$ from $-12a^3b^2 - 8a^4b^3 + 4a^2b$

3. Factor by grouping

Groups:

- (a) $6xy + 9y + 14x + 21$
- (b) $12a^3 - 9a^2 - 40a + 30$

Individually:

- (c) $8xy + 18x + 20y + 45$
- (d) $30p^3 - 35p^2 - 24p + 28$
- (e) $ab - 8b + 4a - 2b^2$
- (f) $7x - 6y^2 + xy - 42y$

Algebra Lab Module 3

From the previous chapter on factoring:

4. Factor the following trinomials. You start by trying to factor any GCF, and then you may use any other method you know such as the AC method or Vietta, etc:

Groups:

- (a) $x^2 + 2x - 63$
- (b) $2a^3 - 28a^2 + 80a$
- (c) $50y^3 + 160x^2y + 128xy^2$

Individually:

- (d) $x^2 + 5w - 66$
- (e) $a^2 - 18a + 81$
- (f) $k^2 + 8k + 16$
- (g) $5p^4 - 40p^3 + 35p^2$
- (h) $25k + 6k^2 + 14$
- (i) $8 + 15n^2 + 26n$
- (j) $7a^3b - 40a^2b^2 - 12ab^3$
- (k) $48x^3 - 72x^2y + 27xy^2$
- (l) $9z^4 + 49w^8 + 42z^2w^4$

5. Factor by using the quadratic formula.

Groups:

- (a) $x^2 - 3x - 7$
- (b) $121y^2 + 4$
- (c) $(5x + 7)(2x - 3) + 2x(x + 15) + 35$
- (d) $5x - 2 - 6x^2$
- (e) $\frac{1}{2}x^2 - \frac{2}{7} - \frac{5}{14}x$

Individually:

- (f) $k(k + 10) + 34$
- (g) $4z + z^2 + 6$
- (h) $-3 - 5x^2 + 7x$
- (i) $-5x - 9 + x^2$
- (j) $y^2 + 8y + 19$

(k) $9a^2 + 49$

(l) $(6x + 5)(x - 3) + 2x(7x + 5) - x + 12$

(m) $x(x - 6) + 10$

(n) $\frac{1}{3}x^2 - \frac{7}{6} - \frac{3}{2}x$

6. Factor the following binomials.

Groups:

(a) $9x^2 - 64$

(b) $27a^3 + 8b^3$

(c) $81k^6 - 24m^9$

Individually:

(d) $49 - 16p^2$

(e) $200w^4 - 18z^6$

(f) $81a^4 - 1$

(g) $8a^6 - 125b^9$

(h) $1000x^{12} + 64y^6$

7. Apply all possible strategies learned and factor them completely.

Groups:

(a) $30y^4 + 70y^3 - 120y^2 - 280y$

(b) $p^2 - q^2 + 10q - 25$

(c) $(x^2 - 2) - 3(x^2 - 2) - 28$

(d) $h^6 - 7h^3 - 8$

Individually:

(e) $k^6 + 26k^3 - 27$

(f) $p^7 + 27p^4 - p^3 - 27$

(g) $24m^7 + 21m^4 - 3m$

(h) $4x^4 - 10x^3 - 36x^2 + 90x$

(i) $a^2 - b^2 + 8b - 16$

(j) $60p^4q + 78p^3q - 180p^2q$

(k) $(x^2 + 2)^2 + 5(x^2 + 2) - 24$

(l) $(x^3 + 12)^2 - 16$

(m) $(a^3 + 34)^2 - 49$

(n) $(p + q)^3 + k^3$

x

(o) $4m^2 + 36m(7n - 1) + 81(7n - 1)^2$

(p) $x^2 - y^2 - x - y$

(q) $x^2 + xy - 2y^2 - y + x$

Chapter 3: Polynomial Equations

1. Solve the following quadratic equations by factoring.

Groups:

(a) $x^2 + 5x = 24$

(b) $8y(y + 3) = 2y - 5$

(c) $40a^2 - 90 = 0$

(d) $3x^2 = 12x$

Individually:

(e) $z^2 - 18 = -7z$

(f) $6x(x + 4) = x - 15$

(g) $32y^2 - 162 = 0$

(h) $x^2 = 25x$

2. Solve by using the square root property: $x^2 + u = 0 \iff x^2 = -u \iff x = \pm\sqrt{-u}$

Groups:

(a) $x^2 = 81$

(b) $5k^2 = 35$

(c) $4y^2 + 64 = 0$

(d) $(x + \sqrt{3})^2 = 32$

(e) $\left(x - \frac{1}{4}\right)^2 = -49/27$

Individually:

(f) $x^2 = 169$

(g) $6v^2 - 30 = 0$

(h) $8y^2 + 32 = 0$

(i) $(k + 9)^2 = 24$

(j) $(x - 9)^2 = 121$

(k) $(z + \sqrt{2})^2 = 28$

(l) $\left(x - \frac{1}{5}\right)^2 = -\frac{19}{8}$