

AUC apCalculus BC

Assignment 05

PROBLEM 10.1. Write the proofs of the following table of Theorems.

$\lim(a_n)$	$\beta > 0$	$\beta = 0$	$\beta < 0$	$+\infty$	$-\infty$
$\alpha > 0$				$+\infty$	$-\infty$
$\alpha = 0$		$\alpha \cdot \beta$		undef	undef
$\alpha < 0$				$-\infty$	$+\infty$
$+\infty$	$+\infty$	undef	$-\infty$	$+\infty$	$-\infty$
$-\infty$	$-\infty$	undef	$+\infty$	$-\infty$	$+\infty$

FIGURE 1. Limits of products of sequences

PROBLEM 10.2. Prove the following theorems:

- (1) $\sqrt[n]{+\infty} = +\infty$.
- (2) $\sqrt[n]{\pm\infty} = \pm\infty$.

PROBLEM 10.3. (eventual boundedness is equivalent to boundedness)

- (1) Prove that if $\exists M$ such that eventually on n , $a_n < M$, then $(\exists M)(\forall n)(a_n < M)$.
- (2) Prove that if $\exists M$ such that eventually on n , $a_n > M$, then $(\exists M)(\forall n)(a_n > M)$.
- (2) Prove that if $\exists M > 0$ such that eventually on n , $|a_n| < M$, then $(\exists M)(\forall n)(|a_n| < M)$.

PROBLEM 10.4. Explain in your own words, why if $(\forall n)(a_n \leq a_{n+1})$, then

$$(\forall m, n \in \mathbb{N}) \quad m < n \Rightarrow a_m \leq a_n?$$