

# AUC apCalculus BC

## Assignment 0

### 1.2 Unary Properties over $\mathbb{N}$ that hold eventually

**Reminder:** Do not start the assignment before you study my lecture and reflect upon what I taught you.

**Definition 1.2.1.** A unary property (or unary predicate)  $P(n)$  holds **eventually** with witness a real number  $r$  iff “for all  $n$ ,  $n > r \Rightarrow P(n)$ ”. For instance “ $n > 7$  holds eventually with witness 8”. It is also true that “ $n > 7$  holds eventually with witness 7.1”, and “ $n > 7$  holds eventually with witness 1234567”.

**Problem 1.2.1.**

*Prove that the following properties hold eventually and provide a witness: Justify everything you write.*

1.  $n + 5 > 10$ .
2.  $3n + 1 > 9$ .
3.  $n^2 + n + 1 > 0$ .
4.  $n^2 + n + 1 > 9$ .
5.  $(n - 1)^{33}(n^2 - 10n + 21)^{31} > 0$ .

**Problem 1.2.2.**

*Consider the sequence  $s_n = 2^{n2^{n^2+1}+1}$ , for  $n > 0$ .*

1. *Prove that  $s$  is a sequence.*
2. *Calculate  $s_2 \cdot s_3$ .*
3. *Calculate  $s_{s_{n^2}}$ .*
4. *Calculate  $s_{s_{s_2}}$ .*