Notetaking Guide

Calculus

NINTH EDITION

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Chapter P Preparation for Calculus

Section P.1 Graphs and Models

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Objective: In this lesson you learned how to identify the characteristics of an equation and sketch its graph.

Important Vocabulary	Define each term or concept.
Graph of an equation	
Intercepts	

I. The Graph of an Equation (Pages 2–3)

What you should learn How to sketch the graph of an equation

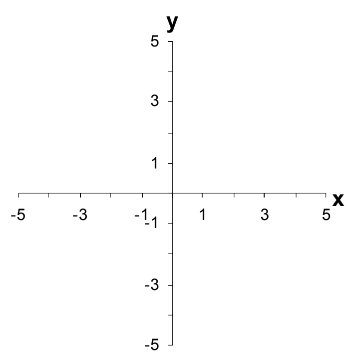
The point (1, 3) is a ______________________ of the equation $-4x + 3y = 5$ because the equation is satisfied when 1 is substituted for __________ and 3 is substituted for __________.

To sketch the graph of an equation using the point-plotting method, ____
____
____
____

One disadvantage of the point-plotting method is _____________
____
____
____

Example 1: Complete the table. Then use the resulting solution points to sketch the graph of the equation $y = 3 - 0.5x$.

x	-4	-2	0	2	4
y					



II. Intercepts of a Graph (Page 4)

> ***What you should learn***
> How to find the intercepts of a graph

The point $(a, 0)$ is a(n) ______________________ of the graph of an equation if it is a solution point of the equation. The point $(0, b)$ is a(n) ______________________ of the graph of an equation if it is a solution point of the equation.

To find the x-intercepts of a graph, ______________________
____

To find the y-intercepts of a graph, ______________________
____

III. Symmetry of a Graph (Pages 5–6)

> ***What you should learn***
> How to test a graph for symmetry with respect to an axis and the origin

Knowing the symmetry of a graph before attempting to sketch it is useful because ______________________________________
____

The three types of symmetry that a graph can exhibit are
____

A graph is **symmetric with respect to the *y*-axis** if, whenever (x, y) is a point on the graph, ___________ is also a point on the graph. This means that the portion of the graph to the left of the y-axis is ____
________________. A graph is **symmetric with respect to the *x*-axis** if, whenever (x, y) is a point on the graph, ___________ is also a point on the graph. This means that the portion of the graph above the x-axis is ________________________________
______________________. A graph is **symmetric with respect to the origin** if, whenever (x, y) is a point on the graph, _____________ is also a point on the graph. This means that the graph is ____
_______________________________________.

The graph of an equation in x and y is symmetric with respect to the y-axis if ____

____________________________.

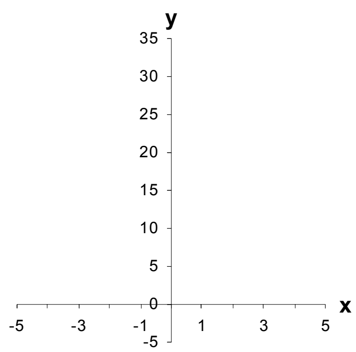
The graph of an equation in x and y is symmetric with respect to the x-axis if ____

____________________________.

The graph of an equation in x and y is symmetric with respect to the origin if ____

_____________________________________.

Example 2: Use symmetry to sketch the graph of the equation $y = 2x^2 + 2$.



IV. Points of Intersection (Page 6)

> ***What you should learn***
> How to find the points of intersection of two graphs

A **point of intersection** of the graphs of two equations is ________

____

You can find the points of intersection of two graphs by _________

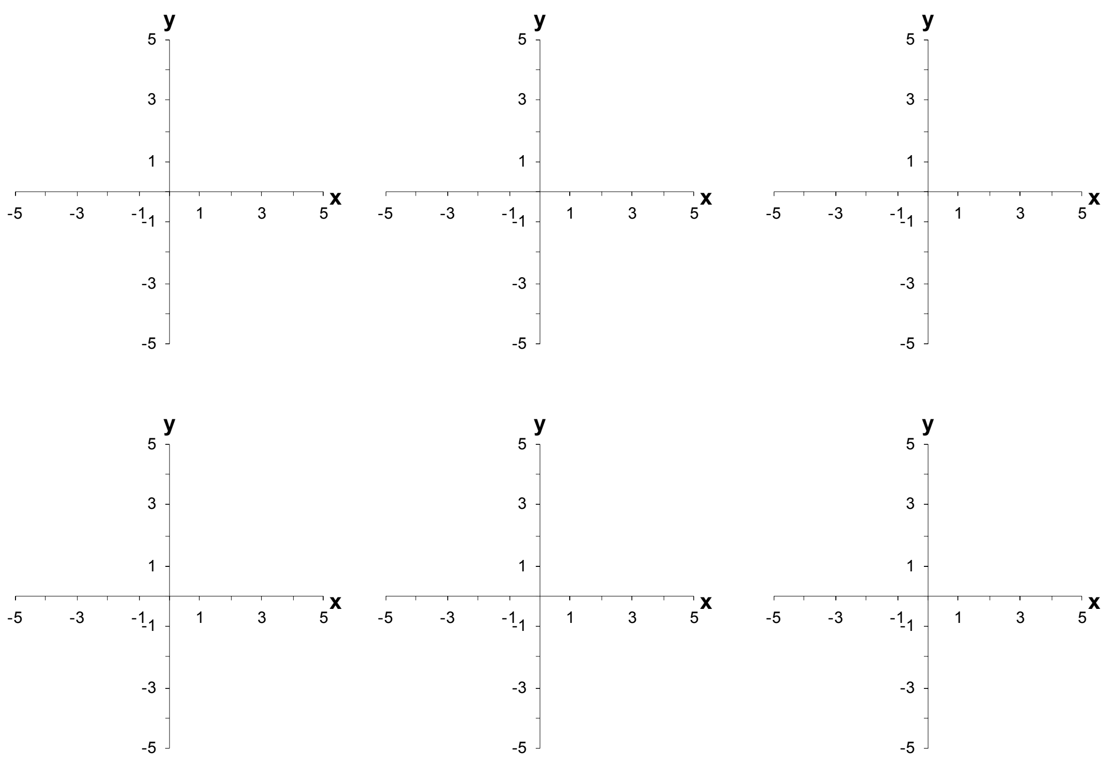
____

Example 3: Find the point of intersection of the graphs of $y = 2x + 10$ and $y = 14 - 3x$.

V. Mathematical Models (Page 7)

> ***What you should learn***
> How to interpret mathematical models for real-life data

In developing a mathematical model to represent actual data, strive for two (often conflicting) goals: ________________ ________________________________.



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Section P.2 Linear Models and Rates of Change

Objective: In this lesson you learned how to find and graph an equation of a line, including parallel and perpendicular lines, using the concept of slope.

Important Vocabulary	Define each term or concept.
Slope	
Parallel	
Perpendicular	

I. The Slope of a Line (Page 10)

> ***What you should learn***
> How to find the slope of a line passing through two points

The **slope** of the nonvertical line passing through the points (x_1, y_1) and (x_2, y_2) is $m =$ ______________________________.

To find the slope of the line through the points $(-2, 5)$ and $(4, -3)$, ____

____

If a line falls from left to right, it has ____________ slope. If a line is horizontal, it has ______________ slope. If a line is vertical, it has ______________ slope. If a line rises from left to right, it has ______________ slope.

II. Equations of Lines (Page 11)

> ***What you should learn***
> How to write the equation of a line with a given point and slope

The **point-slope equation of a line** with slope m, passing through the point (x_1, y_1) is

_______________________________________.

Example 1: Find an equation of the line that passes through the points $(1, 5)$ and $(-3, 7)$.

III. Ratios and Rates of Change (Page 12)

> ***What you should learn***
> How to interpret slope as a ratio or as a rate in a real-life application

In real-life problems, the slope of a line can be interpreted as either ______________, if the x-axis and y-axis have the same unit of measure, or _________________________, if the x-axis and y-axis have different units of measure.

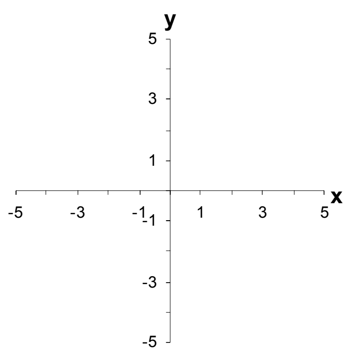
An **average rate of change** is always calculated over ________ ______________________.

IV. Graphing Linear Models (Pages 13–14)

> ***What you should learn***
> How to sketch the graph of a linear equation in slope-intercept form

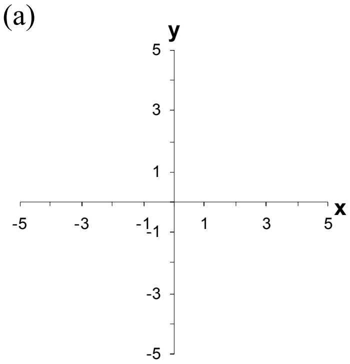
The **slope-intercept** form of the equation of a line is _______________ . The graph of this equation is a line having a slope of ___________ and a y-intercept at (____,____).

Example 1: Explain how to graph the linear equation $y = -2/3x - 4$. Then sketch its graph.

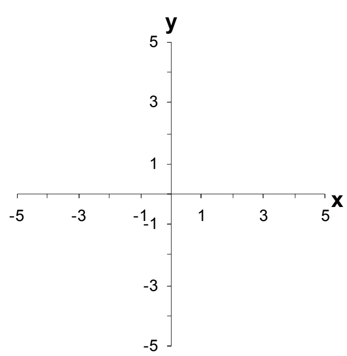


Example 2: Sketch and label the graph of (a) $y = -1$ and (b) $x = 3$.

(a)



(b)



The equation of a vertical line cannot be written in slope-intercept form because ______________________________ ____ A vertical line has an equation of the form _____________.

The equation of any line can be written in **general form,** which is given as ___________________________, where A and B are not both zero.

V. Parallel and Perpendicular Lines (Page 14–15)

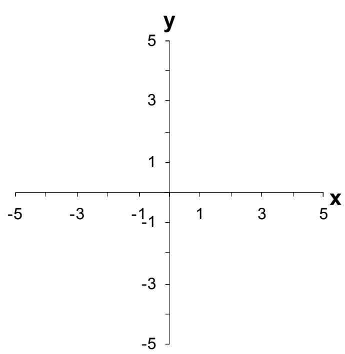
What you should learn
How to write equations of lines that are parallel or perpendicular to a given line

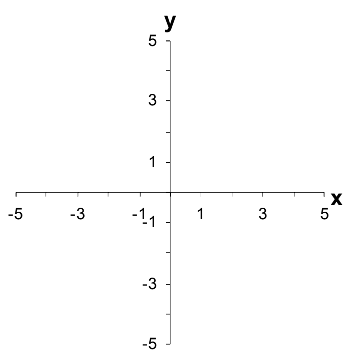
The relationship between the slopes of two lines that are parallel is ____

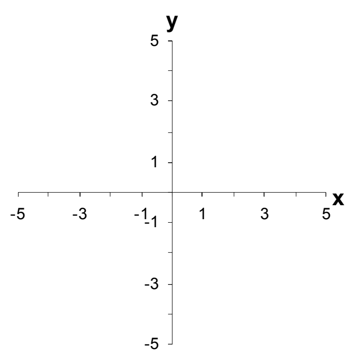
The relationship between the slopes of two lines that are perpendicular is ___________________________________ ______________________________________

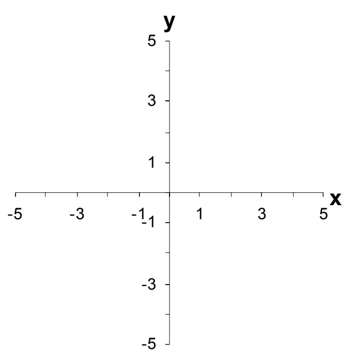
A line that is parallel to a line whose slope is 2 has slope ______.
A line that is perpendicular to a line whose slope is 2 has slope _________.

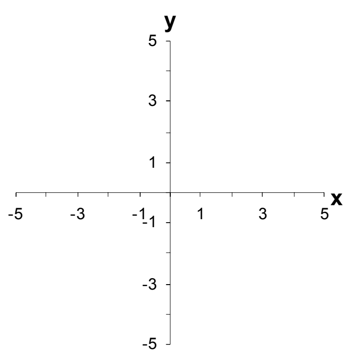
Additional notes

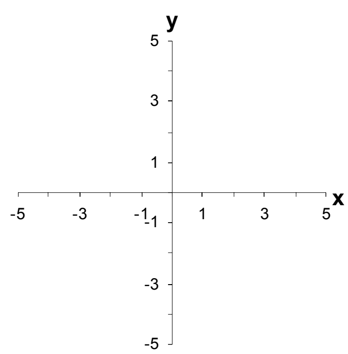












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Section P.3 Functions and Their Graphs

Objective: In this lesson you learned how to evaluate and graph a function and its transformations.

Important Vocabulary Define each term or concept.

Independent variable

Dependent variable

Function

I. Functions and Function Notation (Pages 19–20)

What you should learn
How to use function notation to represent and evaluate a function

Let X and Y be sets of real numbers. A **real-valued function f of a real variable x** from X to Y is ____________________________ ____. In this situation, the **domain** of f is __________________. The number y is the _______________________ of x under f and is denoted by _____________________, which is called the **value of f at x.** The **range** of f is ____________________________ and consists of ____.

In the function $y = 2 + 8x - 3x^2$, which variable is the independent variable? _________________
Which variable is the dependent variable? _________________

Example 1: If $f(w) = 4w^3 - 5w^2 - 7w + 13$, describe how to find $f(-2)$ and then find the value of $f(-2)$.

II. The Domain and Range of a Function (Page 21)

> ***What you should learn***
> How to find the domain and range of a function

The domain of a function can be described explicitly, or it may be described implicitly by ______________________________ ________________. The implied domain is ________________ ____,

whereas an explicitly defined domain is one that is __________ ______________________________________.

A function from X to Y is **one-to-one** if ____________________ ____ ____

A function from X to Y is **onto** if _________________________ ____

III. The Graph of a Function (Page 22)

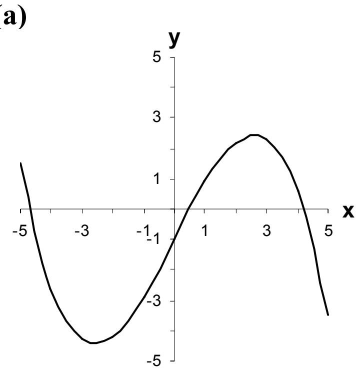
> ***What you should learn***
> How to sketch the graph of a function

The graph of the function $y = f(x)$ consists of ________________ ____.

The **Vertical Line Test** states that _________________________ ____ ____

Example 2: Decide whether each graph represents y as a function of x.

(a)



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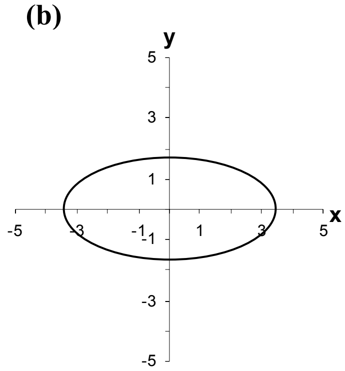
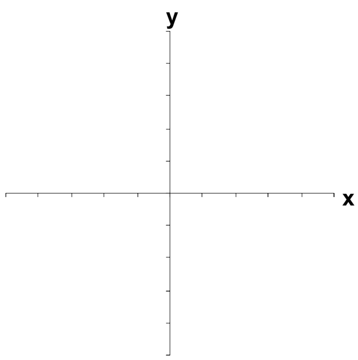
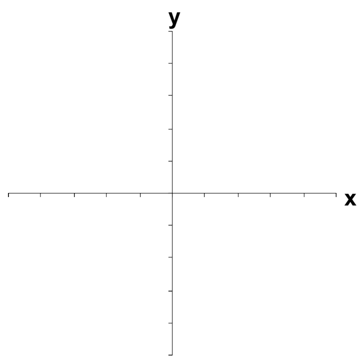


Sketch an example of each of the following eight basic graphs.

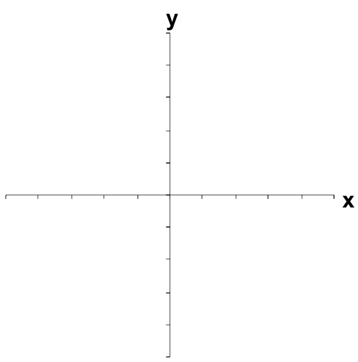
Squaring Function



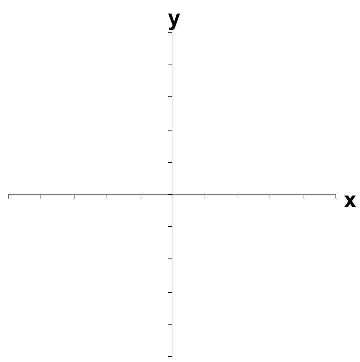
Identity Function



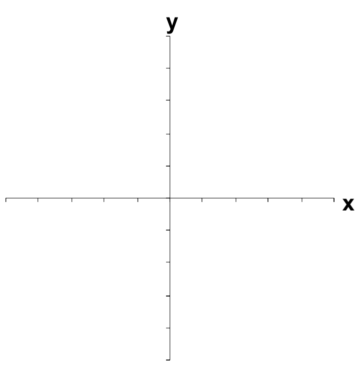
Absolute Value Function



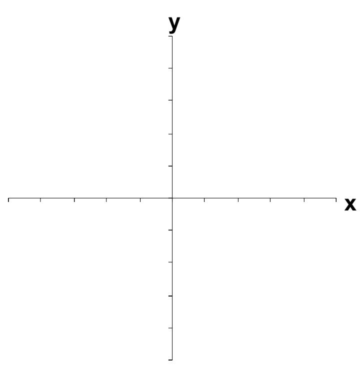
Square Root Function



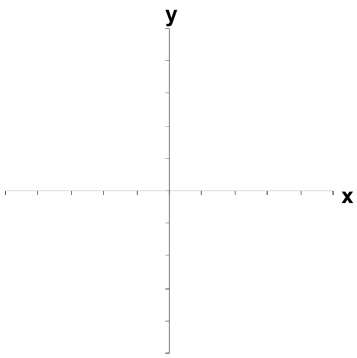
Rational Function



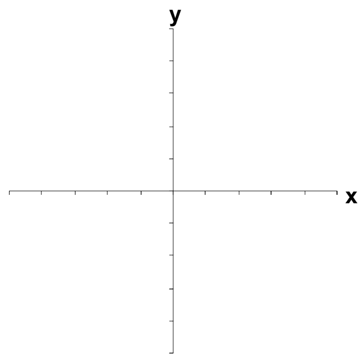
Cubing Function



Sine Function



Cosine Function



IV. Transformations of Functions (Page 23)

> ***What you should learn***
> How to identify different types of transformations of functions

Let c be a positive real number. Complete the following representations of shifts in the graph of $y = f(x)$:

1) Horizontal shift c units to the right: ____________________
2) Horizontal shift c units to the left: ____________________
3) Vertical shift c units downward: ______________________
4) Vertical shift c units upward: ______________________
5) Reflection (about the x-axis): _____________________
6) Reflection (about the y-axis): _____________________
7) Reflection (about the origin): ______________________

V. Classifications and Combinations of Functions (Pages 24–26)

> ***What you should learn***
> How to classify functions and recognize combinations of functions

Elementary functions fall into the following three categories:

____

____

____.

Let n be a nonnegative integer. Then a **polynomial function of *x*** is given as ____

The numbers a_i are ____________________, with a_n the _________ __________________ and a_0 the ______________________ of the polynomial function. If $a_n \neq 0$, then n is the _______________ of the polynomial function.

Just as a rational number can be written as the quotient of two integers, a rational function can be written as ______________ ________________________.

An algebraic function of x is one that _____________________ ____ __________________________________. Functions that are not algebraic are ___________________________.

Two functions can be combined by the operations of

____

to create new functions.

Functions can also be combined through **composition.** The resulting function is called a(n) ______________________.

Let f and g be functions. The function given by $(f \circ g)(x) =$ _______________ is called the **composite** of f with g. The domain of $f \circ g$ is _________________________________ __________________________

Example 3: Let $f(x) = 3x + 4$ and let $g(x) = 2x^2 - 1$. Find (a) $(f \circ g)(x)$ and (b) $(g \circ f)(x)$.

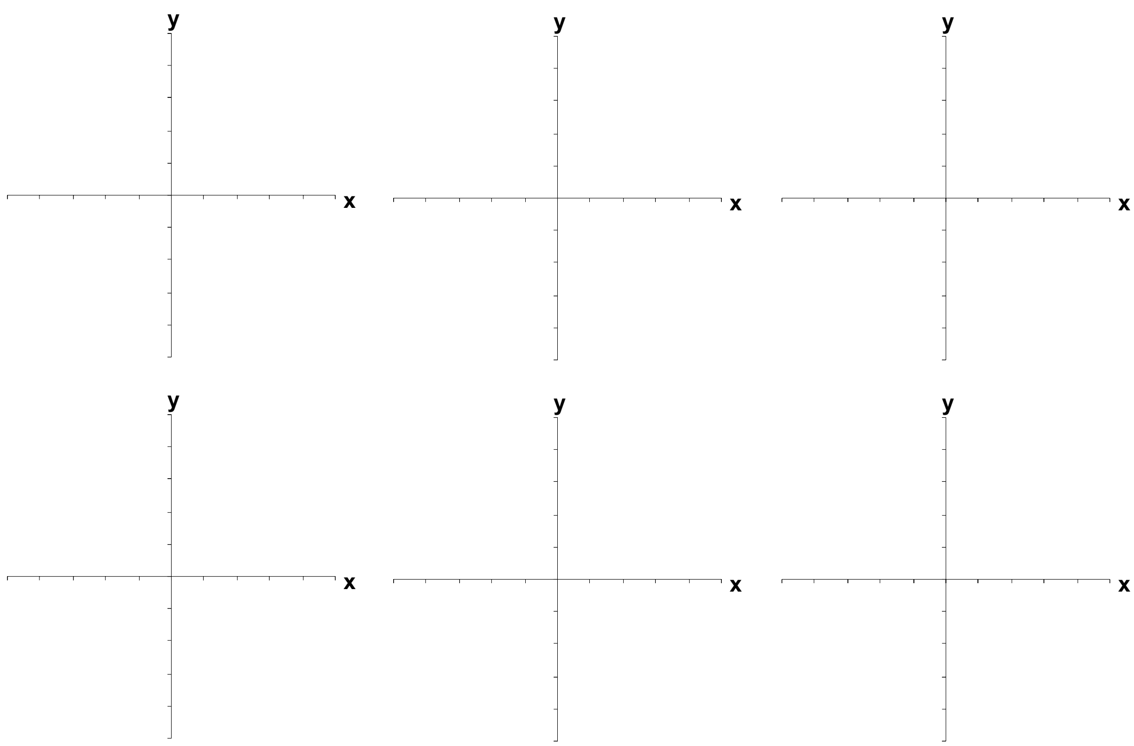
An x-intercept of a graph is defined to be a point $(a, 0)$ at which the graph crosses the x-axis. If the graph represents a function f, the number a is a ______________________. In other words, the zeros of a function f are ____________________________ ______________________.

A function is **even** if _________________________________ _____________________________. A function is **odd** if ____.

The function $y = f(x)$ is **even** if ________________________.

The function $y = f(x)$ is **odd** if ________________________.

Example 4: Decide whether the function $f(x) = 4x^2 - 3x + 1$ is even, odd, or neither.



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Section P.4 Fitting Models to Data

Objective: In this lesson you learned how to fit a mathematical model to a real-life data set.

I. Fitting a Linear Model to Data (Page 31)

What you should learn
How to fit a linear model to a real-life data set

Describe how to find a linear model to represent a set of paired data.

What does the correlation coefficient r indicate?

Example 1: Find a linear model to represent the following data. Round results to the nearest hundredth.

(−2.1, 19.4)	(−3.0, 19.7)	(8.8, 16.9)
(0, 18.9)	(6.1, 17.4)	(−4.0, 20.0)
(3.6, 18.1)	(0.9, 18.8)	(2.0, 18.5)

II. Fitting a Quadratic Model to Data (Page 32)

What you should learn
How to fit a quadratic model to a real-life data set

Example 2: Find a model to represent the following data. Round results to the nearest hundredth.

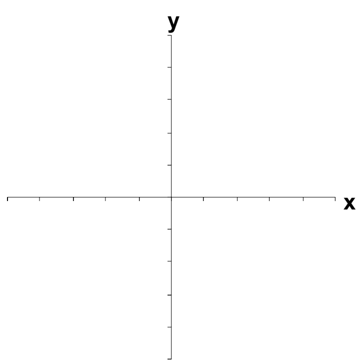
(−5, 68)	(−3, 30)	(−2, 22)
(−1, 11)	(0, 3)	(2, 8)
(4, 23)	(5, 43)	(7, 80)

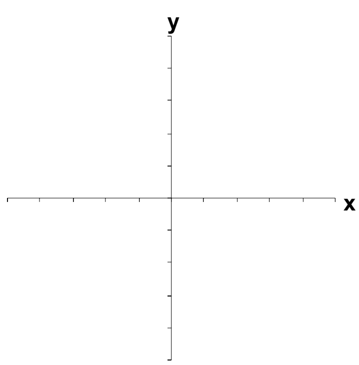
III. Fitting a Trigonometric Model to Data (Page 33)

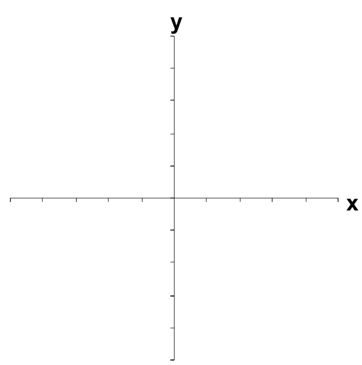
What you should learn
How to fit a trigonometric model to a real-life data set

Example 3: Find a trigonometric function to model the data in the following table.

x	0	$\pi/2$	π	$3\pi/2$	2π
y	2	4	2	0	2







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Chapter 1 Limits and Their Properties

Section 1.1 A Preview of Calculus

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Objective: In this lesson you learned how calculus compares with precalculus.

I. What is Calculus? (Pages 42–44)

What you should learn How to understand what calculus is and how it compares with precalculus

Calculus is ____
____
____
____
____.

List some problem-solving strategies that will be helpful in the study of calculus.

II. The Tangent Line Problem (Page 45)

What you should learn How to understand that the tangent line problem is basic to calculus

In the tangent line problem, you are given ____________________ ______________________________ and are asked to ____________ ____.

Except for cases involving a vertical tangent line, the problem of finding the tangent line at a point P is equivalent to ___________ ____. You can approximate this slope by using a line through ________________ ____. Such a line is called a ________________________.

If $P(c, f(c))$ is the point of tangency and $Q(c + \Delta x, f(c + \Delta x))$ is a second point on the graph of f, the slope of the secant line through these two points can be found using precalculus and is given by $m_{\text{sec}} =$ ____________________ .

As point Q approaches point P, the slope of the secant line approaches the slope of the ________________________. When such a "limiting position" exists, the slope of the tangent line is said to be ____ _________________________________.

III. The Area Problem (Page 46)

> ***What you should learn***
> How to understand that the area problem is also basic to calculus

A second classic problem in calculus is ___________________ ____ ________________________. This problem can also be solved with ______________________________. In this case, the limit process is applied to __________________________________ ____.

Consider the region bounded by the graph of the function $y = f(x)$, the x-axis, and the vertical lines $x = a$ and $x = b$. You can approximate the area of the region with _______________ ____________________________. As you increase the number of rectangles, the approximation tends to become ____________ ____ _________________________. Your goal is to determine the limit of the sum of the areas of the rectangles as _____________ ____.

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Section 1.2 Finding Limits Graphically and Numerically

Objective: In this lesson you learned how to find limits graphically and numerically.

I. An Introduction to Limits (Pages 48–49)

> ***What you should learn***
> How to estimate a limit using a numerical or graphical approach

The notation for a limit is $\lim_{x \to c} f(x) = L$, which is read as

____

The informal description of a limit is as follows: _____________

____

____.

Describe how to estimate the limit $\lim_{x \to -2} \dfrac{x^2 + 4x + 4}{x + 2}$ numerically.

The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of _____________________________________.

II. Limits That Fail to Exist (Pages 50–51)

> ***What you should learn***
> How to learn different ways that a limit can fail to exist

If a function $f(x)$ approaches a different number from the right side of $x = c$ than it approaches from the left side, then ________

____.

If $f(x)$ is not approaching a real number L—that is, if $f(x)$ increases or decreases without bound—as x approaches c, you can conclude that ______________________________________.

The limit of $f(x)$ as x approaches c also does not exist if $f(x)$ oscillates between _________________________________ as x approaches c.

III. A Formal Definition of Limit (Pages 52–54)

> ***What you should learn***
> How to study and use a formal definition of limit

The **ε-δ definition of limit** assigns mathematically rigorous meanings to the two phrases ______________________________ ________________ and ________________________ used in the informal description of limit.

Let ε represent _________________________________. Then the phrase "$f(x)$ becomes arbitrarily close to L" means that $f(x)$ lies in the interval _________________________. Using absolute value, you can write this as _________________________. The phrase "x approaches c" means that there exists a positive number δ such that x lies in either the interval __________________ or the interval __________________. This fact can be concisely expressed by the double inequality _______________________.

State the formal ε-δ definition of limit.

Example 1: Use the ε-δ definition of limit to prove that $\lim_{x\to -2}(10-3x)=16$.

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Section 1.3 Evaluating Limits Analytically

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Objective: In this lesson you learned how to evaluate limits analytically.

I. Properties of Limits (Pages 59–61)

What you should learn
How to evaluate a limit using properties of limits

The limit of $f(x)$ as x approaches c does not depend on the value of f at $x = c$. However, it may happen that the limit is precisely $f(c)$. In such cases, the limit can be evaluated by ______________ ______________________________.

Theorem 1.1 Let b and c be real numbers and let n be a positive integer. Complete each of the following properties of limits.

1. $\lim_{x \to c} b =$ ____________
2. $\lim_{x \to c} x =$ ____________
3. $\lim_{x \to c} x^n =$ ____________

Theorem 1.2 Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

Complete each of the following statements about operations with limits.

1. Scalar multiple: $\lim_{x \to c}[b\,f(x)] =$ ______________
2. Sum or difference: $\lim_{x \to c}[f(x) \pm g(x)] =$ _______________
3. Product: $\lim_{x \to c}[f(x) \cdot g(x)] =$ _____________
4. Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} =$ ________________________
5. Power: $\lim_{x \to c}[f(x)]^n =$ _____________

Example 1: Find the limit: $\lim_{x \to 4} 3x^2$.

The limit of a polynomial function $p(x)$ as $x \to c$ is simply the value of p at $x = c$. This direction substitution property is value for ____

_____________________________________.

Theorem 1.3 If p is a polynomial function and c is a real number, then $\lim_{x\to c} p(x) =$ ______________. If r is a rational function given by $r(x) = p(x)/q(x)$ and c is a real number such that $q(c) \neq 0$, then $\lim_{x\to c} r(x) =$ ____________________________.

Theorem 1.4 Let n be a positive integer. The following limit is valid for all c if n is odd, and is valid for $c > 0$ if n is even:

$\lim_{x\to c} \sqrt[n]{x} =$ ________________

Theorem 1.5 If f and g are functions such that $\lim_{x\to c} g(x) = L$ and $\lim_{x\to L} f(x) = f(L)$, then $\lim_{x\to c} f(g(x)) =$ ______________________.

Theorem 1.6 Let c be a real number in the domain of the given trigonometric function. Complete each of the following limit statements.

1. $\lim_{x\to c} \sin x =$ _______________________
2. $\lim_{x\to c} \cos x =$ _______________________
3. $\lim_{x\to c} \tan x =$ _______________________
4. $\lim_{x\to c} \cot x =$ _______________________
5. $\lim_{x\to c} \sec x =$ _______________________
6. $\lim_{x\to c} \csc x =$ _______________________

Example 2: Find the following limits.

a. $\lim_{x\to 4} \sqrt[4]{5x^2 + 1}$

b. $\lim_{x\to \pi} \cos x$

II. A Strategy for Finding Limits (Page 62)

> ***What you should learn***
> How to develop and use a strategy for finding limits

Theorem 1.7 Let c be a real number and let $f(x) = g(x)$ for all $x \neq c$ in an open interval containing c. If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ _______________ and $\lim\limits_{x \to c} f(x) =$ _________________________.

This theorem states that if two functions agree at all ___________ _____________________, then they have identical limit behavior at $x = c$.

List four steps in the strategy for finding limits.

III. Dividing Out and Rationalizing Techniques (Pages 63–64)

> ***What you should learn***
> How to evaluate a limit using dividing out and rationalizing techniques

An expression such as the meaningless fractional form 0/0 is called a(n) _____________________________ because you cannot, from the form alone, determine the limit. When you try to evaluate a limit and encounter this form, remember that you must rewrite the fraction so that the new denominator _________ __________________________. One way to do this is to ______ ________________________________, using the **dividing out** technique. Another technique is to ____________________ the numerator.

Example 3: Find the following limit: $\lim\limits_{x \to 3} \dfrac{x^2 - 8x + 15}{x - 3}$.

If you apply direct substitution to a rational function and obtain $r(c)=\dfrac{p(c)}{q(c)}=\dfrac{0}{0}$, then by the Factor Theorem of Algebra, you can conclude that $(x-c)$ must be a ____________________ to both $p(x)$ and $q(x)$.

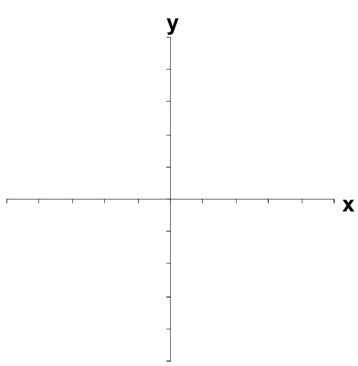
IV. The Squeeze Theorem (Pages 65–66)

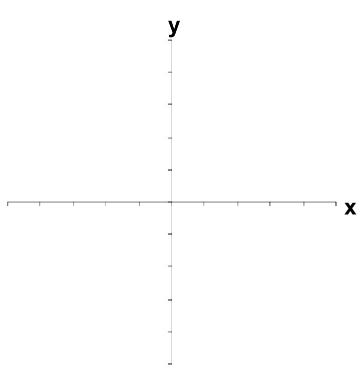
> ***What you should learn***
> How to evaluate a limit using the Squeeze Theorem

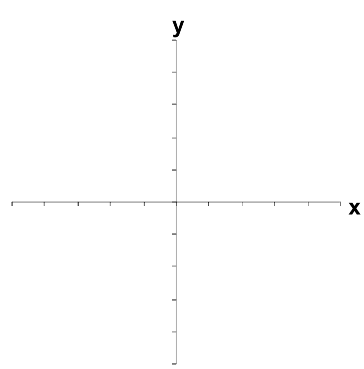
Theorem 1.8 The Squeeze Theorem If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$, then $\lim_{x\to c} f(x)$ exists and is equal to _________________.

Theorem 1.9 Two Special Trigonometric Limits

$\lim_{x\to 0}\dfrac{\sin x}{x} =$ ________ $\qquad$ $\lim_{x\to 0}\dfrac{1-\cos x}{x} =$ _________







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Section 1.4 Continuity and One-Sided Limits

Objective: In this lesson you learned how to determine continuity at a point and on an open interval, and how to determine one-sided limits.

Important Vocabulary Define each term or concept.

Discontinuity

Greatest integer function $f(x) = [\![x]\!]$

I. Continuity at a Point and on an Open Interval (Pages 70–71)

What you should learn
How to determine continuity at a point and continuity on a open interval

To say that a function *f* is continuous at $x = c$ means that there is no ______________________ in the graph of *f* at *c*: the graph is unbroken and there are no ____________________________.

A function *f* is **continuous at *c*** if the following three conditions are met:

1.

2.

3.

If *f* is continuous at each point in the interval (a, b), then it is ____. A function that is continuous on the entire real line $(-\infty, \infty)$ is ____.

A discontinuity at *c* is called **removable** if _________________ ____ ___________________________________.

A discontinuity at c is called **nonremovable** if ____________

____

____.

II. One-Sided Limits and Continuity on a Closed Interval (Pages 72–74)

> ***What you should learn***
> How to determine one-sided limits and continuity on a closed interval

A **one-sided limit** is the limit of a function $f(x)$ at c from either just the ____________ of c or just the ____________ of c.

$\lim\limits_{x \to c^+} f(x) = L$ is a one-sided limit from the _________ and means

____

$\lim\limits_{x \to c^-} f(x) = L$ is a one-sided limit from the _________ and means

____

One-sided limits are useful in taking limits of functions involving _____________________________.

When the limit from the left is not equal to the limit from the right, the (two-sided) limit _______________________________.

Let f be defined on a closed interval $[a, b]$. If f is continuous on the open interval (a, b) and $\lim\limits_{x \to a^+} f(x) = f(a)$ and $\lim\limits_{x \to b^-} f(x) = f(b)$,

then f is ____.

Moreover, f is continuous _________________ at a and continuous ___________________ at b.

III. Properties of Continuity (Pages 75–76)

> ***What you should learn***
> How to use properties of continuity

If b is a real number and f and g are continuous at $x = c$, then the following functions are also continuous at c.

1.

2.

3.

4.

A polynomial function is continuous at ___________________

_________________________.

A rational function is continuous at ______________________

_________________________.

If g is continuous at c and f is continuous at $g(c)$, then the composite function given by $(f \circ g)(x) = f(g(x))$ is continuous

_______________________.

IV. The Intermediate Value Theorem (Pages 77–78)

What you should learn
How to understand and use the Intermediate Value Theorem

Intermediate Value Theorem If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then ____

____.

Explain why the Intermediate Value Theorem is called an **existence theorem.**

The Intermediate Value Theorem states that for a continuous function f, if x takes on all values between a and b, $f(x)$ must ____

____.

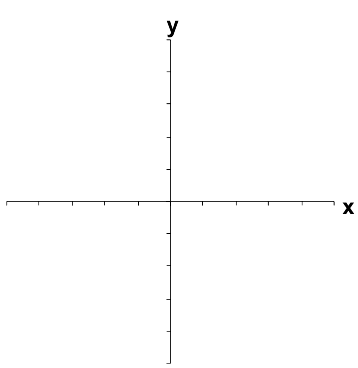
The Intermediate Value Theorem often can be used to locate zeros of a function that is continuous on a closed interval. Specifically, if f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ differ in sign, the Intermediate Value Theorem guarantees _____________

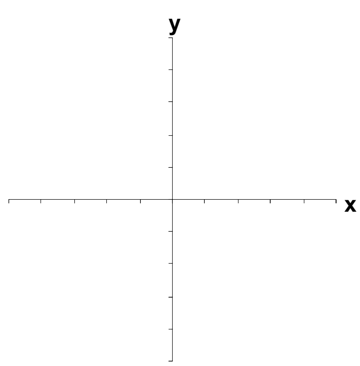
____

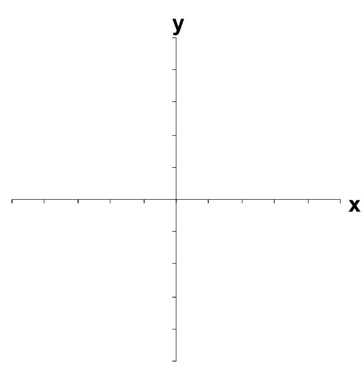
____.

Explain how the **bisection method** can be used to approximate the real zeros of a continuous function.

Additional notes







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Section 1.5 Infinite Limits

Objective: In this lesson you learned how to determine infinite limits and find vertical asymptotes.

I. Infinite Limits (Pages 83–84)

What you should learn How to determine infinite limits from the left and from the right

A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an ________________________.

Let f be a function that is defined at every real number in some open interval containing c (except possibly at c itself). The statement $\lim_{x \to c} f(x) = \infty$ means ____________________________

____

________________________________. Similarly, the statement $\lim_{x \to c} f(x) = -\infty$ means ____________________________

____.

To define the **infinite limit from the left,** replace $0 < |x - c| < \delta$ by ______________________. To define the **infinite limit from the right,** replace $0 < |x - c| < \delta$ by ______________________.

Be sure to see that the equal sign in the statement $\lim f(x) = \infty$ does not meant that ________________________________! On the contrary, it tells you how the limit ___________________ by denoting the unbounded behavior of $f(x)$ as x approaches c.

II. Vertical Asymptotes (Pages 84–87)

What you should learn How to find and sketch the vertical asymptotes of the graph of a function

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a _____________ _______________ of the graph of f.

Let f and g be continuous on an open interval containing c. If $f(c) \neq 0$, $g(c) = 0$, and there exists an open interval containing c such that $g(x) \neq 0$ for all $x \neq c$ in the interval, then the graph of the function given by $h(x) = \dfrac{f(x)}{g(x)}$ has ____________

____.

If both the numerator and denominator are 0 at $x = c$, you obtain the ____ and you cannot determine the limit behavior at $x = c$ without further investigation, such as simplifying the expression.

Example 1: Determine all vertical asymptotes of the graph of

$$f(x) = \frac{x^2 + 9x + 20}{x^2 + 2x - 15}.$$

Theorem 1.15 Let c and L be real numbers and let f and g be functions such that

$$\lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L$$

Complete each of the following statements about operations with limits.

1. Sum or difference: $\lim_{x \to c}[f(x) \pm g(x)] =$ _____________

2. Product: $\lim_{x \to c}[f(x) \cdot g(x)] =$ ________________
   $\lim_{x \to c}[f(x) \cdot g(x)] =$ ________________

3. Quotient: $\lim_{x \to c} \dfrac{g(x)}{f(x)} =$ _____________________

Example 2: Determine the limit: $\lim_{x \to 3}\left(\dfrac{1}{x-3} - 3\right)$.

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Chapter 2 Differentiation

Section 2.1 The Derivative and the Tangent Line Problem

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Objective: In this lesson you learned how to find the derivative of a function using the limit definition and understand the relationship between differentiability and continuity.

Important Vocabulary	Define each term or concept.
Differentiation	
Differentiable	

I. The Tangent Line Problem (Pages 96–99)

What you should learn
How to find the slope of the tangent line to a curve at a point

Essentially, the problem of finding the tangent line at a point *P* boils down to ____ ________________________________. You can approximate this slope using ______________________________ through the point of tangency $(c, f(c))$ and a second point on the curve $(c + \Delta x, f(c + \Delta x))$. The slope of the secant line through these two points is $m_{\text{sec}} = \frac{\qquad}{\qquad}$.

The right side of this equation for the slope of a secant line is called a ___________________________. The denominator Δx is the ________________________, and the numerator $\Delta y = f(c + \Delta x) - f(c)$ is the ______________________.

The beauty of this procedure is that you can obtain more and more accurate approximations of the slope of the tangent line by ____ ______________________________________.

If *f* is defined on an open interval containing *c*, and if the limit $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$ exists, then the line passing

through $(c, f(c))$ with slope m is ___________________________
____.

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is also called ____
__________________________.

Example 1: Find the slope of the graph of $f(x) = 9 - \frac{x}{2}$ at the point (4, 7).

Example 2: Find the slope of the graph of $f(x) = 2 - 3x^2$ at the point (−1, −1).

The definition of a tangent line to a curve does not cover the possibility of a vertical tangent line. If f is continuous at c and $\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = \infty$ or $\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = -\infty$, the vertical line $x = c$ passing through $(c, f(c))$ is _______________
________________________ to the graph of f.

II. The Derivative of a Function (Pages 99–101)

> ***What you should learn***
> How to use the limit definition to find the derivative of a function

The ____________________________________ is given by $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$, provided the limit exists. For all x for which this limit exists, f' is __________________________.

The derivative of a function of x gives the __________________
__________________ to the graph of f at the point $(x, f(x))$, provided that the graph has a tangent line at this point.

A function is **differentiable on an open interval (*a*, *b*)** if ______
____.

Example 3: Find the derivative of $f(t) = 4t^2 + 5$.

III. Differentiability and Continuity (Pages 101–103)

What you should learn
How to understand the relationship between differentiability and continuity

Name some situations in which a function will not be differentiable at a point.

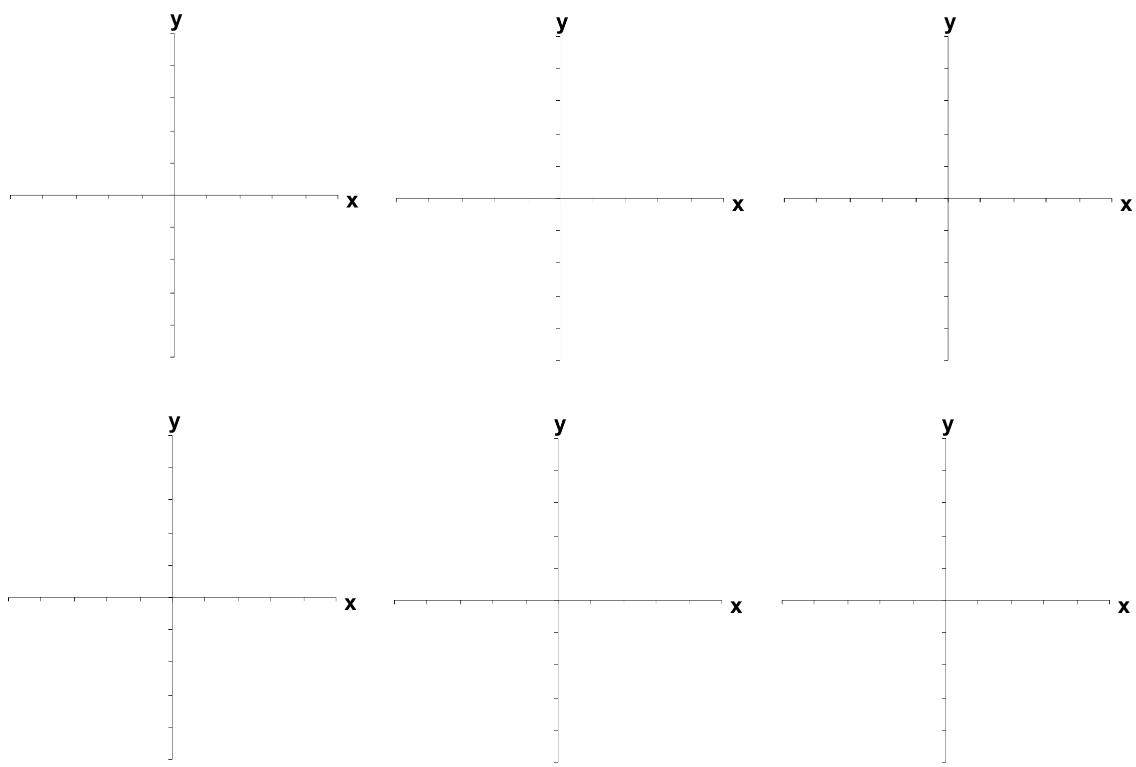
If a function *f* is differentiable at $x = c$, then ________________

__________________________.

Complete the following statements.

1. If a function is differentiable at $x = c$, then it is continuous at $x = c$. So, differentiability ___________________ continuity.
2. It is possible for a function to be continuous at $x = c$ and not be differentiable at $x = c$. So, continuity ________________ _____________ differentiability.

Additional notes



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Section 2.2 Basic Differentiation and Rates of Change

Objective: In this lesson you learned how to find the derivative of a function using basic differentiation rules.

I. The Constant Rule (Page 107)

> ***What you should learn***
> How to find the derivative of a function using the Constant Rule

The derivative of a constant function is __________.

If c is a real number, then $\frac{d}{dx}[c] =$ __________.

II. The Power Rule (Pages 108–109)

> ***What you should learn***
> How to find the derivative of a function using the Power Rule

The **Power Rule** states that if n is a rational number, then the function $f(x) = x^n$ is differentiable and

$\frac{d}{dx}\left[x^n\right] =$ ________________. For f to be differentiable at

$x = 0$, n must be a number such that x^{n-1} is ________________

____.

Also, $\frac{d}{dx}[x] =$ ______________.

Example 1: Find the derivative of the function $f(x) = \frac{1}{x^3}$.

Example 2: Find the slope of the graph of $f(x) = x^5$ at $x = 2$.

III. The Constant Multiple Rule (Pages 110–111)

> ***What you should learn***
> How to find the derivative of a function using the Constant Multiple Rule

The **Constant Multiple Rule** states that if f is a differentiable function and c is a real number then cf is also differentiable and

$\frac{d}{dx}[cf(x)] =$ ______________________.

Informally, the Constant Multiple Rule states that ___________

____

____.

Example 3: Find the derivative of $f(x) = \dfrac{2x}{5}$

The Constant Multiple Rule and the Power Rule can be combined into one rule. The combination rule is

$\dfrac{d}{dx}\left[cx^n\right] =$ ____________________.

Example 4: Find the derivative of $y = \dfrac{2}{5x^5}$

IV. The Sum and Difference Rules (Page 111)

What you should learn
How to find the derivative of a function using the Sum and Difference Rules

The **Sum and Difference Rules** of Differentiation state that the sum (or difference) of two differentiable functions *f* and *g* is itself differentiable. Moreover, the derivative of $f + g$ (or $f - g$) is the sum (or difference) of the derivatives of *f* and *g*.

That is, $\dfrac{d}{dx}\left[f(x) + g(x)\right] =$ ______________________________

and $\dfrac{d}{dx}\left[f(x) - g(x)\right] =$ ______________________________

Example 5: Find the derivative of $f(x) = 2x^3 - 4x^2 + 3x - 1$

V. Derivatives of Sine and Cosine Functions (Page 112)

What you should learn
How to find the derivative of the sine function and of the cosine function

$\dfrac{d}{dx}\left[\sin x\right] =$ ______________________

$\dfrac{d}{dx}\left[\cos x\right] =$ ______________________

Example 6: Differentiate the function $y = x^2 - 2\cos x$.

VI. Rates of Change (Pages 113−114)

> ***What you should learn***
> How to use derivatives to find rates of change

The derivative can also be used to determine _______________
____.

Give some examples of real-life applications of rates of change.

The function s that gives the position (relative to the origin) of an object as a function of time t is called a _________________.

The **average velocity** of an object that is moving in a straight line is found as follows.

Average velocity = ——————— = ———

Example 7: If a ball is dropped from the top of a building that is 200 feet tall, and air resistance is neglected, the height s (in feet) of the ball at time t (in seconds) is given by $s = -16t^2 + 200$. Find the average velocity of the object over the interval [1, 3].

If $s = s(t)$ is the position function for an object moving along a straight line, the (instantaneous) **velocity** of the object at time t is

$v(t) =$ ——————— = ____________________.

In other words, the velocity function is the _____________ the position function. Velocity can be ______________________
_________________. The _____________ of an object is the absolute value of its velocity. Speed cannot be ____________.

Example 8: If a ball is dropped from the top of a building that is 200 feet tall, and air resistance is neglected, the height s (in feet) of the ball at time t (in seconds) is given by $s(t) = -16t^2 + 200$. Find the velocity of the ball when $t = 3$.

The position function for a free-falling object (neglecting air resistance) under the influence of gravity can be represented by the equation ______________________________, where s_0 is the initial height of the object, v_0 is the initial velocity of the object, and g is the acceleration due to gravity. On Earth, the value of g is

____

____.

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Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

Objective: In this lesson you learned how to find the derivative of a function using the Product Rule and Quotient Rule.

What you should learn
How to find the derivative of a function using the Product Rule

I. The Product Rule (Pages 119–120)

The product of two differentiable functions *f* and *g* is itself differentiable. The **Product Rule** states that the derivative of the fg is equal to ____

____

_________________________. That is,

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) .$$

Example 1: Find the derivative of $y = (4x^2 + 1)(2x - 3)$.

The Product Rule can be extended to cover products that have more than two factors. For example, if *f*, *g*, and *h* are differentiable functions of *x*, then

$$\frac{d}{dx}[f(x)g(x)h(x)] = \underline{\hspace{8cm}}$$

Explain the difference between the Constant Multiple Rule and the Product Rule.

II. The Quotient Rule (Pages 121–123)

What you should learn
How to find the derivative of a function using the Quotient Rule

The quotient f / g of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. The derivative of f / g is given by ____________________________

____

____, all divided by ____.

This is called the ______________________, and is given by

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}, \qquad g(x) \neq 0.$$

Example 2: Find the derivative of $y = \dfrac{2x+5}{3x}$.

With the Quotient Rule, it is a good idea to enclose all factors and derivatives ____________________________ and to pay special attention to __________________________________

___________________________.

III. Derivatives of Trigonometric Functions (Pages 123–124)

What you should learn
How to find the derivative of a trigonometric function

$\frac{d}{dx}[\tan x] =$ ______________________

$\frac{d}{dx}[\cot x] =$ ______________________

$\frac{d}{dx}[\sec x] =$ ______________________

$\frac{d}{dx}[\csc x] =$ ______________________

Example 3: Differentiate the function $f(x) = \sin x \sec x$.

IV. Higher-Order Derivatives (Page 125)

> ***What you should learn***
> How to find a higher-order derivative of a function

The derivative of $f'(x)$ is the second derivative of $f(x)$ and is denoted by ________________. The derivative of $f''(x)$ is the ______________________ of $f(x)$ and is denoted by f'''.

These are examples of ____________________________ of $f(x)$.

The following notation is used to denoted the __________________ of the function $y = f(x)$:

$$\frac{d^6 y}{dx^6} \quad D_x^6[y] \quad y^{(6)} \quad \frac{d^6}{dx^6}[f(x)] \quad f^{(6)}(x)$$

Example 4: Find $y^{(5)}$ for $y = 2x^7 - x^5$.

Example 5: On the moon, a ball is dropped from a height of 100 feet. Its height *s* (in feet) above the moon's surface is given by $s = -\frac{27}{10}t^2 + 100$. Find the height, the velocity, and the acceleration of the ball when $t = 5$ seconds.

Example 6: Find y''' for $y = \sin x$.

Additional notes

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Section 2.4 The Chain Rule

Objective: In this lesson you learned how to find the derivative of a function using the Chain Rule and General Power Rule.

I. The Chain Rule (Pages 130–132)

What you should learn
How to find the derivative of a composite function using the Chain Rule

The Chain Rule, one of the most powerful differentiation rules, deals with ____________________ functions.

Basically, the Chain Rule states that if y changes dy/du times as fast as u, and u changes du/dx times as fast as x, then y changes ____________________________ times as fast as x.

The **Chain Rule** states that if $y = f(u)$ is a differentiable function of u, and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x, and

$\frac{dy}{dx} =$ ____________________ or, equivalently,

$\frac{d}{dx}[f(g(x))] =$ _______________________________.

When applying the Chain Rule, it is helpful to think of the composite function $f \circ g$ as having two parts, an *inner part* and an *outer part*. The Chain Rule tells you that the derivative of $y = f(u)$ is the derivative of the ____________________ (at the inner function u) *times* the derivative of the ____________________ ____________________. That is, $y' =$ ________________________.

Example 1: Find the derivative of $y = (3x^2 - 2)^5$.

II. The General Power Rule (Pages 132–133)

> ***What you should learn***
> How to find the derivative of a function using the General Power Rule

The General Power Rule is a special case of the ____________ ____________.

The General Power Rule states that if $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then

$\dfrac{dy}{dx} =$ ______________________ or, equivalently,

$\dfrac{d}{dx}\left[u^n\right] =$ _________________

Example 2: Find the derivative of $y = \dfrac{4}{(2x-1)^3}$.

III. Simplifying Derivatives (Page 134)

> ***What you should learn***
> How to simplify the derivative of a function using algebra

Example 3: Find the derivative of $y = \dfrac{3x^2}{(1-x^3)^2}$ and simplify.

IV. Trigonometric Functions and the Chain Rule
(Pages 135–136)

> ***What you should learn***
> How to find the derivative of a trigonometric function using the Chain Rule

Complete each of the following "Chain Rule versions" of the derivatives of the six trigonometric functions.

$\frac{d}{dx}[\sin u] =$ ____________________

$\frac{d}{dx}[\cos u] =$ ____________________

$\frac{d}{dx}[\tan u] =$ ____________________

$\frac{d}{dx}[\cot u] =$ ____________________

$\frac{d}{dx}[\sec u] =$ ____________________

$\frac{d}{dx}[\csc u] =$ ____________________

Example 4: Differentiate the function $y = \sec 4x$.

Example 5: Differentiate the function $y = x^2 - \cos(2x+1)$.

Additional notes

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Section 2.5 Implicit Differentiation

Objective: In this lesson you learned how to find the derivative of a function using implicit differentiation.

What you should learn How to distinguish between functions written in implicit form and explicit form

I. Implicit and Explicit Functions (Page 141)

Up to this point in the text, most functions have been expressed in **explicit form** $y = f(x)$, meaning that ___________________ ____. However, some functions are only _________________ by an equation.

Give an example of a function in which y is **implicitly** defined as a function of x.

Implicit differentiation is a procedure for taking the derivative of an implicit function when you are unable to _______________ _______________________________.

To understand how to find $\frac{dy}{dx}$ implicitly, realize that the differentiation is taking place ______________________. This means that when you differentiate terms involving x alone, _____ _______________________________. However, when you differentiate terms involving y, you must apply _______________ ___________ because you are assuming that y is defined ________________ as a differentiable function of x.

Example 1: Differentiate the expression with respect to x:
$4x + y^2$

II. Implicit Differentiation (Pages 142–145)

What you should learn
How to use implicit differentiation to find the derivative of a function

Consider an equation involving x and y in which y is a differentiable function of x. List the four guidelines for applying implicit differentiation to find dy/dx.

1.

2.

3.

4.

Example 2: Find dy/dx for the equation $4y^2 - x^2 = 1$.

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Section 2.6 Related Rates

Objective: In this lesson you learned how to find a related rate.

I. Finding Related Variables (Page 149)

> ***What you should learn***
> How to find a related rate

Another important use of the Chain Rule is to find the rates of change of two or more related variables that are changing with respect to ____________________.

Example 1: The variables x and y are differentiable functions of t and are related by the equation $y = 2x^3 - x + 4$. When $x = 2$, $dx/dt = -1$. Find dy/dt when $x = 2$.

II. Problem Solving with Related Rates (Pages 150–153)

> ***What you should learn***
> How to use related rates to solve real-life problems

List the guidelines for solving a related-rate problems.

1.

2.

3.

4.

Example 2: Write a mathematical model for the following related-rate problem situation:
The population of a city is decreasing at the rate of 100 people per month.

Additional notes

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Chapter 3 Applications of Differentiation

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Section 3.1 Extrema on an Interval

Objective: In this lesson you learned how to use a derivative to locate the minimum and maximum values of a function on a closed interval.

Important Vocabulary Define each term or concept.

Relative maximum

Relative minimum

Critical number

I. Extrema of a Function (Page 164)

What you should learn
How to understand the definition of extrema of a function on an interval

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** if ______________________

____________________.

2. $f(c)$ is the **maximum of f on I** if ____________________

____________________.

The minimum and maximum of a function on an interval are the ____________________________, or extrema (the singular for of extrema is ____________________), of the function on the interval. The minimum and maximum of a function on an interval are also called the ______________________________ ____________________, or the _________________________ _____________________, on the interval.

The **Extreme Value Theorem** states that if f is continuous on a closed interval $[a, b]$, then ______________________________ ____.

II. Relative Extrema and Critical Numbers (Pages 165–166)

> ***What you should learn***
> How to understand the definition of relative extrema of a function on an open interval

If f has a relative minimum or relative maximum when $x = c$, then c is a ____________________________ of f.

III. Finding Extrema on a Closed Interval (Pages 167–168)

> ***What you should learn***
> How to find extrema on a closed interval

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1.

2.

3.

4.

Example 1: Find the extrema of the function $f(x) = x^3 + 6x^2 - 15x + 2$ on the interval $[-6, 6]$.

The critical numbers of a function need not produce ___________ ________________________.

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Section 3.2 Rolle's Theorem and the Mean Value Theorem

Objective: In this lesson you learned how many of the results in this chapter depend on two important theorems called Rolle's Theorem and the Mean Value Theorem.

I. Rolle's Theorem (Pages 172–173)

What you should learn
How to understand and use Rolle's Theorem

The Extreme Value Theorem states that a continuous function on a closed interval $[a, b]$ must have __________________________ _______________________________________. Both of these values, however, can occur at ____________________________. **Rolle's Theorem** gives conditions that guarantee the existence of an extreme value in ___________________________________ ___________________.

The statement of Rolle's Theorem says: Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is ________________________ ____.

If the conditions of Rolle's Theorem are satisfied, then there must be at least one x-value between a and b at which the graph of f has ____.
Alternatively, Rolle's Theorem states that if f satisfies the conditions of the theorem, there must be at least one point between a and b at which the derivative is ______________.

II. The Mean Value Theorem (Pages 174–175)

What you should learn
How to understand and use the Mean Value Theorem

The **Mean Value Theorem** states that if f is continuous on ______________________________________ and differentiable on ____, then there exists _______________________________________ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

The Mean Value Theorem has implications for both basic interpretations of the derivative. Geometrically, the theorem guarantees the existence of ____________________________

____

____________________________. In terms of rates of change, the Mean Value Theorem implies that there must be __________

____

____

____________________________.

A useful alternative form of the Mean Value Theorem is as follows: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that

____.

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Section 3.3 Increasing and Decreasing Functions and the First Derivative Test

Objective: In this lesson you learned how to use the first derivative to determine whether a function is increasing or decreasing.

Important Vocabulary Define each term or concept.

Increasing function

Decreasing function

I. Increasing and Decreasing Functions (Pages 179–180)

What you should learn
How to determine intervals on which a function is increasing or decreasing

A function is **increasing** if its graph moves _____________ as x moves ___________________. A function is **decreasing** if its graph moves _____________ as x moves ________________.

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

If $f'(x) > 0$ for all x in (a, b), then f is _______________ on $[a, b]$.

If $f'(x) < 0$ for all x in (a, b), then f is _______________ on $[a, b]$.

If $f'(x) = 0$ for all x in (a, b), then f is _______________ on $[a, b]$.

The first of these tests for increasing and decreasing functions can be interpreted as follows: if the first derivative of a function is positive for all values of x in an interval, then the function is ________________ on that interval.

Interpret the other two tests in a similar way.

Example 1: Find the open intervals on which the function is increasing or decreasing: $f(x) = -x^2 + 10x - 21$

Let *f* be a continuous function on the interval (a, b). List the steps for finding the intervals on which *f* is increasing or decreasing.

1.

2.

3.

A function is **strictly monotonic** on an interval if ____________ ____ ____________________________.

II. The First Derivative Test (Pages 181–185)

What you should learn
How to apply the First Derivative Test to find relative extrema of a function

Let c be a critical number of a function *f* that is continuous on an open interval I containing c. The **First-Derivative Test** states that if *f* is differentiable on the interval (except possibly at c), then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from negative to positive at c, then f has a ______________________________ at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a ______________________________ at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is ______________________ __________________________________.

In your own words, describe how to find the relative extrema of a function *f*.

Example 2: Find all relative extrema of the function $f(x) = x^3 - 7x^2 - 38x + 240$.

Additional notes

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Section 3.4 Concavity and the Second Derivative Test

Objective: In this lesson you learned how to use the second derivative to determine whether the graph of a function is concave upward or downward.

Important Vocabulary Define each term or concept.

Concave upward

Concave downward

Point of inflection

I. Concavity (Pages 190–192)

What you should learn
How to determine intervals on which a function is concave upward or concave downward

Let *f* be differentiable on an open interval *I*. If the graph of *f* is **concave upward**, then the graph of *f* lies ____________ all of its tangent lines on *I*.

Let *f* be differentiable on an open interval *I*. If the graph of *f* is **concave downward,** then the graph of *f* lies ______________ all of its tangent lines on *I*.

As a test for concavity, let *f* be a function whose second derivative exists on an open interval *I*.

1. If $f''(x) > 0$ for all x in I, then the graph of *f* is _________ ____________ in *I*.

2. If $f''(x) < 0$ for all x in I, then the graph of *f* is _________ ___________ in *I*.

In your own words, describe how to apply the Concavity Test.

Example 1: Describe the concavity of the function $f(x) = 1 - 3x^2$.

II. Points of Inflection (Pages 192–193)

What you should learn
How to find any points of inflection of the graph of a function

To locate possible points of inflection, you can determine

____

__________________________.

State Theorem 3.8 for Points of Inflection.

Example 2: Find the points of inflection of $f(x) = -\frac{1}{2}x^4 + 10x^3 - 48x^2 + 4$.

The converse of Theorem 3.8 is ________________________.
That is, it is possible for the second derivative to be 0 at a point
that is ____.

III. The Second-Derivative Test (Page 194)

What you should learn
How to apply the Second Derivative Test to find relative extrema of a function

Let be a function such that $f'(c) = 0$ and the second derivative of *f* exists on an open interval containing *c*. Then the **Second-Derivative Test** states:

1.

2.

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Section 3.5 Limits at Infinity

Objective: In this lesson you learned how to find horizontal asymptotes of the graph of a function.

I. Limits at Infinity (Page 198)

> ***What you should learn***
> How to determine (finite) limits at infinity

To say that a statement is true as x increases without bound means that for some (large) real number M, the statement is true for all x in the interval ________________________.

Let L be a real number. The definition of **limit at infinity** states that

1. $\lim_{x\to\infty} f(x) = L$ means ________________________
____
____.

2. $\lim_{x\to-\infty} f(x) = L$ means ________________________
____
____.

II. Horizontal Asymptotes (Pages 199–203)

> ***What you should learn***
> How to determine the horizontal asymptotes, if any, of the graph of a function

The line $y = L$ is a ____________________________ of the graph of f if $\lim_{x\to-\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$.

Notice that from this definition, if follows that the graph of a function of x can have at most ________________________
____.

If r is a positive rational number and c is any real number, then $\lim_{x\to\infty} \dfrac{c}{x^r} =$ _______________. Furthermore, if x^r is defined when $x < 0$, then $\lim_{x\to-\infty} \dfrac{c}{x^r} =$ _______________.

Example 1: Find the limit: $\lim_{x\to\infty}\left(2+\frac{3}{x^2}\right)$

If an **indeterminate form** ________________ is encountered while finding a limit at infinity, you can resolve this problem by

____

______________________________.

Complete the following guidelines for finding limits at ± ∞ of rational functions.

1. ____
____
____.
2. ____
____
____
____.
3. ____
____
____.

Example 2: Find the limit: $\lim_{x\to\infty}\frac{x^3-1}{1-13x+2x^2-5x^3}$

III. Infinite Limits at Infinity (Page 204)

What you should learn
How to determine infinite limits at infinity

Many function do not approach a finite limit as x increases (or decreases) without bound. ____________________________ are one type of function that does not have a finite limit at infinity.

Let f be a function defined on the interval (a, ∞). The definition of **infinite limits at infinity** states that

1. $\lim_{x \to \infty} f(x) = \infty$ means ______________________
____
______________________________________.

2. $\lim_{x \to \infty} f(x) = -\infty$ means ______________________
____
____
______________________________.

Example 3: Find the limit: $\lim_{x \to \infty} (2x^2 - 9x + 1)$.

Additional notes

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Section 3.6 A Summary of Curve Sketching

Objective: In this lesson you learned how to graph a function using the techniques from Chapters P–3.

I. Analyzing the Graph of a Function (Pages 209–214)

What you should learn   How to analyze the graph of a function

List some of the concepts that you have studied thus far that are useful in analyzing the graph of a function.

List three guidelines for analyzing the graph of a function.

1.

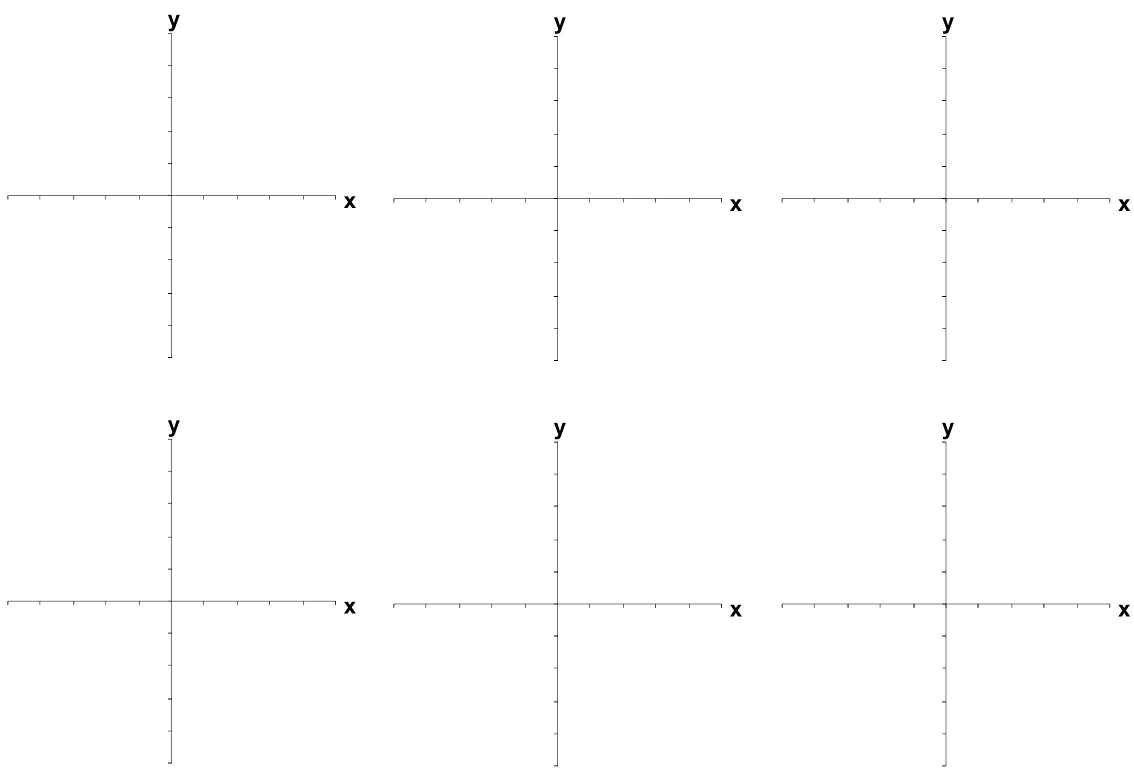
2.

3.

The graph of a rational function (having no common factors and whose denominator is of degree 1 or greater) has a ____________ ___________________ if the degree of the numerator exceeds the degree of the denominator by exactly 1.

To find the slant asymptote, _______________________________ ____ ____.

In general, a polynomial function of degree n can have at most ____________________ relative extrema, and at most ____________________ points of inflection. Moreover, polynomial functions of even degree must have ______________ _______________ relative extremum.



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Section 3.7 Optimization Problems

Objective: In this lesson you learned how to solve optimization problems.

I. Applied Minimum and Maximum Problems
(Pages 218–222)

What you should learn
How to solve applied minimum and maximum problems

What does "optimization problem" mean?

In an optimization problem, the **primary equation** is one that

____.

The feasible domain of a function consists of ______________

____.

A secondary equation is used to _______________________

____.

List the steps for solving optimization problems.

1.

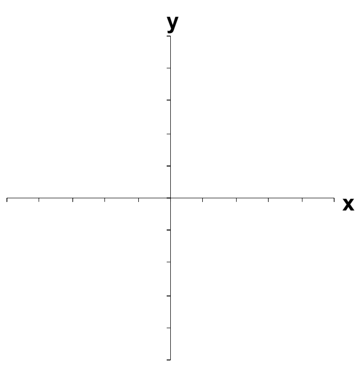
2.

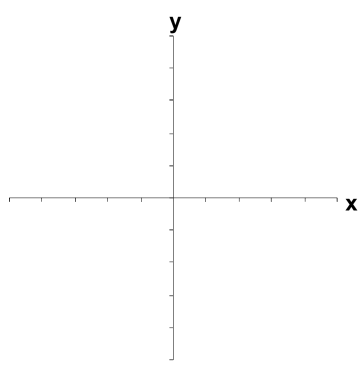
3.

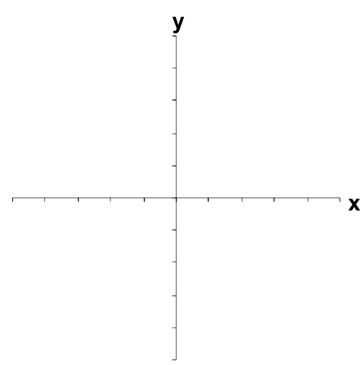
4.

5.

Additional notes







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Section 3.8 Newton's Method

Objective: In this lesson you learned how to use Newton's Method, an approximation technique, to solve problems.

I. Newton's Method (Pages 229–232)

> ***What you should learn***
> How to approximate a zero of a function using Newton's Method

Newton's Method is ________________________________
____, and
it uses ____
____________________________________.

Let $f(c) = 0$, where f is differentiable on an open interval containing c. To use Newton's Method to approximate c, use the following steps.

1.

2.

3.

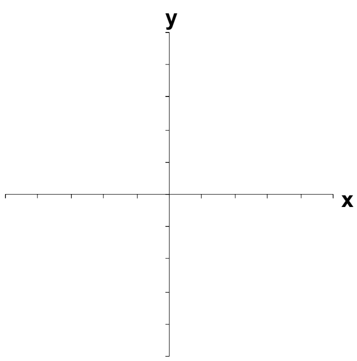
Each successive application of this procedure is called an
__________________________.

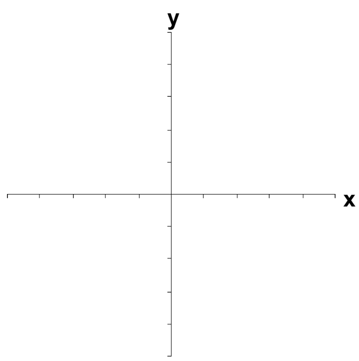
When the approximations given by Newton's Method approach a limit, the sequence $x_1, x_2, x_3, \ldots, x_n, \ldots$ is said to ____________________________. Moreover, if the limit is c, it can be shown that c must be _____________________________.

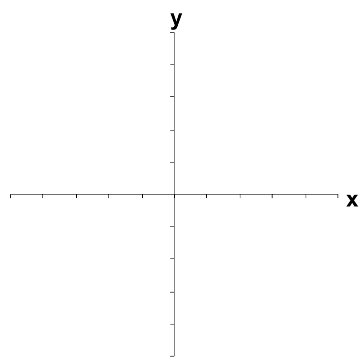
Newton's Method does not always yield a convergent sequence. One way it can fail to do so is if __________________________
or if __________________________________ .

When the first situation is encountered, it can usually be overcome by ____ .

Additional notes







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Section 3.9 Differentials

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Objective: In this lesson you learned how to use approximation techniques to solve problems.

Important Vocabulary Define each term or concept.

Differential of *x*

Differential of *y*

I. Tangent Line Approximations (Page 235)

> ***What you should learn***
> How to understand the concept of a tangent line approximation

Consider a function f that is differentiable at c. The equation for the tangent line at the point $(c, f(c))$ is given by ____, and is called ____ ______________________________________. Because c is a constant, y is a __________________________ of x. Moreover, by restricting the values of x to be sufficiently close to c, the values of y can be used as approximations (to any desired accuracy) of ____. In other words, as $x \to c$, the limit of y is ___________________.

II. Differentials (Page 236)

> ***What you should learn***
> How to compare the value of the differential, dy, with the actual change in y, Δy

When the tangent line to the graph of f at the point $(c, f(c))$ is used as an approximation of the graph of f, the quantity $x - c$ is called the ___________________________ and is denoted by _______________. When Δx is small, the change in y (denoted by Δy) can be approximated as __________________________. For such an approximation, the quantity Δx is traditionally denoted by _____________ and is called the **differential of *x*.** The expression $f'(x)dx$ is denoted by ______________, and is called the **differential of *y*.**

In many types of applications, the differential of y can be used as ____. That is, $\Delta y \approx$ ____________ or $\Delta y \approx$ ___________________.

III. Error Propagation (Page 237)

> ***What you should learn***
> How to estimate a propagated error using a differential

Physicists and engineers tend to make liberal use of the approximation of Δy by dy. One way this occurs in practice is in the ____ _________________________. For example, if you let x represent the measured value of a variable and let $x + \Delta x$ represent the exact value, then Δx is ______________________ ____________________. Finally if the measured value x is used to compute another value $f(x)$, the difference between $f(x + \Delta x)$ and $f(x)$ is the _______________________________________.

IV. Calculating Differentials (Pages 238–239)

> ***What you should learn***
> How to find the differential of a function using differentiation formulas

Each of the differentiation rules that you studied in Chapter 2 can be written in ________________________________.

Suppose u and v are differentiable functions of x. Then by the definition of differentials, you have
$du =$ __________________________ and $dv =$ ___________________________

Complete the following differential forms of common differentiation rules:

Constant Multiple Rule: _______________________

Sum or Difference Rule: _______________________

Product Rule: ______________________________

Quotient Rule: _______________________________

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Chapter 4 Integration

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Section 4.1 Antiderivatives and Indefinite Integration

Objective: In this lesson you learned how to evaluate indefinite integrals using basic integration rules.

Important Vocabulary	Define each term or concept.
Antiderivative	

I. Antiderivatives (Pages 248–249)

> ***What you should learn***
> How to write the general solution of a differential equation

If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form ______________________, for all x in I where C is a constant. The entire family of antiderivatives of a function can be represented by ____ ________________________________. The constant C is called the _____________________________. The family of functions represented by G is the _________________________________.

A **differential equation** in x and y is an equation that ________ ____.

Give an example of a *differential equation* and its **general solution.**

II. Notation for Antiderivatives (Page 249)

> ***What you should learn***
> How to use indefinite integral notation for antiderivatives

The operation of finding all solutions of the equation $dy = f(x)\,dx$ is called __________________________ ______________________________ and is denoted by the symbol $\int$, which is called an ________________________.

The symbol $\int f(x)\,dx$ is the ____________________________.

Use the terms *antiderivative, constant of integration, differential, integral sign,* and *integrand* to label the following notation:

$$\int f(x)\,dx = F(x) + C$$

The differential in the indefinite integral identifies ____________
__________________________________.

The notation $\int f(x)\,dx = F(x) + C$, where C is an arbitrary constant, means that F is __________________________________
____.

III. Basic Integration Rules (Pages 250–252)

> ***What you should learn***
> How to use basic integration rules to find antiderivatives

Complete the following basic integration rules, which follow from differentiation formulas.

1. $\int k\,dx =$ ______________________________________.
2. $\int kf(x)\,dx =$ ________________________________
3. $\int [f(x)+g(x)]\,dx =$ __________________________________
4. $\int [f(x)-g(x)]\,dx =$ __________________________________
5. _________________________ $= \dfrac{x^{n+1}}{n+1} + C, \quad n \neq -1$
6. $\int 0\,dx =$ _______________________
7. $\int \cos x\,dx =$ _____________________________

8. $\int \sin x \, dx =$ ______________________________

9. $\int \sec^2 x \, dx =$ ______________________________

10. $\int \sec x \tan x \, dx =$ ______________________________

11. $\int \csc^2 x \, dx =$ ______________________________

12. $\int \csc x \cot x \, dx =$ ______________________________

Example 1: Find $\int -3 \, dx$.

Example 2: Find $\int 2x^2 \, dx$.

Example 3: Find $\int (1-2x) \, dx$.

IV. Initial Conditions and Particular Solutions
(Pages 253–255)

> ***What you should learn***
> How to find a particular solution of a differential equation

The equation $y = \int f(x) \, dx$ has many solution, each differing from the others ______________________. This means that the graphs of any two antiderivatives of f are __________________ __________________________________.

In many applications, you are given enough information to determine a __________________________________. To do this, you need only know the value of $y = F(x)$ for one value of x, called an ______________________________.

Example 4: Solve the differential equation $\frac{dC}{dx} = -0.2x + 40$, where $C(180) = 89.90$.

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Section 4.2 Area

Objective: In this lesson you learned how to evaluate a sum and approximate the area of a plane region.

What you should learn
How to use sigma notation to write and evaluate a sum

I. Sigma Notation (Pages 259–260)

The sum of n terms $a_1, a_2, a_3, \ldots, a_n$ is written as

$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$, where i is the _______________

_________________, a_i is the ___________________________,

and n and 1 are the ____________________________________

___________________.

Complete the following properties of summation which are derived using the associative and commutative properties of addition and the distributive property of addition over multiplication.

$$\sum_{i=1}^{n} ka_i = \rule{4cm}{0.4pt}$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \rule{4cm}{0.4pt}$$

Now complete the following summation formulas.

1. $\sum_{i=1}^{n} c =$ ______________________

2. $\sum_{i=1}^{n} i =$ ______________________

3. $\sum_{i=1}^{n} i^2 =$ ______________________

4. $\sum_{i=1}^{n} i^3 =$ ______________________

II. Area (Page 261)

> ***What you should learn***
> How to understand the concept of area

In your own words, explain the exhaustion method that the ancient Greeks used to determine formulas for the areas of general regions.

III. Area of a Plane Region (Page 262)

> ***What you should learn***
> How to approximate the area of a plane region

Describe how to approximate the area of a plane region.

IV. Upper and Lower Sums (Pages 263–267)

> ***What you should learn***
> How to find the area of a plane region using limits

Consider a plane region bounded above by the graph of a nonnegative, continuous function $y = f(x)$. The region is bounded below by the ________________, and the left and right boundaries of the region are the vertical lines $x = a$ and $x = b$. To approximate the area of the region, begin by _____________ ____, each of width ___________________________. Because *f* is continuous, the Extreme Value Theorem guarantees the existence of a ____ in each subinterval. The value $f(m_i)$ is _____________________

________________________________ and the value of $f(M_i)$

is ____.

An **inscribed rectangle** __________________ the ith subregion and a **circumscribed rectangle** _______________________ the ith subregion. The height of the ith inscribed rectangle is _____________ and the height of the ith circumscribed rectangle is _______________ . For each i, the area of the inscribed rectangle is ____________________________ the area of the circumscribed rectangle. The sum of the areas of the inscribed rectangles is called ____________________, and the sum of the areas of the circumscribed rectangles is called _________________________.

$$\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$$

$$\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$$

The actual area of the region lies between __________________ ___________________________________.

Let f be continuous and nonnegative on the interval $[a, b]$. The limits as $n \to \infty$ of both the lower and upper sums exist and are ____________________________. That is,

$$\begin{aligned} \lim_{n\to\infty} s(n) &= \lim_{n\to\infty} \sum_{i=1}^{n} f(m_i)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} f(M_i)\Delta x \\ &= \lim_{n\to\infty} S(n) \end{aligned}$$

where $\Delta x = (b-a)/n$ and $f(m_i)$ and $f(M_i)$ are the minimum and maximum values of f on the subinterval.

Definition of the Area of a Region in the Plane

Let f be continuous and nonnegative on the interval $[a, b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \underline{\hspace{4cm}}, \quad x_{i-1} \le c_i \le x_i$$

where $\Delta x = (b - a) / n$.

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Section 4.3 Riemann Sums and Definite Integrals

Objective: In this lesson you learned how to evaluate a definite integral using a limit.

I. Riemann Sums (Pages 271–272)

> ***What you should learn***
> How to understand the definition of a Riemann sum

Let *f* be defined on the closed interval [*a*, *b*], and let Δ be a partition of [*a*, *b*] given by $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$, where Δx_i is the width of the *i*th subinterval. If c_i is any point in the *i*th subinterval $[x_{i-1}, x_i]$, then the sum $\sum_{i=1}^{n} f(c_i)\Delta x_i$, $x_{i-1} \le c_i \le x_i$, is called a ________________________ of *f* for the partition Δ.

The width of the largest subinterval of a partition Δ is the ________________ of the partition and is denoted by ___________________. If every subinterval is of equal width, the partition is ___________________ and the norm is denoted by $\|\Delta\| = \Delta x = \dfrac{b-a}{n}$. For a general partition, the norm is related to the number of subintervals of [*a*, *b*] in the following way: ________________________________. So the number of subintervals in a partition approaches infinity as _____________ ____.

II. Definite Integrals (Pages 273–275)

> ***What you should learn***
> How to evaluate a definite integral using limits

If f is defined on the closed interval $[a, b]$ and the limit of Riemann sums over partitions Δ

$$\lim_{\|\Delta\to 0\|}\sum_{i=1}^{n} f(c_i)\Delta x_i$$

exists, then f is said to be ______________________________ and the limit is denoted by $\lim_{\|\Delta\to 0\|}\sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$. This limit is called the ____. The number a is ____, and the number b is ____.

It is important to see that, although the notation is similar, definite integrals and indefinite integrals are different concepts: a definite integrals is ________________________, where an indefinite integral is ___________________________________.

If a function f is continuous on the closed interval $[a, b]$, then f is ________________________ on $[a, b]$.

Example 1: Evaluate the definite integral $\int_{-1}^{3}(2-x)\,dx$.

If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

Area = $\int$ ______________________

What you should learn
How to evaluate a definite integral using properties of definite integrals

III. Properties of Definite Integrals (Pages 276–278)

If f is defined at $x = a$, then we define $\int_a^a f(x)\,dx =$ __________.

If f is integrable on $[a, b]$, then we define $\int_b^a f(x)\,dx =$ _________________.

If f is integrable on the three closed intervals determined by a, b, and c, then

$\int_a^b f(x)\,dx =$ _______________________________.

If f and g are integrable on $[a, b]$ and k is a constant, then the function kf is integrable on $[a, b]$ and $\int_a^b kf(x)\,dx =$ __________________.

If f and g are integrable on $[a, b]$ and k is a constant, then the function $f \pm g$ is integrable on $[a, b]$ and $\int_a^b [f(x) \pm g(x)]\,dx =$ __________________________.

If f and g are continuous on the closed interval $[a, b]$ and $0 \le f(x) \le g(x)$ for $a \le x \le b$, the area of the region bounded by the graph of f and the x-axis (between a and b) must be _________________________. In addition, this area must be ____________________________________ the area of the region bounded by the graph of g and the x-axis between a and b.

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Section 4.4 The Fundamental Theorem of Calculus

Objective: In this lesson you learned how to evaluate a definite integral using the Fundamental Theorem of Calculus.

What you should learn
How to evaluate a definite integral using the Fundamental Theorem of Calculus

I. The Fundamental Theorem of Calculus (Pages 282–284)

Informally, the Fundamental Theorem of Calculus states that

____
____
____.

The **Fundamental Theorem of Calculus** states that if f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then $\int_a^b f(x)\,dx =$ _______________.

Guidelines for Using the Fundamental Theorem of Calculus

1. Provided you can find an antiderivative of f, you now have a way to evaluate a definite integral without ____________ ______________________________.

2. When applying the Fundamental Theorem, the following notation is convenient. $\int_a^b f(x)\,dx = F(x)\Big]_a^b =$ ____________________.

3. When using the Fundamental Theorem of Calculus, it is not necessary to include a ____________________________.

Example 1: Find $\int_{-2}^{2} (4 - x^2)\,dx$.

Example 2: Find the area of the region bounded by the x-axis and the graph of $f(x) = 2 + e^x$ for $0 \le x \le 6$.

II. The Mean Value Theorem for Integrals (Page 285)

What you should learn
How to understand and use the Mean Value Theorem for Integrals

The **Mean Value Theorem for Integrals** states that if f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that $\int_a^b f(x)\,dx =$

______________________________.

The Mean Value Theorem for Integrals does not specify how to determine c. It merely guarantees ________________________

____.

III. Average Value of a Function (Pages 286–287)

What you should learn
How to find the average value of a function over a closed interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

Average value of f on $[a, b]$ = ———— $\int$

Example 3: Find the average value of $f(x) = 0.24x^2 + 4$ on $[0, 10]$.

IV. The Second Fundamental Theorem of Calculus (Pages 288–290)

What you should learn
How to understand and use the Second Fundamental Theorem of Calculus

The Second Fundamental Theorem of Calculus states that if f is continuous on an open interval I containing a, then, for every x in the interval, $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] =$ ___________________.

V. Net Change Theorem (Pages 291–292)

What you should learn
How to understand and use the Net Change Theorem

The **Net Change Theorem** states that the definite integral of the rate of change of a quantity $F'(x)$ gives the total change, or **net change,** in that quantity of the interval $[a, b]$.

$$\int_a^b F'(x)\,dx = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Example 4: Liquid flows out of a tank at a rate of $40 - 2t$ gallons per minute, where $0 \le t \le 20$. Find the volume of liquid that flows out of the tank during the first 5 minutes.

Additional notes

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Section 4.5 Integration by Substitution

Objective: In this lesson you learned how to evaluate different types of definite and indefinite integrals using a variety of methods.

I. Pattern Recognition (Pages 297–299)

What you should learn How to use pattern recognition to find an indefinite integral

The role of substitution in integration is comparable to the role of ______________________________ in differentiation.

Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$\int f(g(x))g'(x)\,dx =$ _________________________. Letting

$u = g(x)$ gives $du = g'(x)\,dx$ and $\int f(u)\,du =$ _____________.

Example 1: Find $\int (2-3x^2)^3(-6x)\,dx$.

Many integrands contain the variable part of $g'(x)$ but are missing a constant multiple. In such cases, you can ___________ ____.

Example 2: Find $\int 6x^2(4x^3-1)^2\,dx$.

II. Change of Variables (Pages 300–301)

> ***What you should learn***
> How to use a change of variables to find an indefinite integral

With a formal **change of variables,** you completely ____. The change of variable technique uses the _______________ notation for the differential. That is, if $u = g(x)$, then $du =$ _______________________, and the integral takes the form $\int f(g(x))g'(x)\,dx = \int$ ___________________________.

Example 3: Find $\int 6x^2(4x^3 - 1)^2\,dx$ using change of variables.

Complete the list of guidelines for making a change of variables.

1.

2.

3.

4.

5.

6.

III. The General Power Rule for Integration (Page 302)

> ***What you should learn***
> How to use the General Power Rule for Integration to find an indefinite integral

One of the most common u-substitutions involves ____, and is given a special name—the ____. It states that if g is a differentiable function of x, then $\int$ ____.

Equivalently, if $u = g(x)$, then $\int$ ____________________

Example 4: Find $\int (4x^3 - x^2)(12x^2 - 2x)\, dx$.

IV. Change of Variables for Definite Integrals
(Pages 303–304)

> ***What you should learn***
> How to use a change of variables to evaluate a definite integral

When using u-substitution with a definite integral, it is often convenient to ____________________ ____________________ rather than to convert the antiderivative back to the variable x and evaluate the original limits.

Change of Variables for Definite Integrals

If the function $u = g(x)$ has a continuous derivative on the closed interval $[a, b]$ and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)\, dx = \int \qquad \underline{\qquad\qquad\qquad}.$$

Example 5: Find $\int_0^4 2x(2x^2 - 3)^2\, dx$.

V. Integration of Even and Odd Functions (Page 305)

> ***What you should learn***
> How to evaluate a definite integral involving an even or odd function

Occasionally, you can simplify the evaluation of a definite integral over an interval that is symmetric about the *y*-axis or about the origin by ____________________________________

__________________________________.

Let *f* be integrable on the closed interval $[-a, a]$.

If *f* is an ________________ function, then $\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx$.

If *f* is an ________________ function, then $\int_{-a}^{a} f(x)\, dx = 0$.

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Section 4.6 Numerical Integration

Objective: In this lesson you learned how to approximate a definite integral using the Trapezoidal Rule and Simpson's Rule.

I. The Trapezoidal Rule (Pages 311–313)

> ***What you should learn***
> How to approximate a definite integral using the Trapezoidal Rule

In your own words, describe how the Trapezoidal Rule approximates the area under the graph of a continuous function *f*.

The **Trapezoidal Rule** states that if *f* is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx \approx \rule{10cm}{0.4pt}.$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

The approximation of the area under a curve given by the Trapezoidal Rule tends to become ____________________ as *n* increases.

Example 1: Use the Trapezoidal Rule to approximate $\int_1^2 \frac{x}{3-x}\,dx$ using $n = 4$. Round your answer to three decimal places.

II. Simpson's Rule (Pages 313–314)

> ***What you should learn***
> How to approximate a definite integral using Simpson's Rule

In your own words, describe how Simpson's Rule approximates the area under the graph of a continuous function *f*.

For Simpson's Rule, what restriction is there on the value of n?

Simpson's Rule states that if f is continuous on $[a, b]$ and n is even, then

$\int_a^b f(x)\,dx \approx$ ____.

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

Example 2: Use Simpson's Rule to approximate $\int_1^2 \frac{x}{3-x}\,dx$ using $n = 4$. Round your answer to three decimal places.

III. Error Analysis (Page 315)

What you should learn
How to analyze errors in the Trapezoidal Rule and Simpson's Rule

For ____________________ Rule, the error E in approximating $\int_a^b f(x)\,dx$ is given as $|E| \le \frac{(b-a)^5}{180n^4}\left[\max\left|f^{(4)}(x)\right|\right],\ a \le x \le b$.

For ____________________ Rule, the error E in approximating $\int_a^b f(x)\,dx$ is given as $|E| \le \frac{(b-a)^3}{12n^2}\left[\max\left|f''(x)\right|\right],\ a \le x \le b$.

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Chapter 5 Logarithmic, Exponential, and Other Transcendental Functions

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Section 5.1 The Natural Logarithmic Function: Differentiation

Objective: In this lesson you learned the properties of the natural logarithmic function and how to find the derivative of the natural logarithmic function.

Important Vocabulary	Define each term or concept.
Natural logarithmic function	
e	

I. The Natural Logarithmic Function (Pages 324–326)

What you should learn How to develop and use properties of the natural logarithmic function

The domain of the natural logarithmic function is ____________ ____.

The value of ln x is positive for _______________ and negative for ________________________. Moreover, ln (1) = ________, because the upper and lower limits of integration are equal when __________________.

The natural logarithmic function has the following properties:

1.

2.

3.

If a and b are positive numbers and n is rational, then the following properties are true:

1. ln (1) = _________________.

2. ln (ab) = _______________________.

3. $\ln(a^n)$ = ________________________.

4. $\ln\left(\dfrac{a}{b}\right) =$ ______________________.

Example 1: Expand the logarithmic expression $\ln \dfrac{xy^4}{2}$.

II. The Number *e* (Page 327)

> ***What you should learn***
> How to understand the definition of the number *e*

The **base for the natural logarithm** is defined using the fact that the natural logarithmic function is continuous, is one-to-one, and has a range of (–∞, ∞). So, there must a unique real number *x* such that ___________________. This number is denoted by the letter _______________, which has the decimal approximation _____________________________.

III. The Derivative of the Natural Logarithmic Function (Pages 328–330)

> ***What you should learn***
> How to find derivatives of functions involving the natural logarithmic function

Let *u* be a differentiable function of *x*. Complete the following rules of differentiation for the natural logarithmic function:

$\dfrac{d}{dx}[\ln x] =$ ____________________, $x > 0$

$\dfrac{d}{dx}[\ln u] =$ _______________________________, $u > 0$

Example 2: Find the derivative of $f(x) = x^2 \ln x$.

If *u* is a differentiable function of *x* such that $u \neq 0$, then $\dfrac{d}{dx}[\ln|u|] =$ ___________________. In other words, functions of the form $y = \ln|u|$ can be differentiated as if ________________ ____________________________________.

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Section 5.2 The Natural Logarithmic Function: Integration

Objective: In this lesson you learned how to find the antiderivative of the natural logarithmic function.

I. Log Rule for Integration (Pages 334–337)

What you should learn How to use the Log Rule for Integration to integrate a rational function

Let u be a differentiable function of x.

$$\int \frac{1}{x}\,dx = \underline{\qquad\qquad\qquad\qquad}$$

$$\int \frac{u'}{u}\,dx = \int \frac{1}{u}\,du = \underline{\qquad\qquad\qquad\qquad}$$

Example 1: Find $\int \left(1 - \frac{1}{x}\right) dx$.

Example 2: Find $\int \frac{x^2}{3 - x^3}\,dx$.

Example 3: Find $\int \frac{x^2 - 4x + 1}{x}\,dx$.

If a rational function has a numerator of degree greater than

____,

division may reveal a form to which you can apply the Log Rule.

Guidelines for Integration

1.

2.

3.

4.

II. Integrals of Trigonometric Functions (Pages 338–339)

What you should learn
How to integrate trigonometric functions

$\int \sin u \, du =$ ____________________

$\int \cos u \, du =$ ____________________

$\int \tan u \, du =$ ____________________

$\int \cot u \, du =$ ____________________

$\int \sec u \, du =$ ____________________

$\int \csc u \, du =$ ____________________

Example 4: Find $\int \csc 5x \, dx$

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Section 5.3 Inverse Functions

Objective: In this lesson you learned how to determine whether a function has an inverse function.

Important Vocabulary Define each term or concept.

Inverse function

Horizontal Line Test

I. Inverse Functions (Pages 343–344)

> ***What you should learn***
> How to verify that one function is the inverse function of another function

For a function *f* that is represented by a set of ordered pairs, you can form the inverse function of *f* by ______________________ ____.

For a function *f* and its inverse f^{-1}, the domain of *f* is equal to ________________________, and the range of *f* is equal to ________________________.

State three important observations about inverse functions.

1.

2.

3.

To verify that two functions, *f* and *g*, are inverse functions of each other, . . .

Example 1: Verify that the functions $f(x) = 2x - 3$ and $g(x) = \dfrac{x+3}{2}$ are inverse functions of each other.

The graph of f^{-1} is a **reflection** of the graph of *f* in the line ____________________.

The Reflective Property of Inverse Functions states that the graph of *f* contains the point (a, b) if and only if ____.

II. Existence of an Inverse Function (Pages 345–347)

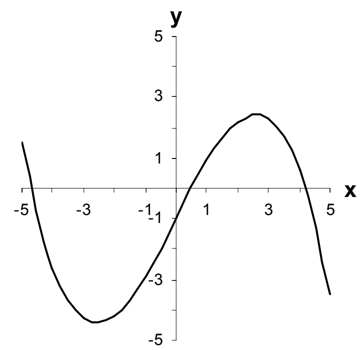
What you should learn
How to determine whether a function has an inverse function

State two reasons why the horizontal line test is valid.

1.

2.

Example 2: Does the graph of the function shown below have an inverse function? Explain.



Complete the following guidelines for finding an inverse function.

1)

2)

3)

4)

5)

Example 3: Find the inverse (if it exists) of $f(x) = 4x - 5$.

III. Derivative of an Inverse Function (Pages 347–348)

> ***What you should learn***
> How to find the derivative of an inverse function

Let *f* be a function whose domain is an interval *I*. If *f* has an inverse function, then the following statements are true.

1.

2.

3.

4.

Let *f* be a function that is differentiable on an interval *I*. If *f* has an inverse function *g*, then *g* is ____________________________

____.

Moreover, $g'(x) = \dfrac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0$.

This last theorem can be interpreted to mean that _____________

____.

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Section 5.4 Exponential Functions: Differentiation and Integration

Objective: In this lesson you learned about the properties of the natural exponential function and how to find the derivative and antiderivative of the natural exponential function.

I. The Natural Exponential Function (Pages 352–353)

What you should learn
How to develop properties of the natural exponential function

The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the ________________________________ ____________________ and is denoted by $f^{-1}(x) = e^x$. That is, $y = e^x$ if and only if __________________________.

Example 1: Solve $e^{x-2} - 7 = 59$ for x. Round to three decimal places.

Example 2: Solve $4 \ln 5x = 28$ for x. Round to three decimal places.

Complete each of the following operations with exponential functions.

1. $e^a e^b =$ ____________________.

2. $\dfrac{e^a}{e^b} =$ ____________________.

List four properties of the natural exponential function.

1.

2.

3.

4.

II. Derivatives of Exponential Functions (Pages 354–355)

> ***What you should learn***
> How to differentiate natural exponential functions

Let u be a differentiable function of x. Complete the following rules of differentiation for the natural exponential function:

$$\frac{d}{dx}\left[e^x\right] = \underline{\hspace{5cm}}.$$

$$\frac{d}{dx}\left[e^u\right] = \underline{\hspace{6cm}}.$$

Example 3: Find the derivative of $f(x) = x^2 e^x$.

III. Integrals of Exponential Functions (Pages 356–357)

> ***What you should learn***
> How to integrate natural exponential functions

Let u be a differentiable function of x.

$$\int e^x \, dx = \underline{\hspace{6cm}}$$

$$\int e^u \, du = \underline{\hspace{5.5cm}}$$

Example 4: Find $\int e^{2x} \, dx$.

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Section 5.5 Bases Other Than *e* and Applications

Objective: In this lesson you learned about the properties, derivatives, and antiderivatives of logarithmic and exponential functions that have bases other than *e*.

> ***What you should learn***
> How to define exponential functions that have bases other than *e*

I. Bases Other than *e* (Pages 362–363)

If a is a positive real number $(a \neq 1)$ and x is any real number, then the **exponential function to the base *a*** is denoted by a^x and is defined by ________________________. If a = 1, then $y = 1^x = 1$ is a ________________________________.

In a situation of radioactive decay, *half-life* is ______________
____
____.

If a is a positive real number $(a \neq 1)$ and x is any positive real number, then the **logarithmic function to the base *a*** is denoted by $\log_a x$ and is defined by $\log_a x =$ ______________________.

Complete the following properties of logarithmic functions to the base a.

1) $\log_a 1 =$ ____________

2) $\log_a (xy) =$ ______________________

3. $\log_a x^n =$ ______________________

4. $\log_a \dfrac{x}{y} =$ ______________________

State the Properties of Inverse Functions

The logarithmic function to the base 10 is called the _________

______________________________.

Example 1: (a) Solve $\log_8 x = \frac{1}{3}$ for x.

(b) Solve $5^x = 0.04$ for x.

II. Differentiation and Integration (Pages 364–365)

> ***What you should learn***
> How to differentiate and integrate exponential functions that have bases other than *e*

To differentiate exponential and logarithmic functions to other bases, you have three options:

1.

2.

3.

Let a be a positive real number $(a \neq 1)$ and let u be a differentiable function of x. Complete the following formulas for the derivatives for bases other than e.

$\frac{d}{dx}\left[a^x\right] =$ __________________________.

$\frac{d}{dx}\left[a^u\right] =$ ________________________________.

$\frac{d}{dx}[\log_a x] =$ _____________________________.

$\frac{d}{dx}[\log_a u] =$ ____________________________________.

Occasionally, an integrand involves an exponential function to a base other than e. When this occurs, there are two options:

(1) ____

__________________________ or (2) integrate directly using the

integration formula $\int a^x dx =$ ____________________________.

Let n be any real number and let u be a differentiable function of x. The Power Rule for Real Exponents gives.

$\frac{d}{dx}[x^n] =$ ____________________.

$\frac{d}{dx}[u^n] =$ ____________________________.

III. Applications of Exponential Functions (Pages 366–367)

What you should learn
How to use exponential functions to model compound interest and exponential growth

Complete the following limit statement:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty}\left(\frac{x+1}{x}\right)^x = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}.$$

Let P be the amount deposited, t the number of years, A the balance after t years, and r the annual interest rate (in decimal form), and n the number of compounding per year. Complete the following compound interest formulas:

Compounded n times per year: ________________________

Compounded continuously: _________________

Example 2: Find the amount in an account after 10 years if \$6000 is invested at an interest rate of 7%,
(a) compounded monthly.
(b) compounded continuously.

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Section 5.6 Inverse Trigonometric Functions: Differentiation

Objective: In this lesson you learned about the properties of inverse trigonometric functions and how to find derivatives of inverse trigonometric functions.

I. Inverse Trigonometric Functions (Pages 373–375)

What you should learn
How to develop properties of the six inverse trigonometric functions

None of the six basic trigonometric functions has ____________ ____________________________. This is true because all six trigonometric functions are ___________________________ __________________________. However, their domains can be redefined in such a way that they will have inverse functions on ____.

For each of the following definitions of inverse trigonometric functions, supply the required restricted domains and ranges.

	Domain	Range
$y = \arcsin x$ iff $\sin y = x$	______________	__________________
$y = \arccos x$ iff $\cos y = x$	______________	__________________
$y = \arctan x$ iff $\tan y = x$	______________	__________________
$y = \operatorname{arccot} x$ iff $\cot y = x$	______________	__________________
$y = \operatorname{arcsec} x$ iff $\sec y = x$	________________	___________________
$y = \operatorname{arccsc} x$ iff $\csc y = x$	________________	____________________

An alternative notation for the inverse sine function is ____________________.

Example 1: Evaluate the function: $\arcsin(-1)$.

Example 2: Evaluate the function: $\arccos\dfrac{1}{2}$.

Example 3: Evaluate the function: arcos (0.85).

State the Inverse Property for the Sine function.

State the Inverse Property for the Cosine function.

State the Inverse Property for the Tangent function.

II. Derivatives of Inverse Trigonometric Functions
(Pages 376–377)

> ***What you should learn***
> How to differentiate an inverse trigonometric function

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{\qquad}{\sqrt{\qquad}}$$

$$\frac{d}{dx}[\arccos u] = \frac{\qquad}{\sqrt{\qquad}}$$

$$\frac{d}{dx}[\arctan u] = \frac{\qquad}{\qquad}$$

$$\frac{d}{dx}[\operatorname{arc}\cot u] = \frac{\qquad}{\qquad}$$

$$\frac{d}{dx}[\operatorname{arc}\sec u] = \frac{\qquad}{\qquad\sqrt{\qquad}}$$

$$\frac{d}{dx}[\operatorname{arc}\csc u] = \frac{\qquad}{\qquad\sqrt{\qquad}}$$

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Section 5.7 Inverse Trigonometric Functions: Integration

Objective: In this lesson you learned how to find antiderivatives of inverse trigonometric functions.

I. Integrals Involving Inverse Trigonometric Functions (Pages 382–383)

> ***What you should learn***
> How to integrate functions whose antiderivatives involve inverse trigonometric functions

Let u be a differentiable function of x, and let $a > 0$.

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \underline{\hspace{6cm}}.$$

$$\int \frac{du}{a^2 + u^2} = \underline{\hspace{7cm}}.$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \underline{\hspace{8cm}}.$$

Example 1: $\displaystyle\int \frac{6x\,dx}{4 + 9x^4}$

II. Completing the Square (Pages 383–384)

> ***What you should learn***
> How to use the method of completing the square to integrate a function

Completing the square helps when ____________________
____.

Example 2: Complete the square for the polynomial: $x^2 + 6x + 3$.

Example 3: Complete the square for the polynomial: $2x^2 + 16x$.

III. Review of Basic Integration Rules (Pages 385–386)

What you should learn
How to review the basic integration rules involving elementary functions

Complete the following selected basic integration rules.

$\int \frac{u'}{u}\,dx = \int \frac{1}{u}\,du =$ ______________________

$\int du =$ __________________________

$\int \cot u\,du =$ ______________________________________

$\int \frac{du}{a^2 + u^2} =$ ________________________________

$\int \cos u\,du =$ ____________________________

$\int \sec^2 u\,du =$ ____________________________

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Section 5.8 Hyperbolic Functions

Objective: In this lesson you learned about the properties of hyperbolic functions and how to find derivatives and antiderivatives of hyperbolic functions.

What you should learn How to develop properties of hyperbolic functions

I. Hyperbolic Functions (Pages 390–392)

Complete the following definitions of the hyperbolic functions.

sinh x = ____________________.

cosh x = ____________________.

tanh x = _________________________.

csch x = _________________________.

sech x = _________________________.

coth x = _________________________.

Complete the following hyperbolic identities.

$\cosh^2 x - \sinh^2 x =$ __________________.

$\tanh^2 x + \operatorname{sech}^2 x =$ __________________.

$\coth^2 x - \operatorname{csch}^2 x =$ __________________.

$\dfrac{-1 + \cosh 2x}{2} =$ ______________________.

$\dfrac{1 + \cosh 2x}{2} =$ ______________________.

2 sinh x cosh x = ______________________.

$\cosh^2 x + \sinh^2 x =$ _____________________.

sinh $(x + y)$= _______________________________.

sinh $(x - y)$= _______________________________.

cosh $(x + y)$= _______________________________.

cosh $(x - y)$= _______________________________.

II. Differentiation and Integration of Hyperbolic Functions (Pages 392–394)

> ***What you should learn***
> How to differentiate and integrate hyperbolic functions

Let u be a differentiable function of x. Complete each of the following rules of differentiation and integration.

$\frac{d}{dx}[\sinh u] =$ ______________________________.

$\frac{d}{dx}[\cosh u] =$ ______________________________.

$\frac{d}{dx}[\tanh u] =$ ______________________________.

$\frac{d}{dx}[\coth u] =$ ______________________________.

$\frac{d}{dx}[\operatorname{sech} u] =$ ______________________________.

$\frac{d}{dx}[\operatorname{csch} u] =$ ______________________________.

$\int \cosh u\, du =$ ______________________________.

$\int \sinh u\, du =$ ______________________________.

$\int \operatorname{sech}^2 u\, du =$ ______________________________.

$\int \operatorname{csch}^2 u\, du =$ ______________________________.

$\int \operatorname{sech} u \tanh u\, du =$ ______________________________.

$\int \operatorname{csch} u \coth u\, du =$ ______________________________.

III. Inverse Hyperbolic Functions (Pages 394–396)

> ***What you should learn***
> How to develop properties of inverse hyperbolic functions

State the inverse hyperbolic function given by each of the following definitions and give the domain for each.

Domain

$\ln\left(x+\sqrt{x^2+1}\right) =$ ________________, ____________.

$\ln\left(x+\sqrt{x^2-1}\right) =$ ________________, ____________.

$\frac{1}{2}\ln\frac{1+x}{1-x} =$ ____________________, ____________.

$\frac{1}{2}\ln\frac{x+1}{x-1} =$ ____________________, ________________.

$\ln\frac{1+\sqrt{1-x^2}}{x} =$ __________________, ____________.

$\ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{|x|}\right) =$ ________________, ________________.

IV. Differentiation and Integration of Inverse Hyperbolic Functions (Pages 396–397)

> ***What you should learn***
> How to differentiate and integrate functions involving inverse hyperbolic functions

Let u be a differentiable function of x. Complete each of the following rules of differentiation and integration.

$\frac{d}{dx}[\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}] = \frac{u'}{\sqrt{u^2+1}}$

$\frac{d}{dx}[\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}] = \frac{u'}{\sqrt{u^2-1}}$

$\frac{d}{dx}[\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}] = \frac{u'}{1-u^2}$

$\frac{d}{dx}[\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}] = \frac{u'}{1-u^2}$

$\frac{d}{dx}[\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}] = \frac{-u'}{u\sqrt{1-u^2}}$

$$\frac{d}{dx}[\ \underline{\hspace{4cm}}\ ] = \frac{-u'}{|u|\sqrt{1+u^2}}$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \underline{\hspace{8cm}}.$$

$$\int \frac{du}{a^2 - u^2} = \underline{\hspace{11cm}}.$$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = \underline{\hspace{11cm}}.$$

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Chapter 6 Differential Equations

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Section 6.1 Slope Fields and Euler's Method

Objective: In this lesson you learned how to sketch a slope field of a differential equation, and find a particular solution.

I. General and Particular Solutions (Pages 406–407)

> ***What you should learn***
> How to use initial conditions to find particular solutions of differential equations

Recall that a **differential equation** in x and y is an equation that

____.

A function $y = f(x)$ is a **solution** of a differential equation if

____

____.

The **general solution** of a differential equation is ____________

____.

The order of a differential equation is determined by _________

____.

Geometrically, the general solution of a first-order differential equation represents a family of curves known as ____________ ____________________, one for each value assigned to the arbitrary constant. Particular solutions of a differential equation are obtained from ______________________________ that give the value of the dependent variable or one of its derivatives for a particular value of the independent variable.

Example 1: For the differential equation $y'' - y' - 2y = 0$, verify that $y = Ce^{2x}$ is a solution, and find the particular solution determined by the initial condition $y = 5$ when $x = 0$.

II. Slope Fields (Pages 408–409)

> ***What you should learn***
> How to use slope fields to approximate solutions of differential equations

Solving a differential equation analytically can be difficult or even impossible. However, there is a ___________________ _______________ you can use to learn a lot about the solution of a differential equation. Consider a differential equation of the form $y' = F(x, y)$ where $F(x, y)$ is some expression in x and y. At each point (x, y) in the xy-plane where F is defined, the differential equation determines the ______________________ of the solution at that point. If you draw a short line segment with slope $F(x, y)$ at selected points (x, y) in the domain of F, then these line segments form a ______________________, or a direction field for the differential equation $y' = F(x, y)$. Each line segment has ____________________________ as the solution curve through that point. A slope field shows ________________ ____ and can be helpful in getting a visual perspective of the directions of the solutions of a differential equation.

A solution curve of a differential equation $y' = F(x, y)$ is simply

____

____.

III. Euler's Method (Page 410)

What you should learn
How to use Euler's Method to approximate solutions of differential equations

Euler's Method is _____________________________________

____

____.

From the given information, you know that the graph of the solution passes through ____________________________ and has a slope of __________________ at this point. This gives a "starting point" for _____________________________________. From this starting point, you can proceed in the direction __________________________________. Using a small step h, move along the tangent line until you arrive at the point (x_1, y_1) where $x_1 =$ ___________ and $y_1 =$ ____________________ . If you think of (x_1, y_1) as a new starting point, you can repeat the process to obtain _______________________________.

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Section 6.2 Differential Equations: Growth and Decay

Objective: In this lesson you learned how to use an exponential function to model growth and decay.

I. Differential Equations (Page 415)

> ***What you should learn***
> How to use separation of variables to solve a simple differential equation

The separation of variables strategy is to ____________________
____
____.

Example 1: Find the general solution of $\frac{dy}{dx} = \frac{3x^2 - 1}{2y + 5}$.

II. Growth and Decay Models (Pages 416–419)

> ***What you should learn***
> How to use exponential functions to model growth and decay in applied problems

In many applications, the rate of change of a variable y is ____________________ to the value of y. If y is a function of time t, the proportion can be written as ____________________.

The **Exponential Growth and Decay Model** states that if y is a differentiable function of t such that $y > 0$ and $y' = ky$, for some constant k, then __________________ where C is the ____________________, and k is the ______________________________.
Exponential growth occurs when _____________ and **exponential decay** occurs when ______________.

Example 2: The rate of change of y is proportional to y. When $t = 0$, $y = 5$. When $t = 4$, $y = 10$. What is the value of y when $t = 2$?

In a situation of radioactive decay, **half-life** is ________________

____

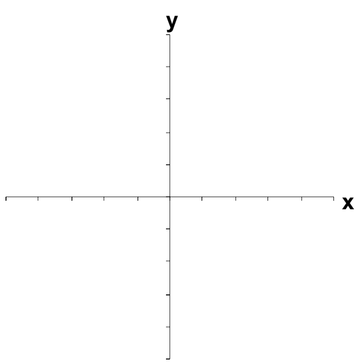
____.

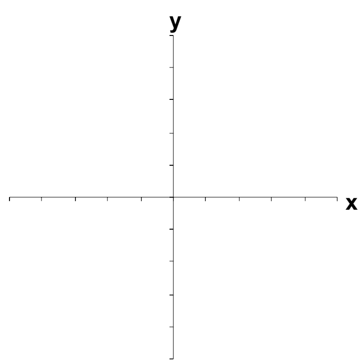
Newton's Law of Cooling states that ________________________

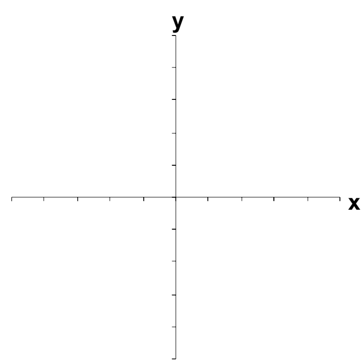
____

____

____.







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Section 6.3 Separation of Variables and the Logistic Equation

Objective: In this lesson you learned how to use separation of variables to solve a differential equation.

I. Separation of Variables (Pages 423–424)

> ***What you should learn***
> How to recognize and solve differential equations that can be solved by separation of variables

Consider a differential equation that can be written in the form $M(x)+N(y)\dfrac{dy}{dx}=0$, where M is a continuous function of x alone and N is a continuous function of y alone. Such equations are said to be ______________________, and the solution procedure is called ____________________________________. For this type of equation, all x terms can be ________________ ______________, all y terms can be ______________________ ______________, and a solution can be obtained by integration.

Give an example of a separable differential equation.

Example 1: Solve the differential equation $2yy' = e^x$ subject to the initial condition $y = 3$ when $x = 0$.

II. Homogeneous Differential Equations (Pages 425–426)

> ***What you should learn***
> How to recognize and solve homogeneous differential equations

Some differential equations that are not separable in x and y can be made separable by ______________________________. This is true for differential equations of the form $y' = f(x, y)$ where f is a ____. The function given by $f(x, y)$ is **homogeneous of degree *n*** if ___________________________________, where n is an integer.

A **homogeneous differential equation** is an equation of the form ______________________________, where M and N are homogenous functions of the same degree.

Example 2: State whether the function $f(x, y) = 6xy^3 + 4x^4 - x^2y^2$ is homogeneous. If so, what is its degree?

If $M(x, y)dx + N(x, y)dy = 0$ is homogeneous, then it can be transformed into a differential equation whose variables are separable by the substitution ________________, where v is a differentiable function of x.

III. Applications (Pages 427–428)

What you should learn
How to use differential equations to model and solve applied problems

Example 3: A new legal requirement is being publicized through a public awareness campaign to a population of 1 million citizens. The rate at which the population hears about the requirement is assumed to be proportional to the number of people who are not yet aware of the requirement. By the end of 1 year, half of the population has heard of the requirement. How many will have heard of it by the end of 2 years?

A common problem in electrostatics, thermodynamics, and hydrodynamics involves finding a family of curves, each of which is ____________________ to all members of a given family of curves. If one family of curves intersects another family of curves at right angles, then the two families are said to be ______________________________, and each curve in one of the families is called an ________________________ of the other family.

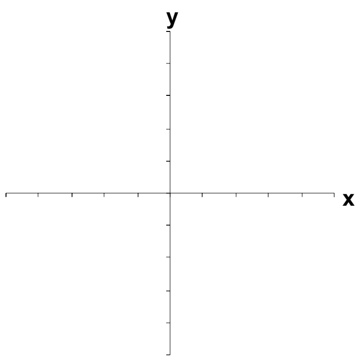
IV. Logistic Differential Equation (Pages 429–430)

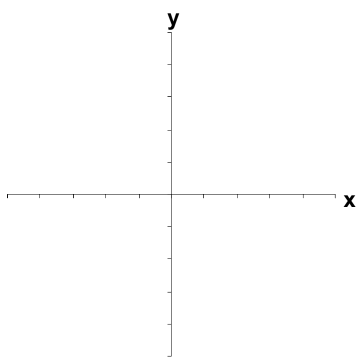
> ***What you should learn***
> How to solve and analyze logistic differential equations

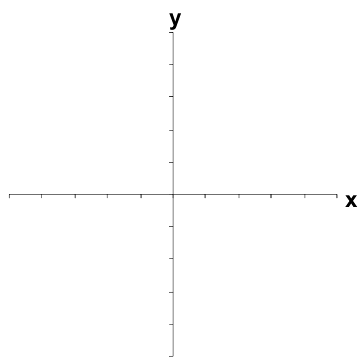
Exponential growth is unlimited, but when describing a population, there often exists some upper limit L past which growth cannot occur. This upper limit L is called the _________ ______________, which is the maximum population $y(t)$ that can be sustained or supported as time t increases. A model that is often used for this type of growth is the ___________________ _________________________ $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$, where k and L are positive constants. A population that satisfies this equation does not grow without bound, but approaches ______________ ________________ as t increases.

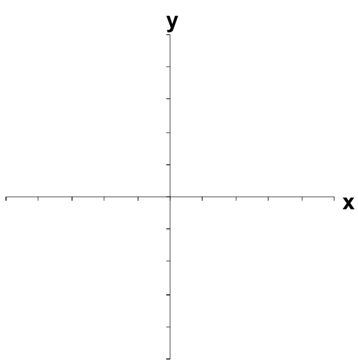
The general solution of the logistic differential equation is of the form $y =$ ______________ .

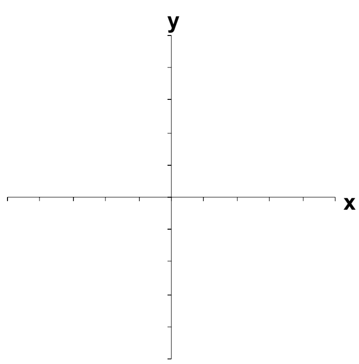
Additional notes

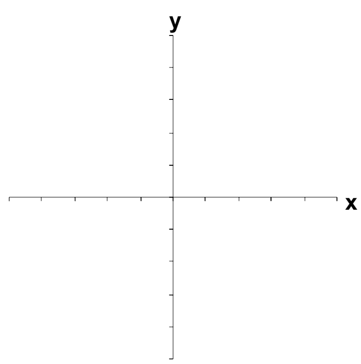












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Section 6.4 First-Order Linear Differential Equations

Objective: In this lesson you learned how to solve a first-order linear differential equation and a Bernoulli differential equation.

I. First-Order Linear Differential Equations (Pages 434–436)

What you should learn
How to solve a first-order linear differential equation

A **first-order linear differential equation** is an equation of the form ______________________________, where P and Q are continuous functions of x. An equation that is written in this form is said to be in ___________________________.

To solve a linear differential equation, ____________________ ____.

Then integrate $P(x)$ and form the expression $u(x) = e^{\int P(x)\,dx}$, which is called a(n) __________________________. The general solution of the equation is $y =$ ____________________.

Example 1: Write $e^x y' = 5 - (2 + e^x)y$ in standard form.

Example 2: Find the general solution of $y' - 3y = e^{6x}$.

II. Applications (Pages 436–438)

What you should learn
How to use linear differential equations to solve applied problems

Give examples of types of problems that can be described in terms of a first-order linear differential equation.

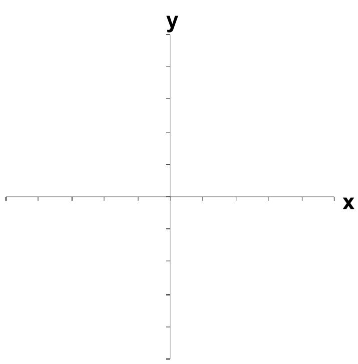
III. Bernoulli Equation (Pages 438–440)

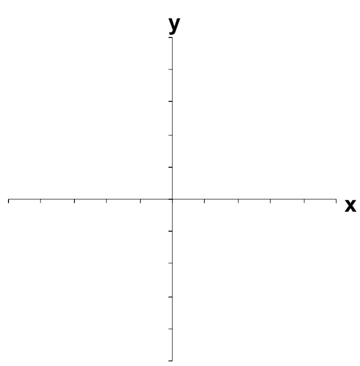
> ***What you should learn***
> How to solve a Bernoulli differential equation

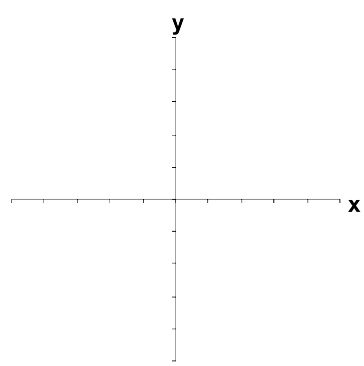
A well-known nonlinear equation, $y' + P(x)y = Q(x)y^n$, that reduces to a linear one with an appropriate substitution is ____.

State the general solution of the Bernoulli equation.

Additional notes







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Chapter 7 Applications of Integration

Section 7.1 Area of a Region Between Two Curves

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Objective: In this lesson you learned how to use a definite integral to find the area of a region bounded by two curves.

I. Area of a Region Between Two Curves (Pages 448–449)

> ***What you should learn***
> How to find the area of a region between two curves using integration

If *f* and *g* are continuous on [*a*, *b*] and $g(x) \leq f(x)$ for all *x* in [*a*, *b*], then the area of the region bounded by the graphs of *f* and *g* and the vertical lines *x* = *a* and *x* = *b* is

$A = \int_a^b$ ______________________.

Example 1: Find the area of the region bounded by the graphs of $y = 6 + 3x - x^2$, $y = 2x - 9$, $x = -2$, and $x = 2$.

II. Area of a Region Between Intersecting Curves (Pages 450–452)

> ***What you should learn***
> How to find the area of a region between intersecting curves using integration

A more common problem involves the area of a region bounded by two intersecting graphs, where the values of *a* and *b* must be ________________________.

Example 2: Find the area of the region bounded by the graphs of $y = x^2 - 5$ and $y = 1 - x$.

If two curves intersect at more than two points. Then to find the area of the region between the graphs, you must ____________

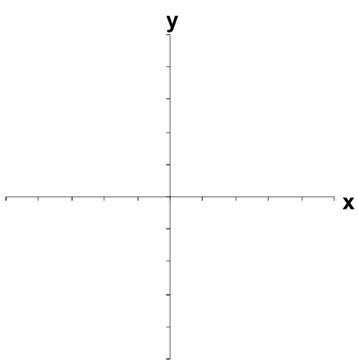
____

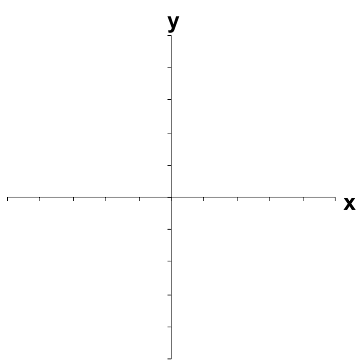
____.

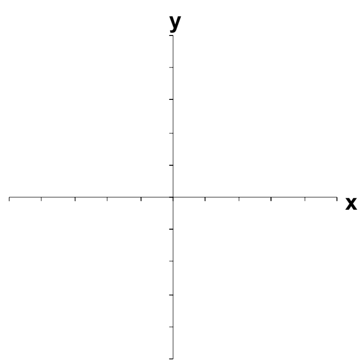
III. Integration as an Accumulation Process (Page 453)

> ***What you should learn***
> How to describe integration as an accumulation process

In this section, the integration formula for the area between two curves was developed by using a ________________________ as the representative element. For each new application in the remaining sections of this chapter, an appropriate representative element will be constructed using ________________________ ________________________. Each integration formula will then be obtained by ____________________________________ these representative elements.







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Section 7.2 Volume: The Disk Method

Objective: In this lesson you learned how to find the volume of a solid of revolution by the disk and washer methods.

I. The Disk Method (Pages 458–460)

> ***What you should learn***
> How to find the volume of a solid of revolution using the disk method

A **solid of revolution** is formed by ________________________
________________________________. The line is called _______
_________________________________. The simplest such solid
is ____,
which is formed by revolving a rectangle about an axis adjacent to one side of the rectangle.

To find the volume of a solid of revolution with the **Disk Method,** use one of the following formulas.

<u>Horizontal axis of revolution:</u>

Volume = ________ $\int$ _____________________.

<u>Vertical axis of revolution:</u>

Volume = ________ $\int$ _____________________.

The simplest application of the disk method involves a plane region bounded by ______________________________________.
If the axis of revolution is the x-axis, the radius $R(x)$ is simply
________________.

Example 1: Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = 0.5x^2 + 4$ and the x-axis, between $x = 0$ and $x = 3$, about the x-axis.

II. The Washer Method (Pages 461–463)

> ***What you should learn***
> How to find the volume of a solid of revolution using the washer method

The Washer Method is used to find the volume of a solid of revolution that has ________________________________.

Consider a region bounded by an outer radius $R(x)$ and an inner radius $r(x)$. The **Washer Method** states that if this region is revolved about its axis of revolution, the volume of the resulting solid is given by

Volume = ________ $\int$ ________________________________.

Note that the integral involving the inner radius represents ______ ______________________________ and is ___________________ ______________ the integral involving the outer radius.

Example 2: Find the volume of the solid formed by revolving the region bounded by the graphs of $f(x) = -x^2 + 5x + 3$ and $g(x) = -x + 8$ about the x-axis.

III. Solids with Known Cross Sections (Pages 463–464)

> ***What you should learn***
> How to find the volume of a solid with a known cross section

With the disk method, you can find the volume of a solid having a circular cross section whose area is $A = \pi R^2$. This method can be generalized to solids of any shape, as long as you know _____ ____.

For cross sections of area $A(x)$ taken perpendicular to the x-axis,

Volume = $\int$ ____________________.

For cross sections of area $A(y)$ taken perpendicular to the y-axis,

Volume = $\int$ ____________________.

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Section 7.3 Volume: The Shell Method

Objective: In this lesson you learned how to find the volume of a solid of revolution by the shell method.

I. The Shell Method (Pages 469–471)

What you should learn
How to find the volume of a solid of revolution using the shell method

To find the volume of a solid of revolution with the **Shell Method,** use one of the following formulas.

Horizontal axis of revolution:

Volume = ________ $\int$ ______________________.

Vertical axis of revolution:

Volume = ________ $\int$ ______________________.

Example 1: Using the shell method, find the volume of the solid formed by revolving the region bounded by the graph of $y = 3 + 2x$ and the x-axis, between $x = 1$ and $x = 4$, about the y-axis.

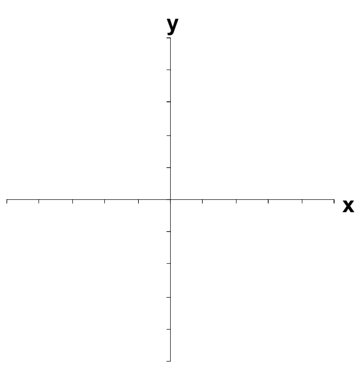
II. Comparison of Disk and Shell Methods (Pages 471–473)

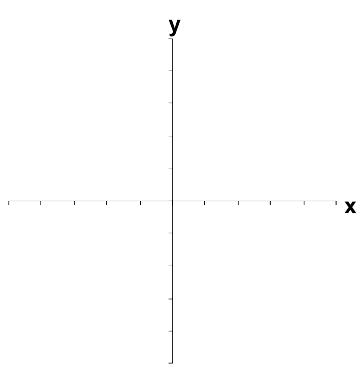
What you should learn
How to compare the uses of the disk method and the shell method

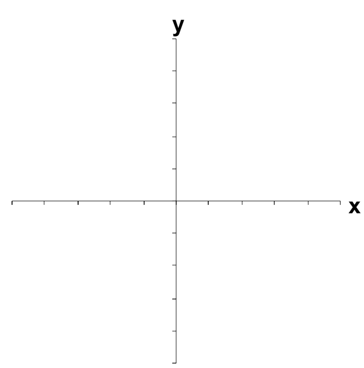
For the disk method, the representative rectangle is always ______________________________ to the axis of revolution.

For the shell method, the representative rectangle is always _________________________ to the axis of revolution.

Additional notes







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Section 7.4 Arc Length and Surfaces of Revolution

Objective: In this lesson you learned how to find the length of a curve and the surface area of a surface of revolution.

I. Arc Length (Pages 478–481)

What you should learn
How to find the arc length of a smooth curve

A **rectifiable** curve is ________________________________
_____________________. A sufficient condition for the graph of a function f to be rectifiable between $(a, f(a))$ and $(b, f(b))$ is that ____. Such a function is continuously differentiable on $[a, b]$, and its graph on the interval $[a, b]$ is a ___________________________.

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The arc length of f between a and b is

$$s = \int \sqrt{\qquad\qquad}$$

Similarly, for a smooth curve given by $x = g(y)$, the arc length of g between c and d is

$$s = \int \sqrt{\qquad\qquad}$$

Example 1: Find the arc length of the graph of $y = 2x^3 - x^2 + 5x - 1$ on the interval [0, 4].

II. Area of a Surface of Revolution (Pages 482–484)

What you should learn
How to find the area of a surface of revolution

If the graph of a continuous function is revolved about a line, the resulting surface is a ________________________________.

Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

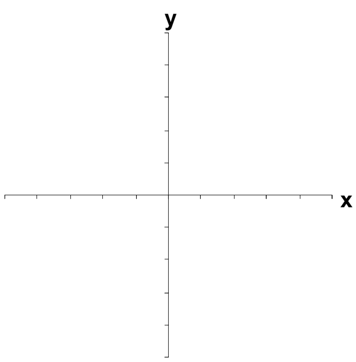
$$s = \quad \int \quad \sqrt{\qquad\qquad}$$

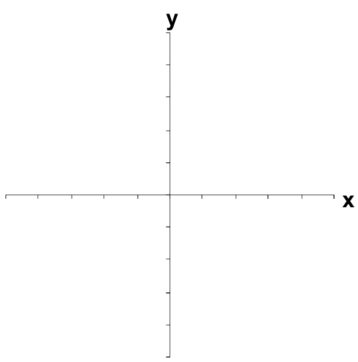
where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is

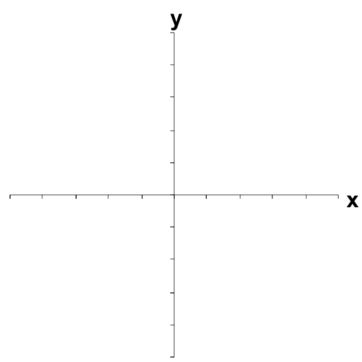
$$s = \quad \int \quad \sqrt{\qquad\qquad}$$

where $r(y)$ is the distance between the graph of g and the axis of revolution.

Example 2: Find the area of the surface formed by revolving the graph of $f(x) = 2x^2$ on the interval [2, 4] about the x-axis.







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Section 7.5 Work

Objective: In this lesson you learned how to find the work done by a constant force and by a variable force.

I. Work Done by a Constant Force (Page 489)

> ***What you should learn***
> How to find the work done by a constant force

Work is done by a force when ______________________ ______________________. If an object is moved a distance *D* in the direction of an applied constant force *F*, then the work *W* done by the force is defined as ______________________.

Give two examples of forces.

A **force** can be thought of as ______________________; a force changes the ______________________ of a body.

In the U.S. measurement system, work is typically expressed in ______________________.

In the centimeter-gram-second (C-G-S) system, the basic unit of force is the ______________—the force required to produce an acceleration of 1 centimeter per second per second on a mass of 1 gram. In this system, work is typically expressed in _______ ______________________ or ______________________.

Example 1: Find the work done in lifting a 100-pound barrel 10 feet in the air.

II. Work Done by a Variable Force (Pages 490–494)

> ***What you should learn***
> How to find the work done by a variable force

If a variable force is applied to an object, calculus is needed to determine the work done, because ______________________ ______________________.

Definition of Work Done by a Variable Force
If an object is moved along a straight line by a continuously varying force $F(x)$, then the **work** W done by the force as the object is moved from $x = a$ to $x = b$ is

$$W = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \Delta W_i$$
$$= \int \underline{\hspace{5cm}}$$

Hooke's Law states that the force F required to compress or stretch a spring (within its elastic limits) is proportional to the distance d that the spring is compressed or stretched from its original length. That is, ________________________ where the constant of proportionality k (the spring constant) depends on the specific nature of the spring.

Newton's Law of Universal Gravitation states that the force F of attraction between two particles of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance d between the two particles. That is, ______________________________.

Coulomb's Law states that the force between two charges q_1 and q_2 in a vacuum is proportional to the product of the charges and inversely proportional to the square of the distance d between the two charges. That is, _______________________ .

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Section 7.6 Moments, Centers of Mass, and Centroids

Objective: In this lesson you learned how to find centers of mass and centroids.

I. Mass (Page 498)

> ***What you should learn***
> How to understand the definition of mass

Mass is ____

____

____.

Force and mass are related by the equations ________________

____________________________.

II. Center of Mass in a One-Dimensional System (Pages 499–500)

> ***What you should learn***
> How to find the center of mass in a one-dimensional system

Consider an idealized situation in which a mass m is concentrated at a point. If x is the distance between this point mass and another point P, the **moment of *m* about the point *P*** is _________________________ and x is the length of the ______________________.

Now imagine a coordinate line on which the origin corresponds to the fulcrum. Suppose several point masses are located on the x-axis. The measure of the tendency of this system to rotate about the origin is the ___________________________________, and it is defined as ___________________________________ . That is M_0 = ________________________________ . If M_0 is 0, the system is said to be __________________________.

For a system that is not in equilibrium, the **center of mass** is defined as ____

____.

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $x_1, x_2, \ldots, x_n$.

The center of mass is $\bar{x}$ = _________________ , where m =

_________________________ is the total mass of the system.

III. Center of Mass in a Two-Dimensional System (Page 501)

> ***What you should learn***
> How to find the center of mass in a two-dimensional system

Let the point masses $m_1, m_2, \ldots, m_n$ be located at (x_1, y_1), (x_2, y_2), . . . , (x_n, y_n). The **moment about the *y*-axis** is M_y = ______________________________ . The **moment about the *x*-axis** is M_x = ______________________________ .

The **center of mass** $(\bar{x}, \bar{y})$, or **center of gravity,** is

$\bar{x}$ = ________________ , and $\bar{y}$ = ________________ ,

where m = ______________________ is the **total mass** of the system.

IV. Center of Mass of a Planar Lamina (Pages 502–504)

> ***What you should learn***
> How to find the center of mass of a planar lamina

A **planar lamina** is ________________________________ ________________________. **Density** is ______________ ____________________________________; however, for planar laminas, density is considered to be ________________ _____________________. Density is denoted by _________.

Let *f* and *g* be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$, and consider the planar lamina of uniform density ρ bounded by the graphs of $y = f(x)$, $y = g(x)$, and $a \leq x \leq b$.

The moment about the *x*-axis is given by

$$M_x = \qquad \int \left[\qquad\qquad \right] [\qquad\qquad]\, dx$$

The moment about the *y*-axis is given by

$$M_y = \qquad \int$$

The **center of mass** $(\bar{x}, \bar{y})$ is given by $\bar{x}$ = ______________ ,

and $\bar{y}$ = ______________ , where $m = \int$ ____________________ .

V. Theorem of Pappus (Page 505)

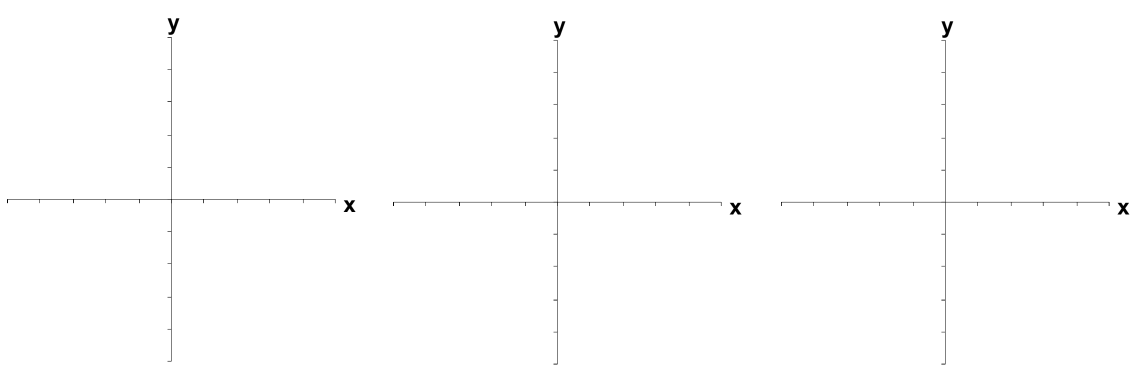
> ***What you should learn***
> How to use the Theorem of Pappus to find the volume of a solid of revolution

State the Theorem of Pappus.

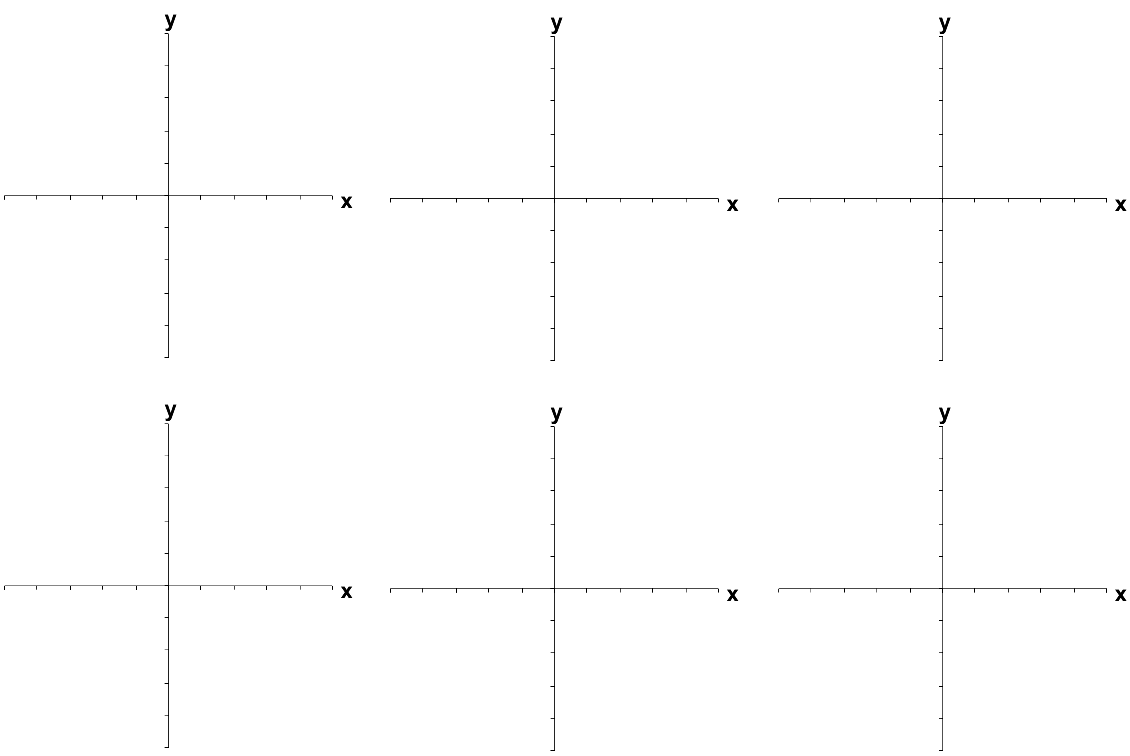
The Theorem of Pappus can be used to find the volume of a torus, which is ____

____

____.



Additional notes



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Section 7.7 Fluid Pressure and Fluid Force

Objective: In this lesson you learned how to find fluid pressure and fluid force.

What you should learn
How to find fluid pressure and fluid force

I. Fluid Pressure and Fluid Force (Pages 509–512)

Pressure is defined as ________________________________ ____. The fluid pressure on an object at a depth h in a liquid is ______________ ______________, where w is the weight-density of the liquid per unit of volume.

When calculating fluid pressure, you can use an important physical law called **Pascal's Principle,** which states that ______ ____ ____.

The fluid force on a submerged *horizontal* surface of area A is

Fluid force = F = ________________________________.

Example 1: Find the fluid force on a horizontal metal disk of diameter 3 feet that is submerged in 12 feet of seawater ($w = 64.0$).

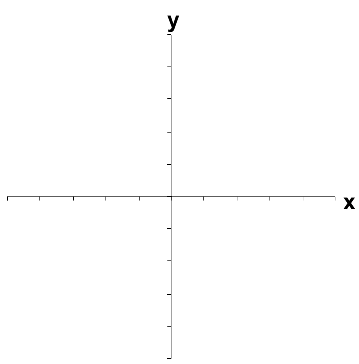
The **force *F* exerted by a fluid** of constant weight-density w (per unit of volume) against a submerged vertical plane region from $y = c$ to $y = d$ is

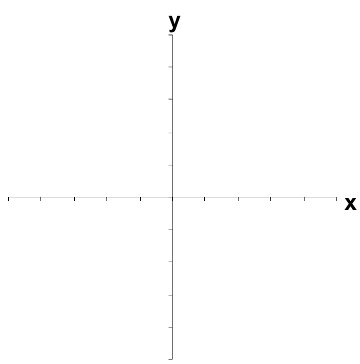
$$F = w \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} h(y_i)L(y_i)\Delta y$$

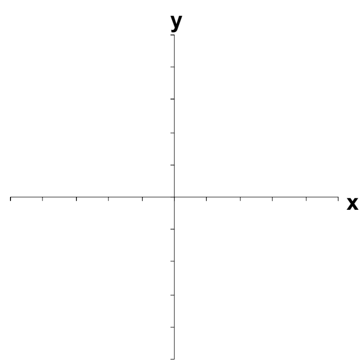
$$= \_\_\_ \int \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

where $h(y)$ is the depth of the fluid at y an $L(y)$ is the horizontal length of the region at y.

Additional notes







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Chapter 8 Integration Techniques, L'Hôpital's Rule, and Improper Integrals

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Section 8.1 Basic Integration Rules

Objective: In this lesson you learned how to fit an integrand to one of the basic integration rules.

I. Fitting Integrands to Basic Rules (Pages 520–523)

What you should learn
How to apply procedures for fitting an integrand to one of the basic integration rules

In this chapter, you study several integration techniques that greatly expand the set of integrals to which the basic integration rules can be applied. A major step in solving any integration problem is ____
________________________________.

Basic Integration Rules

$\int kf(u)\,du =$ ______________________________

$\int [f(u) \pm g(u)]\,du =$ ________________________________

$\int du =$ ________________________________

________________________ $= \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$

$\int \dfrac{du}{u} =$ _________________________

$\int e^u\,du =$ _______________________

$\int a^u\,du =$ ______________________________

$\int \sin u\,du =$ __________________________

$\int \cos u\,du =$ ________________________

$\int \tan u \, du =$ ______________________________

$\int \cot u \, du =$ ______________________________

$\int \sec u \, du =$ ______________________________

$\int \csc u \, du =$ ______________________________

$\int \sec^2 u \, du =$ ______________________

$\int \csc^2 u \, du =$ ______________________

$\int \sec u \tan u \, du =$ ______________________

$\int \csc u \cot u \, du =$ ______________________

$\int \frac{du}{\sqrt{a^2 - u^2}} =$ ______________________

$\int \frac{du}{a^2 + u^2} =$ ______________________

$\int \frac{du}{u\sqrt{u^2 - a^2}} =$ ______________________

Name seven procedures for fitting integrands to basic rules. Give an example of each procedure.

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Section 8.2 Integration by Parts

Objective: In this lesson you learned how to find an antiderivative using integration by parts.

I. Integration by Parts (Pages 527–532)

What you should learn
How to find an antiderivative using integration by parts

The integration technique of **integration by parts** is particularly useful for ____

____.

If u and v are functions of x and have continuous derivatives, then the technique of integration by parts states that

$\int u\,dv =$ ______________________.

List two guidelines for integration by parts:

1.

2.

Example 1: For the indefinite integral $\int x^2 e^{2x}\,dx$, explain which factor you would choose to be dv and which you would choose as u.

Summary of Common Uses of Integration by Parts
List the choices for u and dv in these common integration situations.

1. $\int x^n e^{ax}\,dx$, $\int x^n \sin ax\,dx$, or $\int x^n \cos ax\,dx$

____.

2. $\int x^n \ln x \, dx$, $\int x^n \arcsin ax \, dx$, or $\int x^n \arctan ax \, dx$

____.

3. $\int e^{ax} \sin bx \, dx$ or $\int e^{ax} \cos bx \, dx$

____.

II. Tabular Method (Page 532)

What you should learn
How to use a tabular method to perform integration by parts

In problems involving repeated applications of integration by parts, a tabular method can help organize the work. This method works well for integrals of the form $\int$ ____________________, $\int$ ____________________, and $\int$ ____________________.

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Section 8.3 Trigonometric Integrals

Objective: In this lesson you learned how to evaluate trigonometric integrals.

I. Integrals Involving Powers of Sine and Cosine
(Pages 536–538)

> ***What you should learn***
> How to solve trigonometric integrals involving powers of sine and cosine

In this section you studied techniques for evaluating integrals of the form $\int \sin^m x \cos^n x\, dx$ and $\int \sec^m x \tan^n x\, dx$ where either m or n is a positive integer. To find antiderivatives for these forms, ____.

To break up $\int \sin^m x \cos^n x\, dx$ into forms to which you can apply the Power Rule, use the following identities.

$\sin^2 x + \cos^2 x =$ ____________________

$\sin^2 x =$ ____________________

$\cos^2 x =$ ____________________

List three guidelines for evaluating integrals involving sine and cosine.

Wallis's Formulas state that if n is odd ($n \geq 3$), then

$$\int_0^{\pi/2} \cos^n x \, dx = \underline{\hspace{10cm}}$$

and that if n is even ($n \geq 2$), then

$$\int_0^{\pi/2} \cos^n x \, dx = \underline{\hspace{10cm}}$$

II. Integrals Involving Powers of Secant and Tangent (Pages 539–541)

What you should learn
How to solve trigonometric integrals involving powers of secant and tangent

List five guidelines for evaluating integrals involving secant and tangent of the form $\int \sec^m x \tan^n x \, dx$.

For integrals involving powers of cotangents and cosecants,

____

____.

Another strategy that can be useful when integrating trigonometric functions is ______________________________

____.

III. Integrals Involving Sine-Cosine Products with Different Angles (Page 541)

Complete each of the following product-to-sum identities.

$\sin mx \sin nx =$ ____

$\sin mx \cos nx =$ ____

$\cos mx \cos nx =$ ____

What you should learn
How to solve trigonometric integrals involving sine-cosine products with different angles

Additional notes

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Section 8.4 Trigonometric Substitution

Objective: In this lesson you learned how to use trigonometric substitution to evaluation an integral.

I. Trigonometric Substitution (Pages 545−549)

What you should learn
How to use trigonometric substitution to solve an integral

Now that you can evaluate integrals involving powers of trigonometric functions, you can use trigonometric substitution to evaluate integrals involving the radicals $\sqrt{a^2 - u^2}$, $\sqrt{a^2 + u^2}$, and $\sqrt{u^2 - a^2}$. The objective with trigonometric substitution is

____.

You do this with the ____

____.

Trigonometric substitution (a > 0):

1. For integrals involving $\sqrt{a^2 - u^2}$, let u = ______________.

   Then $\sqrt{a^2 - u^2}$ = ______________, where $-\pi/2 \le \theta \le \pi/2$.

2. For integrals involving $\sqrt{a^2 + u^2}$, let u = ______________.

   Then $\sqrt{a^2 + u^2}$ = ______________, where $-\pi/2 < \theta < \pi/2$.

3. For integrals involving $\sqrt{u^2 - a^2}$, let u = ______________.

   Then $\sqrt{u^2 - a^2}$ = ____________ if $u > a$, where $0 \le \theta < \pi/2$;

   or $\sqrt{u^2 - a^2}$ = ____________ if $u < -a$, where $\pi/2 < \theta \le \pi$.

Special Integration Formulas (a > 0)

$\int \sqrt{a^2 - u^2}\, du$ = ______________________________________

$\int \sqrt{u^2 - a^2}\, du$ = ______________________________________

$\int \sqrt{u^2 + a^2}\,du =$ ______________________________

II. Applications (Page 550)

Give two examples of applications of trigonometric substitution.

What you should learn How to use integrals to model and solve real-life applications

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Section 8.5 Partial Fractions

Objective: In this lesson you learned how to use partial fraction decomposition to integrate rational functions.

What you should learn
How to understand the concept of a partial fraction decomposition

I. Partial Fractions (Pages 554–555)

The **method of partial fractions** is a procedure for ____________ ____ ____.

Decomposition of *N*(*x*)/*D*(*x*) into Partial Fractions

1. **Divide if improper:** If $N(x)/D(x)$ is ____________________ ________________ (that is, if the degree of the numerator is greater than or equal to the degree of the denominator), divide ____ to obtain $\frac{N(x)}{D(x)} =$ ______________________________________, where the degree of $N_1(x)$ is less than the degree of $D(x)$. Then apply steps 2, 3, and 4 to the proper rational expression $N_1(x)/D(x)$.
2. **Factor denominator:** Completely factor the denominator into factors of the form ______________________________ where $ax^2 + bx + c$ is irreducible.
3. **Linear factors:** For each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions.

   ____

4. **Quadratic factors:** For each factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of n fractions.

   ____

II. Linear Factors (Pages 556–557)

> ***What you should learn***
> How to use partial fraction decomposition with linear factors to integrate rational functions

To find the **basic equation** of a partial fraction decomposition, ____ ____ __________

After finding the basic equation, ______________________________ ____ ____ ____.

Example 1: Write the form of the partial fraction decomposition for $\dfrac{x-4}{x^2-8x+12}$.

Example 2: Write the form of the partial fraction decomposition for $\dfrac{2x+1}{x^3-3x^2+x-3}$.

Example 3: Solve the basic equation $5x+3 = A(x-1)+B(x+3)$ for A and B.

III. Quadratic Factors (Pages 558–560)

> ***What you should learn***
> How to use partial fraction decomposition with quadratic factors to integrate rational functions

Guidelines for Solving the Basic Equation
List two guidelines for solving basic equations that involve linear factors.

List four guidelines for solving basic equations that involve quadratic factors.

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Section 8.6 Integration by Tables and Other Integration Techniques

Objective: In this lesson you learned how to evaluate an indefinite integral using a table of integrals and reduction formulas.

I. Integration by Tables (Pages 563−564)

> ***What you should learn***
> How to evaluate an indefinite integral using a table of integrals

Integration by tables is the procedure of integrating by means of ____.

Integration by tables requires ________________________ ____.

A computer algebra system consists, in part, of a database of integration tables. The primary difference between using a computer algebra system and using a table of integrals is ______ ____ ____ ____.

Example 1: Use the integration table in Appendix B to identify an integration formula that could be used to find $\int \frac{x}{3-x}\,dx$, and identify the substitutions you would use.

Example 2: Use the integration table in Appendix B to identify an integration formula that could be used to find $\int 3x^5 \ln x\,dx$, and identify the substitutions you would use.

II. Reduction Formulas (Page 565)

> ***What you should learn***
> How to evaluate an indefinite integral using reduction formulas

An integration table formula of the form

$\int f(x)\,dx = g(x) + \int h(x)\,dx$, in which the right side of the

formula contains an integral, is called a ____________________

_______________ because they ___________________________

____.

III. Rational Functions of Sine and Cosine (Page 566)

> ***What you should learn***
> How to evaluate an indefinite integral involving rational functions of sine and cosine

If you are unable to find an integral in the integration tables that involves a rational expression of sin x and cos x, try using the following special substitution to convert the trigonometric expression to a standard rational expression.

The substitution

$u =$ _________________________ $=$ _______________

yields

$\cos x =$ ______________________________,

$\sin x =$ ______________________________,

and $dx =$ ______________________________.

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Section 8.7 Indeterminate Forms and L'Hôpital's Rule

Objective: In this lesson you learned how to apply L'Hôpital's Rule to evaluate a limit.

I. Indeterminate Forms (Page 569)

> ***What you should learn***
> How to recognize limits that produce indeterminate forms

The forms 0/0 and ∞/∞ are called ___________________ because they ____

____.

Occasionally an indeterminate form may be evaluated by

____

______________________. However, not all indeterminate forms can be evaluated in this manner. This is often true when

____ are involved.

II. L'Hôpital's Rule (Pages 570–575)

> ***What you should learn***
> How to apply L'Hôpital's Rule to evaluate a limit

The **Extended Mean Value Theorem** states that if f and g are differentiable on an open interval (a, b) and continuous on $[a, b]$ such that $g'(x) \neq 0$ for any x in (a, b), then there exists a point c in (a, b) such that $\dfrac{f'(c)}{g'(c)} =$ ______________________________.

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself.

L'Hôpital's Rule states that if the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form 0/0, then

$\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to c}$ ___________________________, provided the

limit on the right exists (or is infinite). This result also applies if the limit of $f(x)/g(x)$ as x approaches c produces any one of the indeterminate forms ___________________________________.

This theorem states that under certain conditions the limit of the quotient $f(x)/g(x)$ is determined by ________________________

____.

Example 1: Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{2x^2 - 3x}$.

Example 2: Evaluate $\lim_{x \to 0} \frac{-3x^2}{\sqrt{x+4} - (x/4) - 2}$.

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Section 8.8 Improper Integrals

Objective: In this lesson you learned how to evaluate an improper integral.

I. Improper Integrals with Infinite Limits of Integration (Pages 580–583)

What you should learn
How to evaluate an improper integral that has an infinite limit of integration

List two properties that make an integral an **improper integral.**

1. ______________________________

2. ______________________________

If an integrand has an **infinite discontinuity,** then ____________ ______________________________.

Complete the following statements about improper integrals having infinite limits of integration.

1. If f is continuous on the interval $[a, \infty)$, then

$$\int_a^{\infty} f(x)\,dx = \underline{\qquad\qquad\qquad\qquad}$$

2. If f is continuous on the interval $(-\infty, b]$, then

$$\int_{-\infty}^{b} f(x)\,dx = \underline{\qquad\qquad\qquad\qquad}$$

3. If f is continuous on the interval $(-\infty, \infty)$, then

$$\int_{-\infty}^{\infty} f(x)\,dx = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

In the first two cases, if the limit exists, then the improper integral ____________________; otherwise, the improper integral ___________________. In the third case, the integral on the left will diverge if ____________________________ ______________________________.

II. Improper Integrals with Infinite Discontinuities
(Pages 583−586)

What you should learn
How to evaluate an improper integral that has an infinite discontinuity

Complete the following statements about improper integrals having infinite discontinuities at or between the limits of integration.

1. If f is continuous on the interval $[a, b)$ and has an infinite discontinuity at b, then
   $\int_a^b f(x)\,dx =$ ______________________________

2. If f is continuous on the interval $(a, b]$ and has an infinite discontinuity at a, then
   $\int_a^b f(x)\,dx =$ ______________________________

3. If f is continuous on the interval $[a, b]$, except for some c in (a, b) at which f has an infinite discontinuity, then
   $\int_a^b f(x)\,dx =$ ______________________________

In the first two cases, if the limit exists, then the improper integral ____________________; otherwise, the improper integral __________________. In the third case, the improper integral on the left diverges if ____________________ ____.

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Chapter 9 Infinite Series

Section 9.1 Sequences

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Objective: In this lesson you learned how to determine whether a sequence converges or diverges.

I. Sequences (Page 596)

What you should learn How to list the terms of a sequence

A **sequence** $\{a_n\}$ is a function whose domain is ______________ ________________________________. The numbers $a_1, a_2, a_3, \ldots,$ $a_n, \ldots$ are the ________________ of the sequence. The number a_n is the ________________________ of the sequence, and the entire sequence is denoted by ________________ .

Example 1: Find the first four terms of the sequence defined by $a_n = n^2 - 4$

II. Limit of a Sequence (Pages 597–600)

What you should learn How to determine whether a sequence converges or diverges

If a sequence **converges**, its terms ________________________ _______________.

Let L be a real number. The **limit** of a sequence $\{a_n\}$ is L, written as $\lim\limits_{n\to\infty} a_n = L$ if for each $\varepsilon > 0$, there exists $M > 0$ such that ____. If the limit L of a sequence exists, then the sequence _____________________. If the limit of a sequence does not exist, then the sequence _____________________________.

If a sequence $\{a_n\}$ agrees with a function f at every positive integer, and if $f(x)$ approaches a limit L as $x\to\infty$, the sequence must ____.

Example 2: Find the limit of each sequence (if it exists) as n approaches infinity.

a. $a_n = n^2 - 4$ b. $a_n = \dfrac{2n^2}{3n - n^2}$

Complete the following properties of limits of sequences. Let $\lim\limits_{n\to\infty} a_n = L$ and $\lim\limits_{n\to\infty} b_n = K$.

1. $\lim\limits_{n\to\infty}(a_n \pm b_n) =$ ____________________
2. $\lim\limits_{n\to\infty} ca_n =$ ____
3. $\lim\limits_{n\to\infty}(a_n b_n) =$ ____________________
4. $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} =$ ____________________________________

If n is a positive integer, then ***n* factorial** is defined as ____. As a special case, **zero factorial** is defined as 0! = ______________.

Another useful limit theorem that can be rewritten for sequences is the **Squeeze Theorem,** which states that if $\lim\limits_{n\to\infty} a_n = L = \lim\limits_{n\to\infty} b_n$ and there exists an integer N such that $a_n \le c_n \le b_n$ for all $n > N$, then $\lim\limits_{n\to\infty} c_n =$ ____________________.

For the sequence $\{a_n\}$, if $\lim\limits_{n\to\infty} |a_n| = 0$ then $\lim\limits_{n\to\infty} a_n =$ ____________.

III. Pattern Recognition for Sequences (Pages 600–601)

> ***What you should learn***
> How to write a formula for the nth term of sequence

Example 3: Determine an nth term for the sequence

$$0, \frac{1}{4}, -\frac{2}{9}, \frac{3}{16}, -\frac{4}{25}, \ldots$$

IV. Monotonic Sequences and Bounded Sequences
(Pages 602–603)

> ***What you should learn***
> How to use properties of monotonic sequences and bounded sequences

A sequence $\{a_n\}$ is **monotonic** if its terms are ________________ ________________________________ or if its terms are ____ .

A sequence $\{a_n\}$ is ____________________________ if there is a real number M such that $a_n \le M$ for all n. The number M is called ___________________________ of the sequence. A sequence $\{a_n\}$ is _________________________ if there is a real number N such that $N \le a_n$ for all n. The number N is called __________________________ of the sequence. A sequence $\{a_n\}$ is _________________________ if it is bounded above and bounded below.

If a sequence $\{a_n\}$ is _________________________________, then it converges.

Additional notes

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Section 9.2 Series and Convergence

Objective: In this lesson you learned how to determine whether an infinite series converges or diverges.

I. Infinite Series (Pages 608–610)

What you should learn
How to understand the definition of a convergent infinite series

If $\{a_n\}$ is an infinite sequence, then the infinite summation

$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ is called an _______________

_______________. The numbers a_1, a_2, a_3, and so on, are the _________________ of the series. The **sequence of partial sums** of the series is denoted by ___________________________________

____ .

If the sequence of partial sums $\{S_n\}$ converges to S, then the infinite series _____________________ to S. This limit is denoted by $\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} a_n = S$, and S is called the ___________

______________. If the limit of the sequence of partial sums $\{S_n\}$ does not exist, then the series ____________________.

A **telescoping series** is of the form $(b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + (b_4 - b_5) + \cdots$, where b_2 is cancelled ____

__________________________. Because the nth partial sum of this series is $S_n = b_1 - b_{n+1}$, it follows that a telescoping series will converge if and only if b_n ____. Moreover, if the series converges, its sum is _________________________.

II. Geometric Series (Pages 610–612)

What you should learn
How to use properties of infinite geometric series

If a is a nonzero real number, then the infinite series

$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots$ is called a ____________

_______________ with ratio r.

An infinite geometric series given by $\sum_{n=0}^{\infty} ar^n$ diverges if

______________________. If ______________________, then the

series converges to the sum $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.

Given the convergent infinite series $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$

and real number c,

$\sum_{n=1}^{\infty} ca_n =$ ______________________

$\sum_{n=1}^{\infty} (a_n + b_n) =$ ______________________

$\sum_{n=1}^{\infty} (a_n - b_n) =$ ______________________

III. *n*th-Term Test for Divergence (Pages 612–613)

> ***What you should learn***
> How to use the *n*th-Term Test for Divergence of an infinite series

The ***n*th Term Test for Divergence** states that if $\lim_{n\to\infty} a_n \neq 0$,

then the series $\sum_{n=1}^{\infty} a_n$ ______________________.

Example 1: Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2}{3n^2 - 1}$ diverges.

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Section 9.3 The Integral Test and *p*-Series

Objective: In this lesson you learned how to determine whether an infinite series converges or diverges.

Important Vocabulary	Define each term or concept.
General harmonic series	

I. The Integral Test (Pages 619–620)

> ***What you should learn***
> How to use the Integral Test to determine whether an indefinite series converges or diverges

The **Integral Test** states that if f is positive, continuous and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either ______________________________________.

Remember that the convergence or divergences of $\sum a_n$ is not affected by deleting ________________________________.
Similarly, if the conditions for the Integral Test are satisfied for all ____________________, you can simply use the integral $\int_N^{\infty} f(x)\,dx$ to test _____________________________________.

II. *p*-Series and Harmonic Series (Pages 621–622)

> ***What you should learn***
> How to use properties of *p*-series and harmonic series

Let p be a positive constant. An infinite series of the form

$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ is called a ____________________.

If $p = 1$, then the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$ is called the

______________________________.

The **Test for Convergence of a *p*-Series** states that the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$ diverges if _______________,

or converges if _________________.

Example 1: Determine whether the series $\sum_{n=1}^{\infty} n^{-\sqrt{2}}$ converges or diverges.

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Section 9.4 Comparison of Series

Objective: In this lesson you learned how to determine whether an infinite series converges or diverges.

I. Direct Comparison Test (Pages 626–627)

What you should learn
How to use the Direct Comparison Test to determine whether a series converges or diverges

This section presents two additional tests for positive-term series which greatly expand the variety of series you are able to test for convergence or divergence; they allow you to ______________
____
____.

Let $0 < a_n \le b_n$ for all n. The **Direct Comparison Test** states that

if $\sum_{n=1}^{\infty} b_n$ ________________, then $\sum_{n=1}^{\infty} a_n$ ________________.

If $\sum_{n=1}^{\infty} a_n$ ________________, then $\sum_{n=1}^{\infty} b_n$ ________________.

Use your own words to give an interpretation of this test.

II. Limit Comparison Test (Pages 628–629)

What you should learn
How to use the Limit Comparison Test to determine whether a series converges or diverges

Suppose that $a_n > 0$ and $b_n > 0$. The **Limit Comparison Test** states that if $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$, where L is *finite* and *positive,* then the two series $\sum a_n$ and $\sum b_n$ either __________________
______________________________.

Describe circumstances under which you might apply the Limit Comparison Test.

The Limit Comparison Test works well for comparing a "messy" algebraic series with a *p*-series. In choosing an appropriate *p*-series, you must choose one with ________________________

____.

In other words, when choosing a series for comparison, you can disregard all but ____

____.

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Section 9.5 Alternating Series

Objective: In this lesson you learned how to determine whether an infinite series converges or diverges.

Important Vocabulary	Define each term or concept.
Alternating series	
Absolutely convergent	
Conditionally convergent	

I. Alternating Series (Pages 633–634)

> ***What you should learn***
> How to use the Alternating Series Test to determine whether an infinite series converges

Alternating series occur in two ways: ______________________
____.

Let $a_n > 0$. The **Alternating Series Test** states that the alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converge if the following two conditions are met:

1.

2.

Example 1: Determine whether the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ converges or diverges.

II. Alternating Series Remainder (Page 635)

> ***What you should learn***
> How to use the Alternating Series Remainder to approximate the sum of an alternating series

For a convergent alternating series, the partial sum S_N can be
____.

If a convergent alternating series satisfies the condition $a_{n+1} \le a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is ______________________ ________________________________. That is,

$|S - S_N| = |R_N| \le a_{N+1}$.

III. Absolute and Conditional Convergence (Pages 636–637)

What you should learn
How to classify a convergent series as absolutely or conditionally convergent

If the series $\Sigma\, |a_n|$ converges, then the series $\Sigma\, a_n$ _____________ ____________________.

Example 2: Is the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ absolutely or conditionally convergent?

IV. Rearrangement of Series (Pages 637–638)

What you should learn
How to rearrange an infinite series to obtain a different sum

The terms of an infinite series can be rearranged without changing the value of the sum of the terms only if ____________ _______________________________________. If the series is ____________________________________, then it is possible that rearranging the terms of the series can change the value of the sum.

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Section 9.6 The Ratio and Root Tests

Objective: In this lesson you learned how to determine whether an infinite series converges or diverges.

I. The Ratio Test (Pages 641–643)

What you should learn
How to use the Ratio Test to determine whether a series converges or diverges

Let $\sum_{n=1}^{\infty} a_n$ be an infinite series with nonzero terms. The **Ratio Test** states that:

1. The series converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ ____________________.

2. The series diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ ____________________ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ ________________.

3. The test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ ____________________.

Example 1: Use the Ratio Test to determine whether the series $\sum_{n=0}^{\infty}\frac{4^n}{n!}$ converges or diverges.

The Ratio Test is particularly useful for series that ___________ ________________, such as those that involve _____________ ______________________.

II. The Root Test (Page 644)

What you should learn
How to use the Root Test to determine whether a series converges or diverges

The Root Test for convergence or divergence of series works especially well for series involving ______________________.

Let $\sum_{n=1}^{\infty} a_n$ be an infinite series. The **Root Test** states that:

1. The series converges absolutely if $\lim_{n\to\infty}\sqrt[n]{|a_n|}$ ____________________.

2. The series diverges if $\lim_{n\to\infty}\sqrt[n]{|a_n|}$ ____________________ or $\lim_{n\to\infty}\sqrt[n]{|a_n|}$ ________________.

3. The test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|}$ ____________________.

III. Strategies for Testing Series (Pages 645–646)

What you should learn
How to review the tests for convergence and divergence of an infinite series

List four guidelines for testing a series for convergence or divergence.

1.

2.

3.

4.

Complete the following selected tests for series.

Test	Series	Converges	Diverges
nth-Term	$\sum_{n=1}^{\infty} a_n$		
	$\sum_{n=0}^{\infty} ar^n$		
	$\sum_{n=1}^{\infty} \frac{1}{n^p}$		
	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$		
Ratio	$\sum_{n=1}^{\infty} a_n$		

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Section 9.7 Taylor Polynomials and Approximations

Objective: In this lesson you learned how to find Taylor or Maclaurin polynomial approximations of elementary functions.

I. Polynomial Approximations of Elementary Functions (Pages 650–651)

> ***What you should learn***
> How to find polynomial approximations of elementary functions and compare them with the elementary functions

To find a polynomial function *P* that approximates another function *f*, ____

____

______________________. The approximating polynomial is said to be ____.

II. Taylor and Maclaurin Polynomials (Pages 652–655)

> ***What you should learn***
> How to find Taylor and Maclaurin polynomial approximations of elementary functions

If *f* has *n* derivatives at *c*, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the ____.

If $c = 0$, then $P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$

is also called the ____.

The accuracy of a Taylor or Maclaurin polynomial approximation is usually better at *x*-values ________________

________________________________. The approximation is usually better for higher-degree Taylor or Maclaurin polynomials than ____.

III. Remainder of a Taylor Polynomial (Pages 656–657)

> ***What you should learn***
> How to use the remainder of a Taylor polynomial

If a function *f* is differentiable through order $n + 1$ in an interval *I* containing *c*, then for each *x* in *I*, Taylor's Theorem states that there exists *z* between *x* and *c* such that $f(x) =$ ________________

____, where $R_n(x)$ is given by $R_n(x) = \dfrac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$. The value

$R_n(x)$ is called the ________________________________.

The practical application of this theorem lies not in calculating $R_n(x)$, but in ____

____.

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Section 9.8 Power Series

Objective: In this lesson you learned how to find the radius and interval of convergence of power series and how to differentiate and integrate power series.

I. Power Series (Pages 661–662)

What you should learn How to understand the definition of a power series

If x is a variable, then an infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots$$ is called a

________________________________. More generally, an infinite series of the form

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots + a_n(x-c)^n + \cdots$$

is called a ____,

where c is a constant.

II. Radius and Interval of Convergence (Pages 662–663)

What you should learn How to find the radius and interval of convergence of a power series

For a power series centered at c, precisely one of the following is true.

1. The series converges only at _______________.
2. There exists a real number $R > 0$ such that the series converges absolutely for ______________________, and diverges for ______________________.
3. The series converges absolutely for ____________________.

The number R is the __________________________________ of the power series. If the series converges only at c, the radius of convergence is ________________, and if the series converges for all x, the radius of convergence is ____________________. The set of all values of x for which the power series converges is the ____ of the power series.

III. Endpoint Convergence (Pages 664–665)

> ***What you should learn***
> How to determine the endpoint convergence of a power series

For a power series whose radius of convergence is a finite number R, each endpoint of the interval of convergence must be ____.

IV. Differentiation and Integration of Power Series (Pages 666–667)

> ***What you should learn***
> How to differentiate and integrate a power series

If the function given by

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$$
$$= a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

has a radius of convergence of $R > 0$, then, on the interval $(c-R, c+R)$, f is ______________________________ ______________. Moreover, the derivative and antiderivative of f are as follows.

1. $f'(x) = \sum_{n=1}^{\infty} n a_n (x-c)^{n-1}$

   $=$ ____

2. $\int f(x)\,dx = C + \sum_{n=0}^{\infty} a_n \dfrac{(x-c)^{n+1}}{n+1}$

   $=$ ____

The radius of convergence of the series obtained by differentiating or integrating a power series is ______________ ______________________________________. The interval of convergence, however, may differ as a result of __________ __________________________________.

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Section 9.9 Representation of Functions by Power Series

Objective: In this lesson you learned how to represent functions by power series.

I. Geometric Power Series (Pages 671–672)

What you should learn How to find a geometric power series that represents a function

Describe two ways for finding a geometric power series.

II. Operations with Power Series (Pages 673–675)

What you should learn How to construct a power series using series operations

Let $f(x) = \sum a_n x^n$ and $g(x) = \sum b_n x^n$.

1. $f(kx) = \sum_{n=0}^{\infty}$ ____________________

2. $f(x^N) = \sum_{n=0}^{\infty}$ ____________________

3. $f(x) \pm g(x) = \sum_{n=0}^{\infty}$ ______________________

The operations described above can change ________________ ____.

Additional notes

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Section 9.10 Taylor and Maclaurin Series

Objective: In this lesson you learned how to find a Taylor or Maclaurin series for a function.

What you should learn
How to find a Taylor or Maclaurin series for a function

I. Taylor Series and Maclaurin Series (Pages 678–682)

The Form of a Convergent Power Series

If f is represented by a power series $f(x) = \sum a_n (x-c)^n$ for all x in an open interval I containing c, then a_n = _______________, and $f(x)$ = ____

____.

The series is called the **Taylor series** for $f(x)$ at c because ______

____

____.

If a function f has derivatives of all orders at $x = c$, then the series $\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$ = ______________________________

____ is called the **Taylor series for $f(x)$ at c.** Moreover, if $c = 0$, then the series is called the ______________________________________.

If $\lim_{n\to\infty} R_n = 0$ for all x in the interval I, then the Taylor series for f

____________________________________, where

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n .$$

Complete the list of guidelines for finding a Taylor series.

1.

2.

3.

II. Binomial Series (Page 683)

> ***What you should learn***
> How to find a binomial series

The **binomial series** for a function of the form $f(x) = (1 + x)^k$ is

____

III. Deriving Taylor Series from a Basic List (Pages 684–686)

> ***What you should learn***
> How to use a basic list of Taylor series to find other Taylor series

Because direct computation of Taylor or Maclaurin coefficients can be tedious, the most practical way to find a Taylor or Maclaurin series is to develop power series for a basic list of elementary functions. From this list, you can determine power series for other functions by the operations of ______________

____

____with known power series.

List power series for the following elementary functions and give the interval of convergence for each.

$\frac{1}{x} =$

$\frac{1}{1+x} =$

$\ln x =$

$e^x =$

$\sin x =$

$\cos x =$

$\arctan x =$

$\arcsin x =$

$(1 + x)^k =$

Additional notes

Additional notes

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Chapter 10 Conics, Parametric Equations, And Polar Coordinates

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Section 10.1 Conics and Calculus

Objective: In this lesson you learned how to analyze and write an equation of a parabola, an ellipse, and a hyperbola.

Important Vocabulary Define each term or concept.

Directrix of a parabola

Focus of a parabola

Tangent of parabola

Foci of an ellipse

Vertices of an ellipse

Major axis of an ellipse

Center of an ellipse

Minor axis of an ellipse

Branches of a hyperbola

Transverse axis of a hyperbola

Conjugate axis of a hyperbola

I. Conic Sections (Page 696)

What you should learn
Understand the definition of a conic section

A **conic section,** or **conic,** is ______________________________
____.

Name the four basic conic sections: ___________________
____.

In the formation of the four basic conics, the intersecting plane does not pass through the vertex of the cone. When the plane does pass through the vertex, the resulting figure is a(n)

________________________________, such as
____
___________________________________.

In this section, each conic is defined as a _________________ of points satisfying a certain geometric property. For example, a circle is the collection of all points (x, y) that are __________________________ from a fixed point (h, k). This locus definition easily produces the standard equation of a circle _________________________.

II. Parabolas (Pages 697–698)

> ***What you should learn***
> How to analyze and write equations of parabolas using properties of parabolas

A **parabola** is _______________________________________
____
____.

The midpoint between the focus and the directrix is the _______________ of a parabola. The line passing through the focus and the vertex is the ______________ of the parabola. The **standard form** of the equation of a parabola with a vertical axis having a vertex at (h, k) and directrix $y = k - p$ is

____

The standard form of the equation of a parabola with a horizontal axis having a vertex at (h, k) and directrix $x = h - p$ is

____

The focus lies on the axis p units (directed distance) from the vertex. The coordinates of the focus are _________________ for a vertical axis or _____________________ for a horizontal axis.

Example 1: Find the standard form of the equation of the parabola with vertex at the origin and focus $(1, 0)$.

A **focal chord** is ____

____.

The specific focal chord perpendicular to the axis of a parabola is called the ____________________.

The reflective property of a parabola states that the tangent line to a parabola at a point P makes equal angles with the following two lines:

1)

2)

III. Ellipses (Pages 699–702)

What you should learn
How to analyze and write equations of ellipses using properties of ellipses

An **ellipse** is ____

____

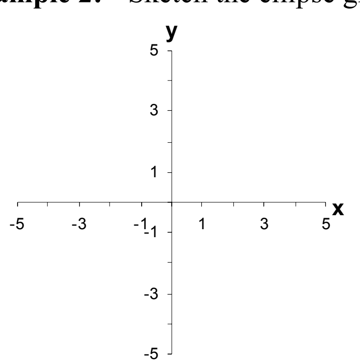
____.

The standard form of the equation of an ellipse with center (h, k) and a horizontal major axis of length $2a$ and a minor axis of length $2b$, where $a > b$, is: ________________________________

The standard form of the equation of an ellipse with center (h, k) and a vertical major axis of length $2a$ and a minor axis of length $2b$, where $a > b$, is: ________________________________

In both cases, the foci lie on the major axis, c units from the center, with $c^2 =$ ______________ .

Example 2: Sketch the ellipse given by $4x^2 + 25y^2 = 100$.



Let P be a point on an ellipse. The Reflective Property of an Ellipse states that ____________________________________
____
__________.

_____________________ measures the ovalness of an ellipse. It is given by the ratio e = _____________. For an elongated ellipse, the value of e is close to ____________. For every ellipse, the value of e lies between ________ and ________.

IV. Hyperbolas (Pages 703–705)

> ***What you should learn***
> How to analyze and write equations of hyperbolas using properties of hyperbolas

A **hyperbola** is ____
____
____.

The line through a hyperbola's two foci intersects the hyperbola at two points called ____________________.

The midpoint of a hyperbola's transverse axis is the ____________________ of the hyperbola.

The standard form of the equation of a hyperbola centered at (h, k) and having a horizontal transverse axis is

____

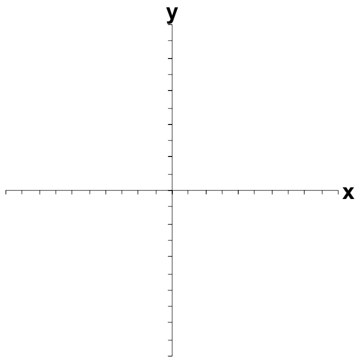
The standard form of the equation of a hyperbola centered at (h, k) and having a vertical transverse axis is

____

The vertices are a units from the center and the foci are c units from the center. Moreover, a, b, and c are related by the equation _____________________.

The **asymptotes** of a hyperbola with a horizontal transverse axis are ____.
The **asymptotes** of a hyperbola with a vertical transverse axis are ____.

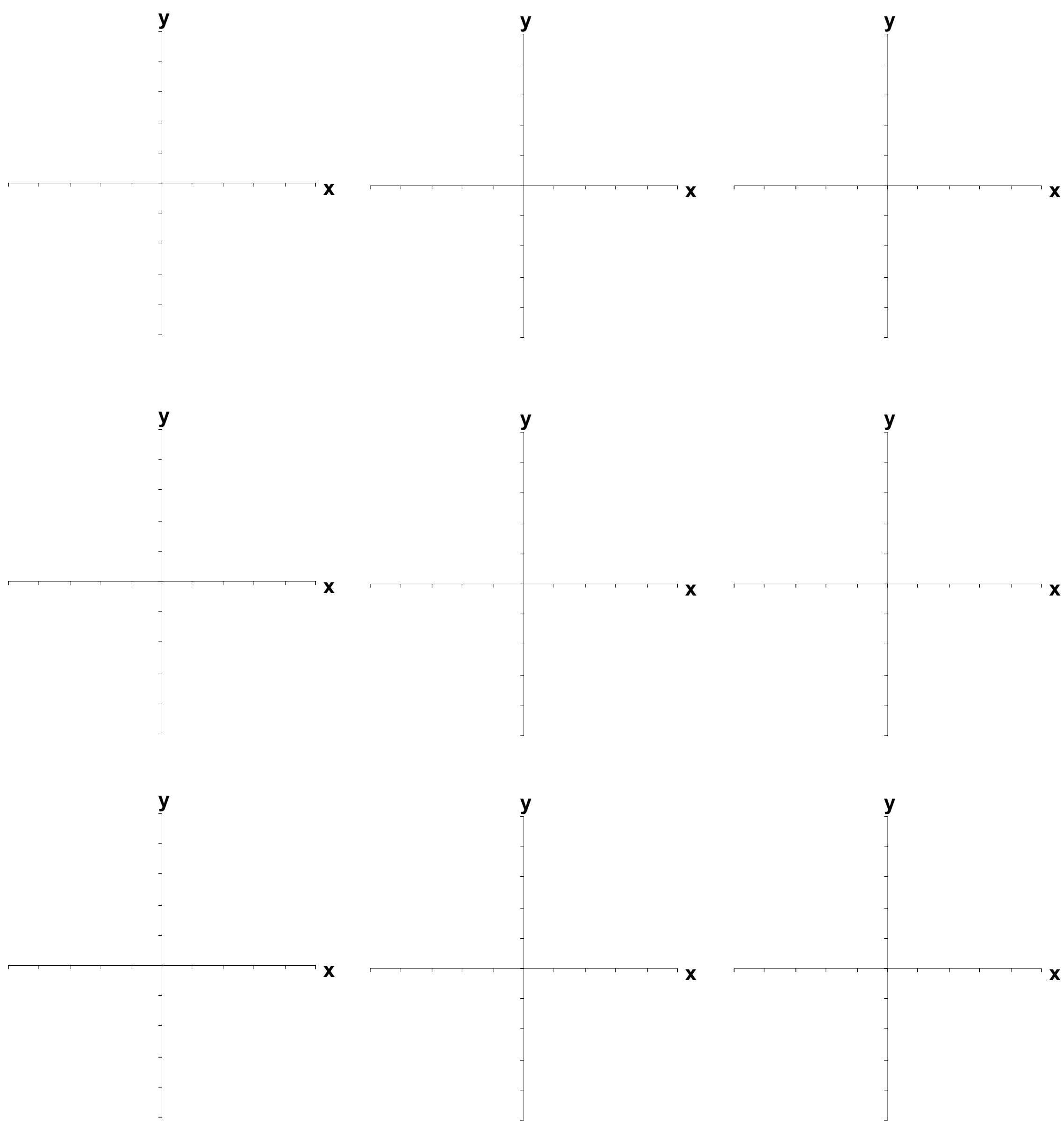
Example 3: Sketch the graph of the hyperbola given by $y^2 - 9x^2 = 9$.



The **eccentricity** of a hyperbola is $e =$ _______________, where the values of e are _______________________.

Additional notes

Additional notes



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Section 10.2 Plane Curves and Parametric Equations

Objective: In this lesson you learned how to sketch a curve represented by parametric equations.

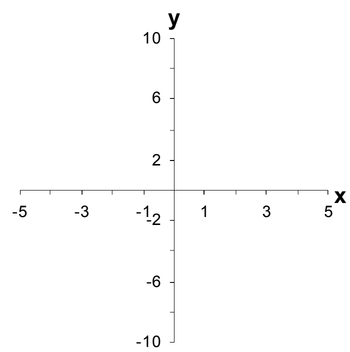
I. Plane Curves and Parametric Equations (Pages 711–712)

What you should learn
How to sketch the graph of a curve given by a set of parametric equations

If f and g are continuous functions of t on an interval I, then the equations $x = f(t)$ and $y = g(t)$ are called ________________ ________________ and t is called the ________________.
The set of points (x, y) obtained as t varies over the interval I is called the ____.
Taken together, the parametric equations and the graph are called a ________________________, denoted by C.

When sketching (by hand) a curve represented by a set of parametric equations, you can plot points in the ____________.
Each set of coordinates (x, y) is determined from a value chosen for the ____________________. By plotting the resulting points in the order of increasing values of t, the curve is traced out in a specific direction, called the ___________________ of the curve.

Example 1: Sketch the curve described by the parametric equations $x = t - 3$ and $y = t^2 + 1$, $-1 \le t \le 3$.



II. Eliminating the Parameter (Pages 713–714)

What you should learn
How to eliminate the parameter in a set of parametric equations

Eliminating the parameter is the process of _______________ ____ ____.

Describe the process used to eliminate the parameter from a set of parametric equations.

When converting equations from parametric to rectangular form, the range of x and y implied by the parametric equations may be ______________ by the change to rectangular form. In such instances, the domain of the rectangular equation must be

____

___________________________________.

To eliminate the parameter in equations involving trigonometric functions, try using the identity __________________________.

III. Finding Parametric Equations (Pages 715–716)

What you should learn
How to find a set of parametric equations to represent a curve

Describe how to find a set of parametric equations for a given graph.

A curve C represented by $x = f(t)$ and $y = g(t)$ on an interval I is called __________________ if f' and g' are continuous on I and not simultaneously 0, except possibly at the endpoints of I. The curve C is called **piecewise smooth** if _____________________

____.

IV. The Tautochrone and Brachistochrone Problems (Page 717)

What you should learn
Understand two classic calculus problems, the tautochrone and brachistochrone problems

Describe the tautochrone problem and the brachistochrone problem in your own words.

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Section 10.3 Parametric Equations and Calculus

Objective: In this lesson you learned how to use a set of parametric equations to find the slope of a tangent line to a curve and the arc length of a curve.

I. Slope and Tangent Lines (Pages 721–723)

> ***What you should learn***
> How to find the slope of a tangent line to a curve by a set of parametric equations

If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of C at (x, y) is $\dfrac{dy}{dx} =$ ____________ , $\dfrac{dx}{dt} \neq$ __________.

Example 1: For the curve given by the parametric equations $x = t - 3$ and $y = t^2 + 1$, $-1 \leq t \leq 3$, find the slope at the point (−3, 1).

II. Arc Length (Pages 723–725)

> ***What you should learn***
> How to find the arc length of a curve given by a set of parametric equations

If a smooth curve C is given by $x = f(t)$ and $y = g(t)$ such that C does not intersect itself on the interval $a \leq t \leq b$ (except possibly at the endpoints), then the arc length of C over the interval is given by

$$s = \int_a^b \sqrt{\qquad\qquad\qquad}\, dt = \int_a^b \sqrt{\qquad\qquad\qquad}\, dt$$

In the preceding section you saw that if a circle rolls along a line, a point on its circumference will trace a path called a ____________________. If the circle rolls around the circumference of another circle, the path of the point is an ________________________.

III. Area of Surface of Revolution (Page 726)

> ***What you should learn***
> How to find the area of a surface of revolution (parametric form)

If a smooth curve C given by $x = f(t)$ and $y = g(t)$ does not cross itself on the interval $a \le t \le b$, then the area S of the surface of revolution formed by revolving C about the coordinate axes is given by

1. $S = \qquad \int \qquad \sqrt{\qquad}$ Revolution about the ____________: $g(t) \ge 0$

2. $S = \qquad \int \qquad \sqrt{\qquad}$ Revolution about the ____________: $f(t) \ge 0$

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Section 10.4 Polar Coordinates and Polar Graphs

Objective: In this lesson you learned how to sketch the graph of an equation in polar form, find the slope of a tangent line to a polar graph, and identify special polar graphs.

I. Polar Coordinates (Page 731)

What you should learn
How to understand the polar coordinate system

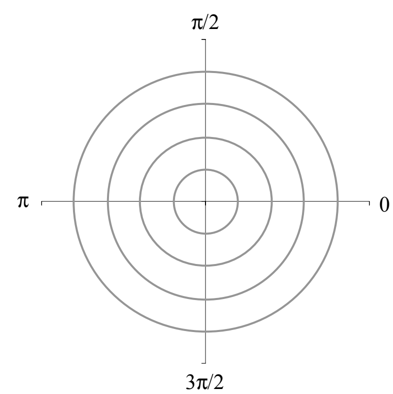
To form the **polar coordinate system** in the plane, fix a point O, called the ______________ or ________________, and construct from O an initial ray called the __________________. Then each point P in the plane can be assigned _______________________ (r, θ) as follows:

1) $r =$ _______________________________________

2) $\theta =$ ____
____

In the polar coordinate system, points do not have a unique representation. In general, the point (r, θ) can be represented as _____________________ or ________________________, where n is any integer. Moreover, the pole is represented by $(0, \theta)$, where θ is ______________________.

Example 1: Plot the point $(r, \theta) = (-2, 11\pi/4)$ on the polar coordinate system.



Example 2: Find another polar representation of the point $(4, \pi/6)$.

II. Coordinate Conversion (Page 732)

> ***What you should learn***
> How to rewrite rectangular coordinates and equations in polar form and vice versa

The polar coordinates (r, θ) of a point are related to the rectangular coordinates $(x, y)$of the point as follows . . .

Example 3: Convert the polar coordinates $(3, 3\pi/2)$ to rectangular coordinates.

III. Polar Graphs (Pages 733–734)

> ***What you should learn***
> How to sketch the graph of an equation given in polar form

One way to sketch the graph of a polar equation is to

____

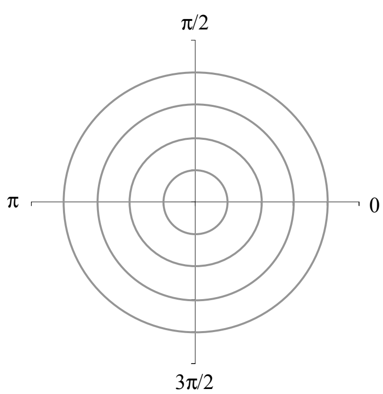
____.

To convert a rectangular equation to polar form, ___________

____.

Example 4: Find the rectangular equation corresponding to the polar equation $r = \dfrac{-5}{\sin\theta}$.

Example 5: Sketch the graph of the polar equation $r = 3\cos\theta$.



IV. Slope and Tangent Lines (Pages 735–736)

> ***What you should learn***
> How to find the slope of a tangent line to a polar graph

If *f* is a differentiable function of θ, then the slope of the tangent line to the graph of $r = f(\theta)$ at the point (r, θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \underline{\hspace{6cm}},$$

provided that $\frac{dx}{d\theta} \neq 0$ at (r, θ).

Solutions to $\frac{dy}{d\theta} = 0$ yield ______________________________, provided that $\frac{dx}{d\theta} \neq 0$. Solutions to $\frac{dx}{d\theta} = 0$ yield ___________ _______________, provided that $\frac{dy}{d\theta} \neq 0$.

If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is ____________ ____. This theorem is useful because it states that ______________________________ ____.

V. Special Polar Graphs (Page 737)

> ***What you should learn***
> How to identify several types of special polar graphs

List the general equations that yield each of the following types of special polar graphs:

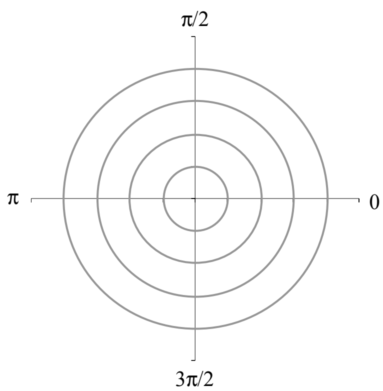
Limaçons:

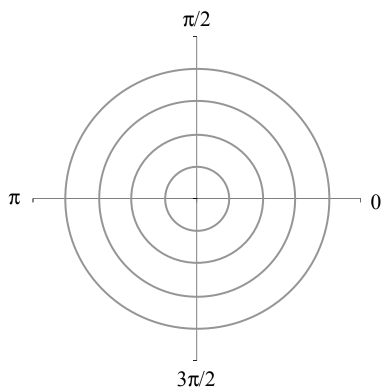
Rose curves:

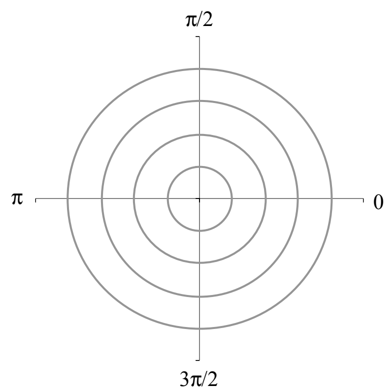
Circles:

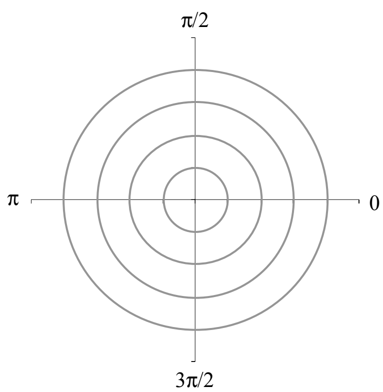
Lemniscates:

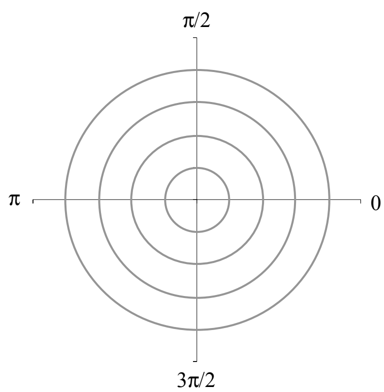
Additional notes

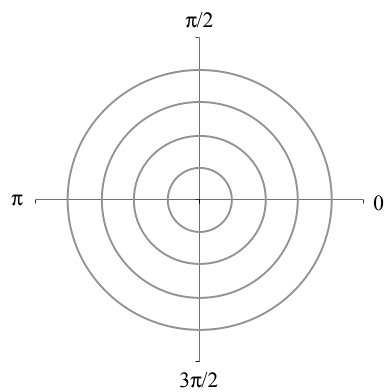












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Section 10.5 Area and Arc Length in Polar Coordinates

Objective: In this lesson you learned how to find the area of a region bounded by a polar graph and the arc length of a polar graph.

I. Area of a Polar Region (Pages 741–742)

What you should learn
How to find the area of a region bounded by a polar graph

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, $0 < \beta - \alpha \le 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by ____ .

II. Points of Intersection of Polar Graphs (Pages 743–744)

What you should learn
How to find the points of intersection of two polar graphs

Explain why care must be taken in determining the points of intersection of two polar graphs.

III. Arc Length in Polar Form (Page 745)

What you should learn
How to find the arc length of a polar graph

Let f be a function whose derivative is continuous on an interval $\alpha \le \theta \le \beta$. The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is ____ .

IV. Area of a Surface of Revolution (Page 746)

> ***What you should learn***
> How to find the area of a surface of revolution (polar form)

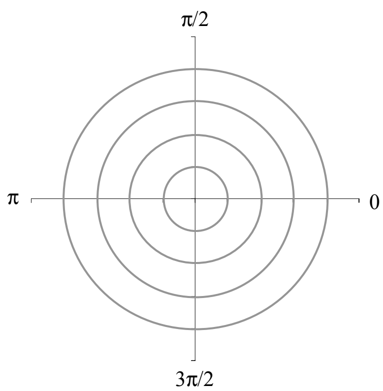
Let f be a function whose derivative is continuous on an interval $\alpha \le \theta \le \beta$. The area of the surface formed by revolving the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the indicated line is as follows.

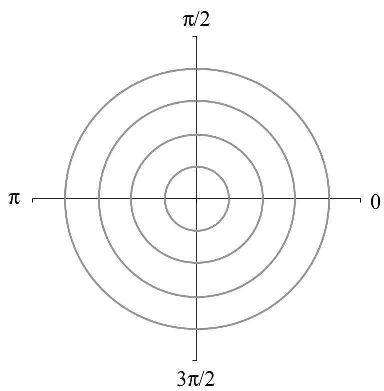
1. About the polar axis:

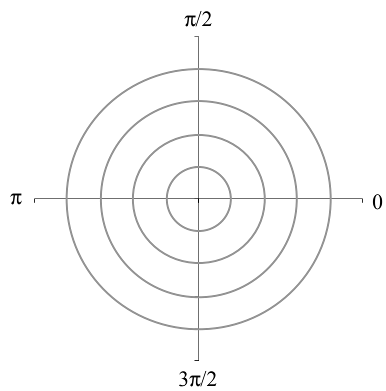
____

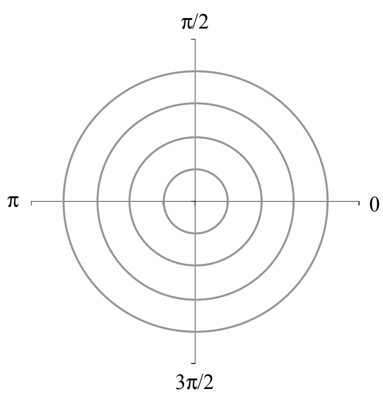
2. About the line $\theta = \dfrac{\pi}{2}$:

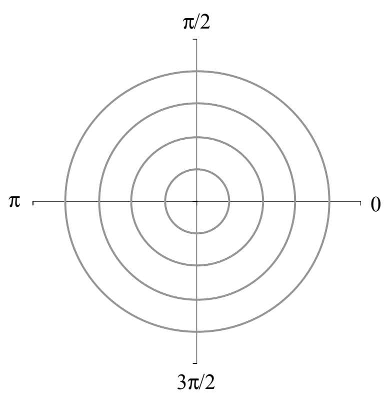
____

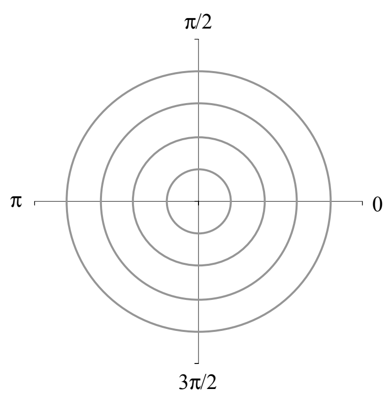












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Section 10.6 Polar Equations of Conics and Kepler's Laws

Objective: In this lesson you learned how to analyze and write a polar equation of a conic.

What you should learn How to analyze and write polar equations of conics

I. Polar Equations of Conics (Pages 750–752)

Let F be a fixed point (*focus*) and D be a fixed line (*directrix*) in the plane. Let P be another point in the plane and let e (*eccentricity*) be the ratio of the distance between P and F to the distance between P and D. The collection of all points P with a given eccentricity is a ____________________.

The conic is an ellipse if ______________. The conic is a parabola if ______________. Finally, the conic is a hyperbola if ____________.

For each type of conic, the pole corresponds to the ______________.

The graph of the polar equation ________________________ is a conic with a vertical directrix to the right of the pole, where $e > 0$ is the eccentricity and $| d |$ is the distance between the focus (pole) and the directrix.

The graph of the polar equation ________________________ is a conic with a vertical directrix to the left of the pole, where $e > 0$ is the eccentricity and $| d |$ is the distance between the focus (pole) and the directrix.

The graph of the polar equation ________________________ is a conic with a horizontal directrix above the pole, where $e > 0$ is the eccentricity and $| d |$ is the distance between the focus (pole) and the directrix.

The graph of the polar equation ________________________ is a conic with a horizontal directrix below the pole, where $e > 0$ is the eccentricity and $| d |$ is the distance between the focus (pole) and the directrix.

Example 1: Identify the type of conic from the polar equation $r = \dfrac{36}{10 + 12\sin\theta}$, and describe its orientation.

II. Kepler's Laws (Pages 753−754)

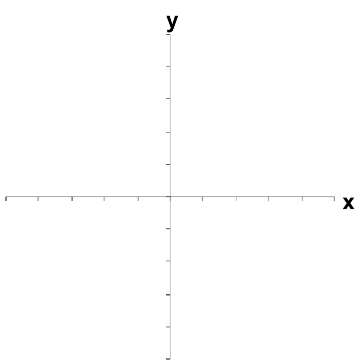
What you should learn
How to understand and use Kepler's Laws of planetary motion

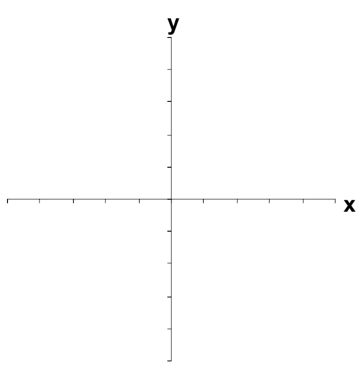
List Kepler's Laws, which can be used to describe the orbits of the planets about the sun.

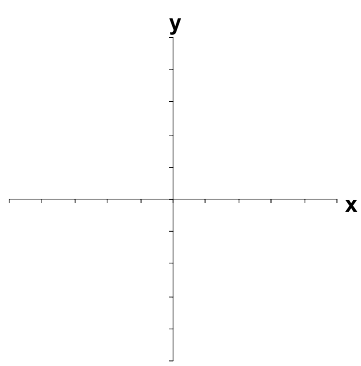
1.

2.

3.







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Chapter 11 Vectors and the Geometry of Space

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Section 11.1 Vectors in the Plane

Objective: In this lesson you learned how to represent vectors, perform basic vector operations, and represent vectors graphically.

Important Vocabulary	Define each term or concept.
Vector v in the plane	
Standard position	
Zero vector	
Unit vector	
Standard unit vectors	

I. Component Form of a Vector (Pages 764–765)

What you should learn How to write the component form of a vector

A **directed line segment** has an ____________________ and a ______________________.

The **magnitude** of the directed line segment $\overrightarrow{PQ}$, denoted by ________________, is its ________________. The length of a directed line segment can be found by ____________________ _____________________.

If **v** is a vector in the plane whose initial point is at the origin and whose terminal point is (v_1, v_2), then the ________________ ________________ is given by $\mathbf{v} = \langle v_1, v_2 \rangle$, where the coordinates v_1 and v_2 are called the _____________________.

If P (p_1, p_2) and Q (q_1, q_2) are the initial and terminal points of a directed line segment, the component form of the vector **v** represented by $\overrightarrow{PQ}$ is _____________ = ____________________.

The **length** (or magnitude) **of v** is:

$\|\mathbf{v}\| = \sqrt{} = \sqrt{}$

If $\mathbf{v} = \langle v_1, v_2 \rangle$, $\mathbf{v}$ can be represented by the _______________ _______________________________________ from $P(0, 0)$ to $Q(v_1, v_2)$.

The length of $\mathbf{v}$ is also called the _____________________.

Example 1: Find the component form and length of the vector $\mathbf{v}$ that has (1, 7) as its initial point and (4, 3) as its terminal point.

II. Vector Operations (Pages 766–769)

What you should learn
How to perform vector operations and interpret the results geometrically

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let c be a scalar. Then the **vector sum** of $\mathbf{u}$ and $\mathbf{v}$ is the vector:

$\mathbf{u} + \mathbf{v} =$ ________________________

and the **scalar multiple** of c and $\mathbf{u}$ is the vector:

$c\mathbf{u} =$ _____________________________ .

Furthermore, the **negative** of $\mathbf{v}$ is the vector

$-\mathbf{v} =$ ________________________

and the **difference** of $\mathbf{u}$ and $\mathbf{v}$ is

$\mathbf{u} - \mathbf{v} =$ ________________________

Geometrically, the scalar multiple of a vector $\mathbf{v}$ and a scalar c is ____.

If c is positive, $c\mathbf{v}$ has the _____________ direction as $\mathbf{v}$, and if c is negative, $c\mathbf{v}$ has the _____________ direction.

To add two vectors geometrically, ________________________ ____ ____.

The vector $\mathbf{u} + \mathbf{v}$, called the ________________________, is ____ _______________________________.

Example 2: Let $\mathbf{u} = \langle 1, 6 \rangle$ and $\mathbf{v} = \langle -4, 2 \rangle$. Find:
(a) $3\mathbf{u}$ **(b)** $\mathbf{u} + \mathbf{v}$

Let **u**, **v**, and **w** be vectors in the plane, and let c and d be scalars. Complete the following properties of vector addition and scalar multiplication:

1. $\mathbf{u} + \mathbf{v} =$ _______________
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} =$ ____________________
3. $\mathbf{u} + \mathbf{0} =$ ____________
4. $\mathbf{u} + (-\mathbf{u}) =$ ____________
5. $c(d\mathbf{u}) =$ ________________
6. $(c + d)\mathbf{u} =$ ________________
7. $c(\mathbf{u} + \mathbf{v}) =$ ________________
8. $1(\mathbf{u}) =$ ____________ ; $0(\mathbf{u}) =$ ____________

Any set of vectors, with an accompanying set of scalars, that satisfies these eight properties is a ________________________.

Let **v** be a vector and let c be a scalar. Then
$\|c\mathbf{v}\| =$ ___________________

To find a unit vector **u** that has the same direction as a given nonzero vector **v**, _____________________________________
____.

In this case, the vector **u** is called a ________________________ ____________________. The process of multiplying **v** by $1/\|\mathbf{v}\|$ to get a unit vector is called _____________________________.

Example 3: Find a unit vector in the direction of $\mathbf{v} = \langle -8, 6 \rangle$.

III. Standard Unit Vectors (Pages 769–770)

What you should learn
How to write a vector as a linear combination of standard unit vectors

Let $\mathbf{v} = \langle v_1, v_2 \rangle$. Then the standard unit vectors can be used to represent $\mathbf{v}$ as $\mathbf{v} =$ ________________ , where the scalar v_1 is called the ________________________________ and the scalar v_2 is called the ______________________________. The vector sum $v_1\mathbf{i} + v_2\mathbf{j}$ is called a _________________________ of the vectors $\mathbf{i}$ and $\mathbf{j}$.

Example 4: Let $\mathbf{v} = \langle -5, 3 \rangle$. Write $\mathbf{v}$ as a linear combination of the standard unit vectors $\mathbf{i}$ and $\mathbf{j}$.

Example 5: Let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{w} = 2\mathbf{i} + 9\mathbf{j}$. Find $\mathbf{v} + \mathbf{w}$.

If $\mathbf{u}$ is a unit vector and θ is the angle (measured counterclockwise) from the positive x-axis to $\mathbf{u}$, the terminal point of $\mathbf{u}$ lies on the unit circle and $\mathbf{u} =$ __________________ = _________________________.

Now, if $\mathbf{v}$ is any nonzero vector that makes an angle θ with the positive x-axis, it has the same direction as $\mathbf{u}$ and $\mathbf{v} =$ ______________________ = ________________________.

IV. Applications of Vectors (Pages 770–771)

What you should learn
How to use vectors to solve problems involving force or velocity

Describe several real-life applications of vectors.

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Section 11.2 Space Coordinates and Vectors in Space

Objective: In this lesson you learned how to plot points in a three-dimensional coordinate system and analyze vectors in space.

Important Vocabulary	Define each term or concept.
Sphere	
Standard unit vector notation in space	
Parallel vectors in space	

I. Coordinates in Space (Pages 775–776)

What you should learn How to understand the three-dimensional rectangular coordinate system

A **three-dimensional coordinate system** is constructed by

____

____.

Taken as pairs, the axes determine three coordinate planes: the ______________, the ______________, and the ______________.

These three coordinate planes separate the three-space into eight __________________. The first of these is the one for which ___ ____.

In the three-dimensional system, a point P in space is determined by an ordered triple (x, y, z), where x, y, and z are as follows . . .

x = ____,

y = ____,

and z = ____.

A three-dimensional coordinate system can have either a ______ _______________ or a ______________________ orientation. To determine the orientation of a system, ______________________

____

____
____
____.

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) given by the Distance Formula in space is

$$d = \sqrt{\qquad\qquad\qquad\qquad\qquad}$$

The midpoint of the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) given by the **Midpoint Formula in Space** is

$$\left(\qquad\qquad\qquad\qquad \right)$$

Example 1: For the points (2, 0, – 4) and (– 1, 4, 6), find
(a) the distance between the two points, and
(b) the midpoint of the line segment joining them.

The **standard equation of a sphere** whose center is (x_0, y_0, z_0) and whose radius is r is __________________________________ .

Example 2: Find the center and radius of the sphere whose equation is $x^2 + y^2 + z^2 - 4x + 2y + 8z + 17 = 0$.

II. Vectors in Space (Pages 777–779)

> ***What you should learn***
> How to analyze vectors in space

In space, vectors are denoted by ordered triples of the form

__________________________ .

The **zero vector in space** is denoted by __________________.

If **v** is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, the **component form** of **v** is given by

____

____

____.

Two vectors are equal if and only if ________________________

______________________________________.

The length of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ is:

$\| \mathbf{u} \| = \sqrt{}$ ____________________

A unit vector **u** in the direction of **v** is ____________________.

The sum of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$\mathbf{u} + \mathbf{v} =$ ____________________________ .

The scalar multiple of the real number c and $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ is

$c\mathbf{u} =$ _____________________ .

Example 3: Determine whether the vectors $\langle 6, 1, -3 \rangle$ and $\langle -2, -1/3, 1 \rangle$ are parallel.

To use vectors to determine whether three points P, Q, and R in space are collinear, ___________________________________

_____________________________________.

III. Application (Page 779)

Describe a real-life application of vectors in space.

What you should learn
How to use three-dimensional vectors to solve real-life problems

Additional notes

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Section 11.3 The Dot Product of Two Vectors

Objective: In this lesson you learned how to find the dot product of two vectors in the plane or in space.

Important Vocabulary Define each term or concept.

Angle between two nonzero vectors

Orthogonal

I. The Dot Product (Pages 783–784)

What you should learn
How to use properties of the dot product of two vectors

The **dot product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

________________________.

The dot product of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$\mathbf{u} \bullet \mathbf{v} =$ ____________________________.

The dot product of two vectors yields a ___________________.

Let **u**, **v**, and **w** be vectors in the plane or in space and let c be a scalar. Complete the following properties of the dot product:

1. $\mathbf{u} \bullet \mathbf{v} =$ _______________
2. $\mathbf{0} \bullet \mathbf{v} =$ ____________
3. $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) =$ _____________________
4. $\mathbf{v} \bullet \mathbf{v} =$ ________________
5. $c(\mathbf{u} \bullet \mathbf{v}) =$ _______________ = ________________

Example 1: Find the dot product: $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$.

Example 2: Find the dot product of the vectors $\langle -1, 4, -2 \rangle$ and $\langle 0, -1, 5 \rangle$.

II. Angle Between Two Vectors (Pages 784–785)

What you should learn
How to find the angle between two vectors using the dot product

If θ is the angle between two nonzero vectors **u** and **v**, then θ can be determined from _____________________________________.

Example 3: Find the angle between $\mathbf{v} = \langle 5, -4 \rangle$ and $\mathbf{w} = \langle 9, -2 \rangle$.

An alternative way to calculate the dot product between two vectors **u** and **v**, given the angle θ between them, is ______________________________.

Two vectors **u** and **v** are orthogonal if ________________.

Two nonzero vectors are orthogonal if and only if ____________ _____________________________________.

Example 4: Are the vectors $\mathbf{u} = \langle 1, -4 \rangle$ and $\mathbf{v} = \langle 6, 2 \rangle$ orthogonal?

III. Direction Cosines (Page 786)

> ***What you should learn***
> How to find the direction cosines of a vector in space

For a vector in the plane, it is convenient to measure direction in terms of the angle, measured counterclockwise, from __________ _________________________________. In space it is more convenient to measure direction in terms of _________________ ____ ________________. The angles α, β, and γ are the _____________ ________________, and cos α, cos β, and cos γ are the ________ __________________________.

The measure of α, the angle between **v** and **i**, can be found from ________________ ____________. The measure of β, the angle between **v** and **j**, can be found from ______________ _________. The measure of γ, the angle between **v** and **k**, can be found from ________________ ____________.

Any nonzero vector **v** in space has the normalized form $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} =$ _____________________________________.

The sum of the squares of the directions cosines

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma =$ _______________.

IV. Projections and Vector Components (Pages 787–788)

> ***What you should learn***
> How to find the projection of a vector onto another vector

Let **u** and **v** be nonzero vectors. Moreover, let $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$, where $\mathbf{w}_1$ is parallel to **v**, and $\mathbf{w}_2$ is orthogonal to **v**. The vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ are called _________________________________.
The vector $\mathbf{w}_1$ is called the **projection of u onto v** and is denoted by ____________________. The vector $\mathbf{w}_2$ is given by ___________________, and is called the __________________ ___________________________.

Let **u** and **v** be nonzero vectors. The projection of **u** onto **v** is given by $\text{proj}_{\mathbf{v}}\,\mathbf{u} =$ _____________________________.

V. Work (Page 789)

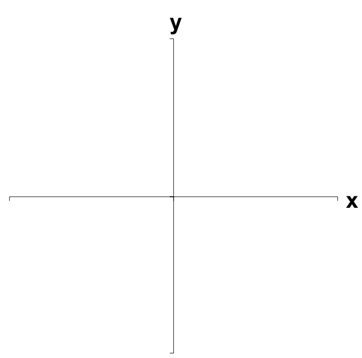
> ***What you should learn***
> How to use vectors to find the work done by a constant force

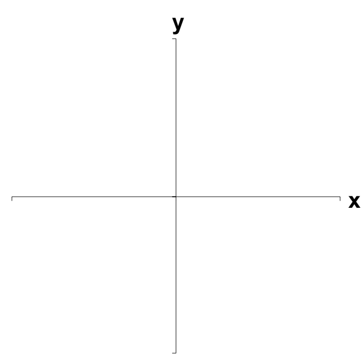
The **work** W done by a constant force **F** as its point of application moves along the vector $\overrightarrow{PQ}$ is given by either of the following:

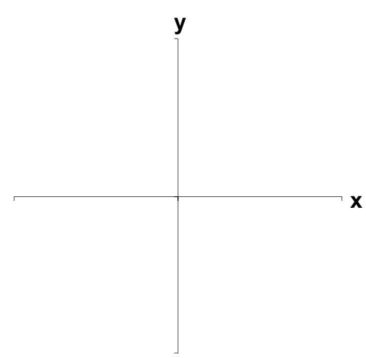
1.

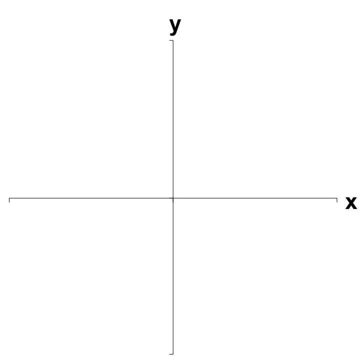
2.

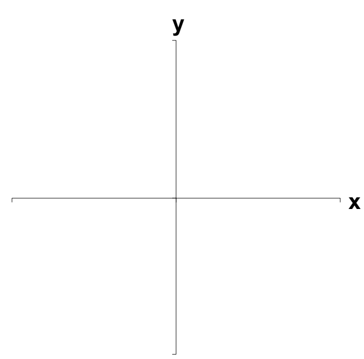
Additional notes

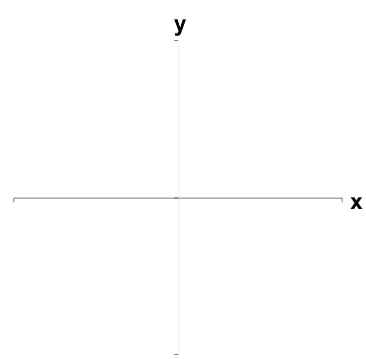












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Section 11.4 The Cross Product of Two Vectors in Space

Objective: In this lesson you learned how to find the cross product of two vectors in space.

What you should learn How to find the cross product of two vectors in space

I. The Cross Product (Pages 792–796)

A vector in space that is orthogonal to two given vectors is called their ____________________.

Let $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ be two vectors in space. The cross product of **u** and **v** is the vector

$\mathbf{u} \times \mathbf{v} =$ ____

Describe a convenient way to remember the formula for the cross product.

Example 1: Given $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find the cross product $\mathbf{u} \times \mathbf{v}$.

Let **u**, **v**, and **w** be vectors in space and let c be a scalar. Complete the following properties of the cross product:

1. $\mathbf{u} \times \mathbf{v} =$ _________________
2. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) =$ ______________________
3. $c(\mathbf{u} \times \mathbf{v}) =$ ________________________
4. $\mathbf{u} \times \mathbf{0} =$ ________________
5. $\mathbf{u} \times \mathbf{u} =$ ___________
6. $\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) =$ __________________

Complete the following geometric properties of the cross product, given **u** and **v** are nonzero vectors in space and θ is the angle between **u** and **v**.

1. $\mathbf{u} \times \mathbf{v}$ is orthogonal to __________________.

2. $\| \mathbf{u} \times \mathbf{v} \| =$ ____________________________.

3. $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ if and only if _________________________________

_________________________________.

4. $\| \mathbf{u} \times \mathbf{v} \| =$ area of the parallelogram having _______________

_____________________.

II. The Triple Scalar Product (Pages 796–797)

> ***What you should learn***
> How to use the triple scalar product of three vectors in space

For vectors **u**, **v**, and **w** in space, the dot product of **u** and $\mathbf{v} \times \mathbf{w}$ is called the ____________________________ of **u**, **v**, and **w**, and is found as

$$\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) = \left| \quad \right|$$

The volume V of a parallelepiped with vectors **u**, **v**, and **w** as adjacent edges is __________________________.

Example 2: Find the volume of the parallelepiped having $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, and $\mathbf{w} = 4\mathbf{i} - 3\mathbf{k}$ as adjacent edges.

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Section 11.5 Lines and Planes in Space

Objective: In this lesson you learned how to find equations of lines and planes in space, and how to sketch their graphs.

I. Lines in Space (Pages 800–801)

> ***What you should learn***
> How to write a set of parametric equations for a line in space

Consider the line L through the point $P(x_1, y_1, z_1)$ and parallel to the vector $\mathbf{v} = \langle a, b, c\rangle$. The vector $\mathbf{v}$ is ________________ for the line L, and a, b, and c are ________________________.

One way of describing the line L is ______________________

____

____.

A line L parallel to the vector $\mathbf{v} = \langle a, b, c\rangle$ and passing through the point $P = (x_1, y_1, z_1)$ is represented by the following **parametric equations,** where t is the parameter:

____

If the direction numbers a, b, and c are all nonzero, you can eliminate the parameter t to obtain the **symmetric equations** of the line:

____

II. Planes in Space (Pages 801–803)

> ***What you should learn***
> How to write a linear equation to represent a plane in space

The plane containing the point (x_1, y_1, z_1) and having normal vector $\mathbf{n} = \langle a, b, c\rangle$ can be represented by the **standard form** of the equation of a plane, which is

____

By regrouping terms, you obtain the **general form** of the equation of a plane in space:

____

Given the general form of the equation of a plane it is easy to find a normal vector to the plane, ______________________

____.

Two distinct planes in three-space either are _______________ or _________________________.

If two distinct planes intersect, you can determine the angle θ between them from the angle between their normal vectors. If vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are normal to the two intersecting planes, the angle θ between the normal vectors is equal to the angle between the two planes and is given by

____

Consequently, two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are

1. ____________________ if $\mathbf{n}_1 \bullet \mathbf{n}_2 = 0$.
2. _________________ if $\mathbf{n}_1$ is a scalar multiple of $\mathbf{n}_2$.

III. Sketching Planes in Space (Page 804)

What you should learn
How to sketch the plane given by a linear equation

If a plane in space intersects one of the coordinate planes, the line of intersection is called the _____________ of the given plane in the coordinate plane.

To sketch a plane in space, ______________________________
____
____.

The plane with equation $3y - 2z + 1 = 0$ is parallel to
_____________________.

IV. Distances Between Points, Planes, and Lines (Pages 805–807)

What you should learn
How to find the distances between points, planes, and lines in space

The **distance between a plane and a point** Q (not in the plane) is

____

where P is a point in the plane and $\mathbf{n}$ is normal to the plane.

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Section 11.6 Surfaces in Space

Objective: In this lesson you learned how to recognize and write equations for cylindrical and quadric surfaces, and surfaces of revolution.

I. Cylindrical Surfaces (Pages 812–813)

What you should learn
How to recognize and write equations for cylindrical surfaces

Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a ________________. C is called the ______________ __________________________ of the cylinder, and the parallel lines are called ___________________.

The equation of a cylinder whose rulings are parallel to one of the coordinate axes contains only ________________________ ____.

II. Quadric Surfaces (Pages 813–817)

What you should learn
How to recognize and write equations for quadric surfaces

Quadric surfaces are __________________________________ ________________________________.

The equation of a **quadric surface** in space is _______ ____. The general form of the equation is ________________________ ___________________________________. There are six basic types of quadric surfaces: _____________________________ ____ ____.

The intersection of a surface with a plane is called ___________ ______________________________________. To visualize a surface in space, it is helpful to ______________________ _____________________________. The traces of quadric surfaces are __________________.

To classify a quadric surface, ____________________________

____

____

____. For a quadric surface not centered at the origin, you can form the standard equation by ____________________________________.

Example 1: Classify and name the center of the surface given by $4x^2 + 36y^2 - 9z^2 + 8x - 144y + 18z + 139 = 0$.

III. Surfaces of Revolution (Page 818–819)

> ***What you should learn***
> How to recognize and write equations for surfaces of revolution

Consider the graph of the radius function $y = r(z)$ in the yz-plane. If this graph is revolved about the z-axis, it forms a ____________________________________. The trace of the surface in the plane $z = z_0$ is a circle whose radius is $r(z_0)$ and whose equation is ____________________________ .

If the graph of a radius function r is revolved about one of the coordinate axes, the equation of the resulting surface of revolution has one of the following forms.

1. Revolved about the ___________________: $y^2 + z^2 = [r(x)]^2$
2. Revolved about the ___________________: $x^2 + z^2 = [r(y)]^2$
3. Revolved about the ___________________: $x^2 + y^2 = [r(z)]^2$

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Section 11.7 Cylindrical and Spherical Coordinates

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Objective: In this lesson you learned how to use cylindrical or spherical coordinates to represent surfaces in space.

What you should learn How to use cylindrical coordinates to represent surfaces in space

I. Cylindrical Coordinates (Pages 822–824)

The **cylindrical coordinate system** is an extension of ________ ____.

In a **cylindrical coordinate system,** a point P in space is represented by an ordered triple ____________________. (r, θ) is a polar representation of ______________________________ ____________________. z is the directed distance from _______ _________________.

To convert from rectangular to cylindrical coordinates, or vice versa, use the following conversion guidelines for polar coordinates.

Cylindrical to rectangular:

_______________ _______________ _______________

Rectangular to cylindrical:

_______________ _______________ _______________

The point (0, 0, 0) is called the _________________. Because the representation of a point in the polar coordinate system is not unique, it follows that __________________________________ ____.

Example 1: Convert the point $(r, \theta, z) = \left(2, \dfrac{\pi}{2}, 5\right)$ to rectangular coordinates.

Cylindrical coordinates are especially convenient for representing ____ ____.

Give an example of a cylindrical coordinate equation for a vertical plane containing the z-axis. ____________________

Give an example of a cylindrical coordinate equation for a horizontal plane. ____________________

II. Spherical Coordinates (Pages 825–826)

What you should learn
How to use spherical coordinates to represent surfaces in space

In a spherical coordinate system, a point P in space is represented by an ordered triple ____________________.

1. ρ is the distance between ______________________________.
2. θ is the same angle used in ______________________________ _____________________________________.
3. ϕ is the angle between _________________________________ ____.

To convert from spherical to rectangular coordinates, use:

_________________ ___________________ ____________

To convert from rectangular to spherical coordinates, use:

_________________ ______________ _________________

To convert from spherical to cylindrical coordinates ($r \geq 0$), use:

__________________ ___________ ___________________

To convert from cylindrical to spherical coordinates ($r \geq 0$), use:

_______________ ___________ ______________________

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Chapter 12 Vector-Valued Functions

Section 12.1 Vector-Valued Functions

Course Number
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Objective: In this lesson you learned how to analyze and sketch a space curve represented by a vector-valued function and how to apply the concepts of limits and continuity to vector-valued functions.

I. Space Curves and Vector-Valued Functions (Pages 834–836)

What you should learn How to analyze and sketch a space curve given by a vector-valued function

A **space curve** *C* is the set of all ordered triples____________ ______ together with their defining parametric equations ______ ______________________________ where *f*, *g*, and *h* are continuous functions of *t* on an interval *I*.

A function of the form $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ in a plane or $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ in space is a ____________________ ______________, where the **component functions** *f*, *g*, and *h* are real-valued functions of the parameter *t*. Vector-valued functions are sometimes denoted as _________________________ or ____ ________________________________.

Vector-valued functions serve dual roles in the representation of curves. By letting the parameter *t* represent time, you can use a vector-valued function to represent _______________________ ___________. Or, in the more general case, you can use a vector-valued function to __________________________________. In either case, the terminal point of the position vector **r**(*t*) coincides with ____ _____________________________________. The arrowhead on the curve indicates the curve's __________________ by pointing in the direction of increasing values of *t*.

Unless stated otherwise, the **domain** of a vector-valued function **r** is considered to be ____________________________________ ____.

II. Limits and Continuity (Pages 837–838)

> ***What you should learn***
> How to extend the concepts of limits and continuity to vector-valued functions

Definition of the Limit of a Vector-Valued Function

1. If **r** is a vector-valued function such that $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$,

   then ____ ,

   provided f and g have limits as $t \to a$.
2. If **r** is a vector-valued function in space such that $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, then

   ____ ,

   provided f, g, and h have limits as $t \to a$.

If $\mathbf{r}(t)$ approaches the vector **L** as $t \to a$, the length of the vector $\mathbf{r}(t) - \mathbf{L}$ approaches ____________.

A vector-valued function **r** is **continuous at the point** given by $t = a$ if ____ ______________________________. A vector-valued function **r** is **continuous on an interval** I if ______________________________ ____.

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Section 12.2 Differentiation and Integration of Vector-Valued Functions

Objective: In this lesson you learned how to differentiate and integrate vector-valued functions.

I. Differentiation of Vector-Valued Functions
(Pages 842–845)

What you should learn How to differentiate a vector-valued function

The **derivative of a vector-valued function r** is defined by

________________________________ for all t for

which the

limit exists. If $\mathbf{r}'(t)$ exists, then **r** is ____________________.

If $\mathbf{r}'(t)$ exists for all t in an open interval I, then **r** is

____.

Differentiability of vector-valued functions can be extended to

closed intervals by ________________________________.

If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$, where f and g are differentiable functions of

t, then ____________________________________.

If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, where f, g, and h are differentiable

functions of t, then ________________________________.

Example 1: Find $\mathbf{r}'(t)$ for the vector-valued function given by $\mathbf{r}(t) = (1 - t^2)\mathbf{i} + 5\mathbf{j} + \ln t\mathbf{k}$.

The parameterization of the curve represented by the vector-

valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ is **smooth on an open**

interval I if ____

____.

Let **r** and **u** be differentiable vector-valued functions of t, let w be a differentiable real-valued function of t, and let c be a scalar.

1. $D_t[c\mathbf{r}(t)]$ = ______________________.
2. $D_t[\mathbf{r}(t) \pm \mathbf{u}(t)]$ = ___________________________.

3. $D_t[w(t)\mathbf{r}(t)] =$ ____________________________________.

4. $D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] =$ ____________________________________.

5. $D_t[\mathbf{r}(t) \times \mathbf{u}(t)] =$ ____________________________________.

6. $D_t[\mathbf{r}(w(t))] =$ ______________________________.

7. If $\mathbf{r}(t) \cdot \mathbf{r}(t) = c$, then ______________________________.

II. Integration of Vector-Valued Functions (Pages 846–847)

What you should learn
How to integrate a vector-valued function

If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$, where f and g are continuous on $[a, b]$, then the ____ is

$$\int \mathbf{r}(t)\,dt = \left[\int f(t)\,dt\right]\mathbf{i} + \left[\int g(t)\,dt\right]\mathbf{j}$$ and its **definite integral**

over the interval ____________________ is

$$\int_a^b \mathbf{r}(t)\,dt = \left[\int_a^b f(t)\,dt\right]\mathbf{i} + \left[\int_a^b g(t)\,dt\right]\mathbf{j}.$$

If $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, where f, g, and h are continuous on $[a, b]$, then the ____________________________________ is

$$\int \mathbf{r}(t)\,dt = \left[\int f(t)\,dt\right]\mathbf{i} + \left[\int g(t)\,dt\right]\mathbf{j} + \left[\int h(t)\,dt\right]k$$ and its

definite integral over the interval ____________________ is

$$\int_a^b \mathbf{r}(t)\,dt = \left[\int_a^b f(t)\,dt\right]\mathbf{i} + \left[\int_a^b g(t)\,dt\right]\mathbf{j} + \left[\int_a^b h(t)\,dt\right]k.$$

The antiderivative of a vector-valued function is a family of vector-valued functions all differing by _____________

__________________________.

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Section 12.3 Velocity and Acceleration

Objective: In this lesson you learned how to describe the velocity and acceleration associated with a vector-valued function and how to use a vector-valued function to analyze projectile motion.

I. Velocity and Acceleration (Pages 850–853)

> ***What you should learn***
> How to describe the velocity and acceleration associated with a vector-valued function

If x and y are twice-differentiable function of t, and $\mathbf{r}$ is a vector-valued function given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, then the velocity vector, acceleration vector, and speed at time t are as follows.

1. **Velocity** = $\mathbf{v}(t)$ =_________________________________.
2. **Acceleration** = $\mathbf{a}(t)$ =_______________________________.
3. **Speed** = $\|\mathbf{v}(t)\|$ = _______________________________.

List the corresponding definitions for velocity, acceleration, and speed along a space curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$.

Example 1: Find the velocity vector and acceleration vector of a particle that moves along the plane curve C given by $\mathbf{r}(t) = \cos t\mathbf{i} - 2t\mathbf{j}$.

II. Projectile Motion (Pages 854–855)

> ***What you should learn***
> How to use a vector-valued function to analyze projectile motion

Neglecting air resistance, the path of a projectile launched from an initial height h with initial speed v_0 and angle of elevation θ is described by the vector function

____

where g is the acceleration due to gravity.

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Section 12.4 Tangent Vectors and Normal Vectors

Objective: In this lesson you learned how to find tangent vectors and normal vectors.

I. Tangent Vectors and Normal Vectors (Pages 859–862)

> ***What you should learn***
> How to find a unit tangent vector at a point on a space curve

Let C be a smooth curve represented by $\mathbf{r}$ on an open interval I. The **unit tangent vector** $\mathbf{T}(t)$ at t is defined to be ____________ ____.

Recall that a curve is smooth on an interval if ________________ ____. So, "smoothness" is sufficient to guarantee that _________________ ___________________________________.

The tangent line to a curve at a point is ____________________ ____.

Let C be a smooth curve represented by $\mathbf{r}$ on an open interval I. If $\mathbf{T}'(t) \neq \mathbf{0}$, then the **principal unit normal vector** at t is defined to be ____.

II. Tangential and Normal Components of Acceleration (Pages 862–865)

> ***What you should learn***
> How to find the tangential and normal components of acceleration

For an object traveling at a constant speed, the velocity and acceleration vectors _______________________________. For an object traveling at a variable speed, the velocity and acceleration vectors ____.

If $\mathbf{r}(t)$ is the position vector for a smooth curve C and $\mathbf{N}(t)$ exists, then the acceleration vector $\mathbf{a}(t)$ lies ________________________ ____.

If $\mathbf{r}(t)$ is the position vector for a smooth curve C [for which $\mathbf{N}(t)$ exists], then the **tangential component of acceleration** a_{T} and the **normal component of acceleration** a_{N} are as follows.

$a_{\mathbf{T}} =$ ________________________________

$a_{\mathbf{N}} =$ ____

Note that $a_{\mathbf{N}} \geq 0$. The normal component of acceleration is also called the ____.

Additional notes

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Section 12.5 Arc Length and Curvature

Objective: In this lesson you learned how to find the arc length and curvature of a curve.

I. Arc Length and Curvature (Pages 869–870)

What you should learn How to find a unit tangent vector at a point on a space curve

If C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, on an interval $[a, b]$, then the arc length of C on the interval is

____.

Example 1: Find the arc length of the curve given by $\mathbf{r}(t) = \sin t\mathbf{i} - 2t\mathbf{j} + t^2\mathbf{k}$, from $t = 0$ to $t = 4$.

II. Arc Length Parameter (Pages 870–871)

What you should learn How to find the tangential and normal components of acceleration

Let C be a smooth curve given by $\mathbf{r}(t)$ defined on the closed interval $[a, b]$. For $a \le t \le b$, the arc length function is given by

____.

The arc length s is called the ______________________________.

If C is a smooth curve given by $\mathbf{r}(s) = x(s)\mathbf{i} + y(s)\mathbf{j}$ or $\mathbf{r}(s) = x(s)\mathbf{i} + y(s)\mathbf{j} + z(s)\mathbf{k}$ where s is the arc length parameter, then ____________________________. Moreover, if t is *any* parameter for the vector-valued function $\mathbf{r}$ such that $\|\mathbf{r}'(t)\| = 1$, then t ____.

III. Curvature (Pages 872–875)

What you should learn How to find the tangential and normal components of acceleration

Curvature is the measure of ______________________________ ____________________.

Let C be a smooth curve (in the plane or in space) given by $\mathbf{r}(s)$, where s is the arc length parameter. The **curvature** K at s is given by ____________________________.

Describe the curvature of a circle.

.

If C is a smooth curve given by $\mathbf{r}(t)$, then two additional formulas for finding the curvature K of C at t are

$K =$ ________________________________, or

$K =$ ____.

If C is the graph of a twice-differentiable function given by $y = f(x)$, then the curvature K at the point (x, y) is given by

$K =$ ____.

Let C be a curve with curvature K at point P. The circle passing through point P with radius $r = 1/K$ is called the **circle of curvature** if ____
____
________________________. The radius is called the ________ __________________ at P, and the center of the circle is called the __________________________________.

If $\mathbf{r}(t)$ is the position vector for a smooth curve C, then the acceleration vector is given by

__________________________________, where K is the curvature of C and ds/dt is the speed.

IV. Application (Pages 876–877)

> ***What you should learn***
> How to find the tangential and normal components of acceleration

A moving object with mass m is in contact with a stationary object. The total force required to produce an acceleration **a** along a given path is

The portion of this total force that is supplied by the stationary object is called the ______________________________.

Additional notes

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Chapter 13 Functions of Several Variables

Section 13.1 Introduction to Functions of Several Variables

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Objective: In this lesson you learned how to sketch a graph, level curves, and level surfaces.

Important Vocabulary Define each term or concept.

Function of two variables

Domain of a function of two variables

Range of a function of two variables

I. Functions of Several Variables (Pages 886–887)

What you should learn
How to understand the notation for a function of several variables

For the function given by $z = f(x, y)$, x and y are called the ________________________________ and z is called the ____________________ of f.

Example 1: For $f(x, y) = \sqrt{100 - 2x^2 - 6y}$, evaluate $f(3, 3)$.

Example 2: For $f(x, y, z) = 2x + 5y^2 - z^3$, evaluate $f(4, 3, 2)$.

II. The Graph of a Function of Two Variables (Page 888)

What you should learn
How to sketch the graph of a function of two variables

The **graph** of a function f of two variables is ____________ ____ ____________________________________.

The graph of $z = f(x, y)$ is a surface whose projection onto the xy-plane is ________________________________. To each point (x, y) in D there corresponds ________________________ _________________, and conversely, to each point (x, y, z) on the surface there corresponds ____________________________.

To sketch a surface in space by hand, it helps to use __________ ____.

III. Level Curves (Pages 889–891)

> ***What you should learn***
> How to sketch level curves for a function of two variables

A second way to visualize a function of two variables is to use a ___________________________ in which the scalar $z = f(x, y)$ is assigned to the point (x, y). A scalar field can be characterized by ______________________ or ______________________ along which the value of $f(x, y)$ is _________________________.

Name a few applications of level curves.

A contour map depicts ____________________________________ ____. Much space between level curves indicates that ____________________ _________________, whereas little space indicates ___________ _______________________.

What is the **Cobb-Douglas production function**?

Let x measure the number of units of labor and let y measure the number of units of capital. Then the number of units produced is modeled by the function

____

Example 3: A manufacturer estimates that its production (measured in units of a product) can be modeled by $f(x, y) = 400x^{0.3}y^{0.7}$, where the labor x is measured in person-hours and the capital y is measured in thousands of dollars. What is the production level when $x = 500$ and $y = 200$?

IV. Level Surfaces (Pages 891–892)

> ***What you should learn***
> How to sketch level curves for a function of three variables

The concept of a level curve can be extended by one dimension to define a _______________________________. If f is a function of three variables and c is a constant, the graph of the equation $f(x, y, z) = c$ is ______________________________ ____________________________.

V. Computer Graphics (Pages 892–893)

> ***What you should learn***
> How to use computer graphs to graph a function of two variables

The problem of sketching the graph of a surface can be simplified by ______________________________.

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Section 13.2 Limits and Continuity

Objective: In this lesson you learned how to find a limit and determine continuity.

I. Neighborhoods in the Plane (Page 898)

> ***What you should learn***
> How to understand the definition of a neighborhood in the plane

Using the formula for the distance between two points (x, y) and (x_0, y_0) in the plane, you can define the δ-neighborhood about (x_0, y_0) to be ____.

When this formula $\left\{(x, y): \sqrt{(x-x_0)^2+(y-y_0)^2}<\delta\right\}$

contains the less than inequality, <, the disk is called ________________. When it contains the less than or equal to inequality, ≤, the disk is called ____________________.

A point (x_0, y_0) in a plane region R is an **interior point** of R if there exists ____ ____________________________. If every point in R is an interior point, then R is ____________________. A point (x_0, y_0) is a **boundary point** of R if ______________________ ____ ________________________. By definition, a region must contain its interior points, but it need not contain ____________ ____________________. If a region contains all its boundary points, the region is ________________. A region that contains some but not all of its boundary points is __________________ ________________________.

II. Limit of a Function of Two Variables (Pages 899–901)

> ***What you should learn***
> How to understand and use the definition of the limit of a function of two variables

Let f be a function of two variables defined, except possibly at (x_0, y_0), on an open disk centered at (x_0, y_0), and let L be a real number. Then $\lim_{(x,y)\to(x_0,y_0)} f(x,y) =$ ______________ if for each

$\varepsilon > 0$ there corresponds ____________________ such that

$|f(x,y) - L| < \varepsilon$ whenever $0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$.

For a function of two variables, the statement $(x, y) \to (x_0, y_0)$ means ____

____. If the value of $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ is not the same for all possible approaches, or **paths,** to (x_0, y_0), ______________________

_____________.

Example 1: Evaluate $\lim\limits_{(x,y)\to(4,-1)} \dfrac{x^2 + 16y}{3x - 4y}$.

III. Continuity of a Function of Two Variables (Pages 902–903)

> ***What you should learn***
> How to extend the concept of continuity to a function of two variables

A function f of two variables is **continuous at a point (x_0, y_0)** in an open region R if ____________________________________

____.

The function f is ____________________________________ if it is continuous at every point in R.

Discuss the difference between **removable** and **nonremovable** discontinuities.

If k is a real number and f and g are continuous at (x_0, y_0), then the following functions are continuous at (x_0, y_0).

1.

2.

3.

4.

If h is continuous at (x_0, y_0) and g is continuous at $h(x_0, y_0)$, then the composite function given by $(g \circ h)(x, y) = g(h(x, y))$ is

____. That is,

$$\lim_{(x,y)\to(x_0,y_0)} g(h(x,y)) = g(h(x_0,y_0)).$$

IV. Continuity of a Function of Three Variables (Page 904)

What you should learn
How to extend the concept of continuity to a function of three variables

A function f of three variables is **continuous at a point** $(\boldsymbol{x_0, y_0, z_0})$ in an open region R if ____________________________

____

____.

That is, $\lim_{(x,y,z)\to(x_0,y_0,z_0)} f(x,y,z) = f(x_0,y_0,z_0)$. The function f is

____ if it is continuous at every point in R.

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Section 13.3 Partial Derivatives

Objective: In this lesson you learned how to find and use a partial derivative.

I. Partial Derivatives of a Function of Two Variables
(Pages 908–911)

What you should learn
How to find and use partial derivatives of a function of two variables

The process of determining the rate of change of a function *f* with respect to one of its several independent variables is called ______________________________________, and the result is referred to as the ___________________________ of *f* with respect to the chosen independent variable.

If $z = f(x, y)$, then the **first partial derivatives** of *f* with respect to *x* and *y* are the functions f_x and f_y, defined by

$$f_x(x, y) = \lim_{\Delta x \to 0} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$f_y(x, y) = \lim_{\Delta y \to 0} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

provided the limit exists.

This definition indicates that if $z = f(x, y)$, then to find f_x, you consider ____ _________________. Similarly, to find f_y, you consider _______ ____.

List the equivalent ways of denoting the first partial derivatives of $z = f(x, y)$ with respect to *x*.

List the equivalent ways of denoting the first partial derivatives of $z = f(x, y)$ with respect to *y*.

The values of the first partial derivatives at the point (a, b) are denoted by

$$\left.\frac{\partial z}{\partial x}\right|_{(a,b)} = \underline{\hspace{3cm}} \qquad \text{and} \qquad \left.\frac{\partial z}{\partial y}\right|_{(a,b)} = \underline{\hspace{3cm}}$$

Example 1: Find $\partial z/\partial x$ for the function $z = 20 - 2x^2 + 3xy + 5x^2y^2$.

For the function $z = f(x, y)$, if $y = y_0$, then $z = f(x, y_0)$ represents the curve formed by intersecting ________________ ___________________________________. On this curve, the partial derivative $f_x(x_0, y_0)$ represents ______________________ ____.

Informally, the values of $\partial f / \partial x$ and $\partial f / \partial y$ at the point (x_0, y_0, z_0) denote ______________________________________ ____, respectively.

Example 2: Find the slope of the surface given by $z = 20 - 2x^2 + 3xy + 5x^2y^2$ at the point (1, 1, 26) in the y-direction.

II. Partial Derivatives of a Function of Three or More Variables (Pages 911–912)

What you should learn
How to find and use partial derivatives of a function of three or more variables

The function $w = f(x, y, z)$ has _______________ partial derivatives, each of which is formed by _____________________ ___________________________________.

The partial derivative of w with respect to x is denoted by _______________________________. To find the partial derivative of w with respect to x, consider __________________ to be constant and differentiate with respect to ____________.

What you should learn
How to find higher-order partial derivatives of a function of two or three variables

III. Higher-Order Partial Derivatives (Pages 912–913)

As with ordinary derivatives, it is possible to take __________ __________________________ partial derivatives of a function of several variables, provided such derivatives exist. Higher-order derivatives are denoted by ______________________________ ______________________________.

The notation $\frac{\partial^2 f}{\partial x \partial y}$ indicates to differentiate first with respect to __________ and then with respect to __________.

The notation $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ indicates to differentiate first with respect to __________ and then with respect to __________.

The notation f_{yx} indicates to differentiate first with respect to __________ and then with respect to __________.

The cases represented in the three examples of notation given above are called ____.

Example 3: Find the value of $f_{xy}(2, -3)$ for the function $f(x, y) = 20 - 2x^2 + 3xy + 5x^2y^2$.

If f is a function of x and y such that f_{xy} and f_{yx} are continuous on an open disk R, then, for every (x, y) in R,
$f_{xy}(x, y) =$ ____________________________.

Additional notes

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Section 13.4 Differentials

Objective: In this lesson you learned how to find and use a total differential and determine differentiability.

I. Increments and Differentials (Page 918)

> ***What you should learn***
> How to understand the concepts of increments and differentials

The **increments of x and y** are __________________, and the **increment of z** is given by ______________________________ __________________.

If $z = f(x, y)$ and Δx and Δy are increments of x and y, then the **differentials** of the independent variables x and y are __________ ____________________, and the total differential of the dependent variable z is

____ .

II. Differentiability (Page 919)

> ***What you should learn***
> How to extend the concept of differentiability to a function of two variables

A function f given by $z = f(x, y)$ is **differentiable** at (x_0, y_0) if Δz can be written in the form

____

where both ε_1 and $\varepsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$. The function f is ______________________________________ if it is differentiable at each point in R.

If f is a function of x and y, where f_x and f_y are continuous in an open region R, then f is _________________________________.

III. Approximation by Differentials (Pages 920–922)

> ***What you should learn***
> How to use a differential as an approximation

The partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$ can be interpreted as ____.

This means that $dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$ represents ______________

____

__________________________________. Because a plane in space is represented by a linear equation in the variables x, y, and z, the approximation of Δz by dz is called a _________________ ____________________.

If a function of x and y is differentiable at (x_0, y_0), then ________ ____.

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Section 13.5 Chain Rules for Functions of Several Variables

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Objective: In this lesson you learned how to use the Chain Rules and find a partial derivative implicitly.

I. Chain Rules for Functions of Several Variables
(Pages 925–929)

What you should learn
How to use the Chain Rules for functions of several variables

Let $w = f(x, y)$, where *f* is a differentiable function of *x* and *y*.

The Chain Rule for One Independent Variable states that _______
____
____
____.

Example 1: Let $w = 2xy + 3xy^3$, where $x = 1 - 2t$ and $y = 2\sin t$. Find *dw*/*dt*.

Let $w = f(x, y)$, where *f* is a differentiable function of *x* and *y*.

The Chain Rule for Two Independent Variables states that _____
____
____
____
____
____.

II. Implicit Partial Differentiation (Pages 929–930)

What you should learn
How to find partial derivatives implicitly

If the equation $F(x, y) = 0$ defines *y* implicitly as a differentiable function of *x*, then $\dfrac{dy}{dx} =$ ____________________, $F_y(x, y) \neq 0$. If the equation $F(x, y, z) = 0$ defines *z* implicitly as a differentiable function of x and y, then

$\frac{\partial z}{\partial x} =$ ______________________________________, and

$\frac{\partial z}{\partial y} =$ ______________________________________,

$F_z(x, y, z) \neq 0$.

Example 2: Find dy/dx, given $2x^2 + xy + y^2 - x - 2y = 0$.

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Section 13.6 Directional Derivatives and Gradients

Objective: In this lesson you learned how to find and use a directional derivative and a gradient.

I. Directional Derivative (Pages 933–935)

What you should learn
How to find and use directional derivatives of a function of two variables

Let f be a function of two variables x and y, and let $\mathbf{u} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$ be a unit vector. Then the ________________ ____, denoted by $D_{\mathbf{u}}f$, is $D_{\mathbf{u}}f(x, y) = \lim_{t\to 0} \frac{f(x + t\cos\theta, y + t\sin\theta) - f(x, y)}{t}$, provided this limit exists.

A simpler working formula for finding a directional derivative states that if f is a differentiable function of x and y, then the directional derivative of f in the direction of the unit vector $\mathbf{u} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$ is ___________________________________.

II. The Gradient of a Function of Two Variables (Pages 936–937)

What you should learn
How to find the gradient of a function of two variables

Let $z = f(x, y)$ be a function of x and y such that f_x and f_y exist. Then the **gradient of *f*,** denoted by _________________, is the vector ____.
Note that for each (x, y), the gradient $\nabla f(x, y)$ is a vector in _____________________________________.

If f is a differentiable function of x and y, then the directional derivative of f in the direction of the unit vector $\mathbf{u}$ is __________________________________.

III. Applications of the Gradient (Pages 937–940)

What you should learn
How to use the gradient of a function of two variables in applications

In many applications, you may want to know in which direction to move so that $f(x, y)$ increases most rapidly. This direction is

called ____,
and it is given by the ______________________________.

Let f be differentiable at the point (x, y). State three properties of the gradient at that point.

1.

2.

3.

If f is differentiable at (x_0, y_0) and $\nabla f(x_0, y_0) \neq \mathbf{0}$, then $\nabla f(x_0, y_0)$ is
____.

IV. Functions of Three Variables (Page 941)

What you should learn
How to find directional derivatives and gradients of functions of three variables

Let f be a function of x, y, and z, with continuous first partial derivatives. The **directional derivative of *f*** in the direction of a unit vector $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is given by
____.

The **gradient of *f*** is defined to be __________________________
_____________________________________.

Properties of the gradient are as follows.

1.

2.

3.

4.

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Section 13.7 Tangent Planes and Normal Lines

Objective: In this lesson you learned how to find and use a directional derivative and a gradient.

What you should learn How to find equations of tangent planes and normal lines to surfaces

I. Tangent Plane and Normal line to a Surface (Pages 945–949)

For a surface S given by $z = f(x, y)$, you can convert to the general form by defining F as $F(x, y, z) =$ _________________.

Because $f(x, y) - z = 0$, you can consider S to be __________ ____.

Example 1: For the function given by $F(x, y, z) = 12 - 3x^2 + y^2 - 4z^2$, describe the level surface given by $F(x, y, z) = 0$.

Let F be differentiable at the point $P(x_0, y_0, z_0)$ on the surface S given by $F(x, y, z) = 0$ such that $\nabla F(x_0, y_0, z_0) \neq \mathbf{0}$.

1. The plane through P that is normal to $\nabla F(x_0, y_0, z_0)$ is called ____________________________________.
2. The line through P having the direction of $\nabla F(x_0, y_0, z_0)$ is called ____.

If F is differentiable at (x_0, y_0, z_0), then an equation of the tangent plane to the surface given by $F(x, y, z) = 0$ at (x_0, y_0, z_0) is

____

To find the equation of the tangent plane at a point on a surface given by $z = f(x, y)$, you can define the function F by $F(x, y, z) = f(x, y) - z$. Then S is given by the level surface $F(x, y, z) = 0$, and an equation of the tangent plane to S at the point (x_0, y_0, z_0) is

____

II. The Angle of Inclination of a Plane (Pages 949–950)

> ***What you should learn***
> How to find the angle of inclination of a plane in space

Another use of the gradient $\nabla F(x, y, z)$ is ________________

____.

The **angle of inclination** of a plane is defined to be __________

____

________________. The angle of inclination of a horizontal plane is defined to be ______________. Because the vector **k** is normal to the *xy*-plane, you can use the formula for the cosine of the angle between two planes to conclude that the angle of inclination of a plane with normal vector **n** is given by ________

____.

III. A Comparison of the Gradients $\nabla f(x, y)$ and $\nabla F(x, y, z)$ (Page 950)

> ***What you should learn***
> How to compare the gradients $\nabla f(x, y)$ and $\nabla F(x, y, z)$

If F is differentiable at (x_0, y_0, z_0) and $\nabla F(x_0, y_0, z_0) \neq \mathbf{0}$, then $\nabla F(x_0, y_0, z_0)$ is ____________________ to the level surface through (x_0, y_0, z_0).

When working with the gradients $\nabla f(x, y)$ and $\nabla F(x, y, z)$, be sure to remember that $\nabla f(x, y)$ is a vector in ______________________ and $\nabla F(x, y, z)$ is a vector in ________________________.

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Section 13.8 Extrema of Functions of Two Variables

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Objective: In this lesson you learned how to find absolute and relative extrema.

What you should learn
How to find absolute and relative extrema of a function of two variables

I. Absolute Extrema and Relative Extrema (Pages 954–956)

Let *f* be a continuous function of two variables *x* and *y* defined on a closed bounded region *R* in the *xy*-plane. The Extreme Value Theorem states that ______________________________ ______________________________ ______________________________.

Let *f* be a function defined on a region *R* containing (x_0, y_0). The function *f* has a **relative maximum** at (x_0, y_0) if ____________ ______________________________ .

The function *f* has a **relative minimum** (x_0, y_0) if ____________ ______________________________ .

To say that *f* has a relative maximum at (x_0, y_0) means that the point (x_0, y_0, z_0) is ______________________________ ______________________________.

Let *f* be defined on an open region *R* containing (x_0, y_0). The point (x_0, y_0) is a **critical point** of *f* if one of the following is true.

1.

2.

If *f* has a relative extremum at (x_0, y_0) on an open region *R*, then (x_0, y_0) is a ______________________________.

Example 1: Find the relative extrema of
$f(x, y) = 3x^2 + 2y^2 - 36x + 24y - 9$.

II. The Second Partials Test (Pages 957–959)

What you should learn
How to use the Second Partials Test to find relative extrema of a function of two variables

The critical points of a function of two variables do not always yield relative maximum or relative minima. Some critical points yield ____________________________, which are neither relative maxima nor relative minima.

For the Second-Partials Test for Relative Extrema, let f have continuous second partial derivatives on an open region containing (a, b) for which $f_x(a, b) = 0$ and $f_y(a, b) = 0$. To test for relative extrema of f, consider the quantity

$$d = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2.$$

1. If $d > 0$ and $f_{xx}(a, b) > 0$, then f has __________________ ________________________.
2. If $d > 0$ and $f_{xx}(a, b) < 0$, then f has __________________ ________________________.
3. If $d < 0$, then ____________________________________.
4. If $d = 0$, then _________________________________.

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Section 13.9 Applications of Extrema of Functions of Two Variables

Course Number

Instructor

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Objective: In this lesson you learned how to solve an optimization problem and how to use the method of least squares.

I. Applied Optimization Problems (Pages 962–963)

What you should learn
How to solve optimization problems involving functions of several variables

Give an example of a real-life situation in which extrema of functions of two variables play a role.

Describe the process used to optimize the function of two or more variables.

In many applied problems, the domain of the function to be optimized is a closed bounded region. To find minimum or maximum points, you must not only test critical points, but also

____

__________________________________.

II. The Method of Least Squares (Pages 964–966)

What you should learn
How to use the method of least squares

In constructing a model to represent a particular phenomenon, the goals are ____________________________________.

As a measure of how well the model $y = f(x)$ fits the collection of points $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$, ______________________

____

__________________________________. This sum is called the ____________________________________ and is given by

______________________________. Graphically, S can be

interpreted as ____

____

______________________. If the model is a perfect fit, then

$S =$ _____________. However, when a perfect fit is not feasible,

you can settle for a model that ___________________________.

The linear model that minimizes S is called ______________________

___________________________.

The **least squares regression line** for $\{(x_1, y_1), (x_2, y_2), \ldots,$

$(x_n, y_n)\}$ is given by _________________________, where

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \quad \text{and} \quad b = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i\right).$$

Example 1: Find the least squares regression line for the data in the table.

x	1	3	4	8	11	12
y	16	21	24	27	29	33

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Section 13.10 Lagrange Multipliers

Objective: In this lesson you learned how to solve a constrained optimization problem using a Lagrange multiplier.

What you should learn
How to understand the Method of Lagrange Multipliers

I. Lagrange Multipliers (Pages 970–971)

The ____ offers a way to solve constrained optimization problems.

Let f and g have continuous first partial derivatives such that f has an extremum at a point (x_0, y_0) on the smooth constraint curve $g(x, y) = c$. Lagrange's Theorem states that if $\nabla g(x_0, y_0) \neq \mathbf{0}$, then there is a real number λ such that ____________________________.

The scalar λ, the lowercase Greek letter lambda, is called a ___________________________________.

Let f and g satisfy the hypothesis of Lagrange's Theorem, and let f have a minimum or maximum subject to the constraint $g(x, y) = c$. To find the minimum or maximum of f, use the following steps.

1.

2.

II. Constrained Optimization Problems (Pages 972–974)

> ***What you should learn***
> How to use Lagrange multipliers to solve constrained optimization problems

Economists call the Lagrange multiplier obtained in a production function the ____, which tells the number of additional units of product that can be produced if one additional dollar is spent on production.

III. The Method of Lagrange Multipliers with Two Constraints (Page 975)

> ***What you should learn***
> How to use the Method of Lagrange Multipliers with two constraints

For an optimization problem involving two constraint functions *g* and *h*, you need to introduce __________________________ ____________________, and then solve the equation ________________________________.

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Chapter 14 Multiple Integration

Section 14.1 Iterated Integrals and Area in the Plane

Course Number

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Objective: In this lesson you learned how to evaluate an iterated integral and find the area of a plane region.

I. Iterated Integrals (Pages 984–985)

> ***What you should learn***
> How to evaluate an iterated integral

To extend definite integrals to functions of several variables, you can apply the Fundamental Theorem of Calculus to one variable while ____.

An "integral of an integral" is called a(n) ___________________. The _____________________ limits of integration can be variable with respect to the outer variable of integration. The ______________________ limits of integration must be constant with respect to both variables of integration. The limits of integration for an iterated integral identify two sets of boundary intervals for the variables, which determine the ______________ ___________________ of the iterated integral.

Example 1: Evaluate $\int_{-3}^{0}\int_{0}^{y}(6x-2y)\,dx\,dy$.

II. Area of a Plane Region (Pages 986–989)

> ***What you should learn***
> How to use an iterated integral to find the area of a plane region

One of the applications of the iterated integral is _____________ __________________________________.

When setting up a double integral to find the area of a region in a plane, placing a representative rectangle in the region R helps determine both ____. A vertical rectangle implies the order ______________________, with the inside limits corresponding to the _________________ ___________________________________. This type of region is called ______________________________, because the outside limits of integration represent the ________________ ____________________________. Similarly, a horizontal rectangle implies the order __________________, with the inside limits corresponding to the ________________________ ______________________. This type of region is called ______ ______________________, because the outside limits represent the ____.

Example 2: Use a double integral to find the area of a rectangular region for which the bounds for x are $-6 \le x \le 1$ and the bound for y are $-3 \le y \le 8$.

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Section 14.2 Double Integrals and Volume

Objective: In this lesson you learned how to use a double integral to find the volume of a solid region.

I. Double Integrals and Volume of a Solid Region
(Pages 992–994)

What you should learn How to use a double integral to represent the volume of a solid region

If f is defined on a closed, bounded region R in the xy-plane, then the ____ is given by

$\iint_R f(x,y)\,dA = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(x_i, y_i)\Delta A_i$, provided the limit exists. If the limit exists, then f is ________________________ over R.

A double integral can be used to find the volume of a solid region that lies between __________________________________

____.

If f is integrable over a plane region R and $f(x, y) \geq 0$ for all (x, y) in R, then the volume of the solid region that lies above R and below the graph of f is defined as ________________________ .

II. Properties of Double Integrals (Page 994)

What you should learn How to use properties of double integrals

Let f and g be continuous over a closed, bounded plane region R, and let c be a constant.

1. $\iint_R cf(x,y)\,dA =$ ________ $\iint_R$ ________________________

2. $\iint_R [f(x,y) \pm g(x,y)]\,dA = \iint_R$ ______________ $\iint_R$ ____________

3. $\iint_R f(x,y)\,dA \geq 0$, if _____________________

4. $\iint\limits_R f(x,y)\,dA \geq \iint\limits_R g(x,y)\,dA$, if ______________________

5. $\iint\limits_R f(x,y)\,dA = \iint\limits_{R_1} f(x,y)\,dA + \iint\limits_{R_2} f(x,y)\,dA$, where R is ______
___________________________________ _____ ____________.

III. Evaluation of Double Integrals (Pages 995–999)

What you should learn
How to evaluate a double integral as an iterated integral

Normally, the first step in evaluating a double integral is ______
____.

Explain the meaning of Fubini's Theorem.

In your own words, explain how to find the volume of a solid.

IV. Average Value of a Function (Pages 999–1000)

What you should learn
How to find the average value of a function over a region

If f is integrable over the plane region R, then the _____________
________________________ is $\frac{1}{A}\iint\limits_R f(x,y)\,dA$, where A is the area of R.

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Section 14.3 Change of Variables: Polar Coordinates

Objective: In this lesson you learned how to write and evaluate double integrals in polar coordinates.

I. Double Integrals in Polar Coordinates (Pages 1004–1008)

What you should learn
How to write and evaluate double integrals in polar coordinates

Some double integrals are much easier to evaluate in ________ __________ than in rectangular form, especially for regions such as ____.

A **polar sector** is defined as ____________________________ ________________________________.

Let R be a plane region consisting of all points $(x, y) = (r\cos\theta, r\sin\theta)$ satisfying the conditions $0 \le g_1(\theta) \le r \le g_2(\theta)$, $\alpha \le \theta \le \beta$, where $0 \le (\beta - \alpha) \le 2\pi$. If g_1 and g_2 are continuous on $[\alpha, \beta]$ and f is continuous on R, then

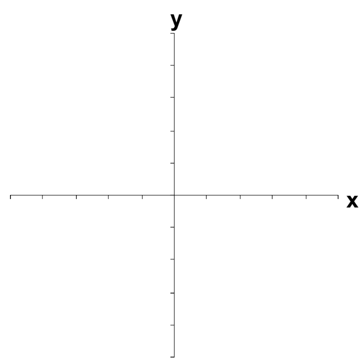
____ .

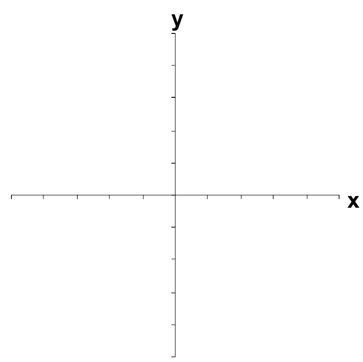
If $z = f(x, y)$ is nonnegative on R, then the integral

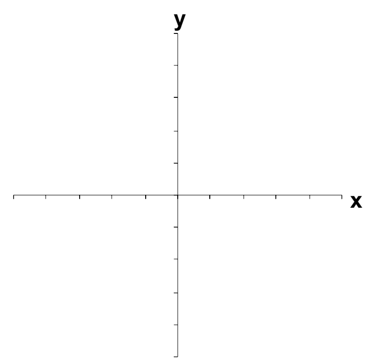
$$\iint_R f(x, y)\, dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$$ can be

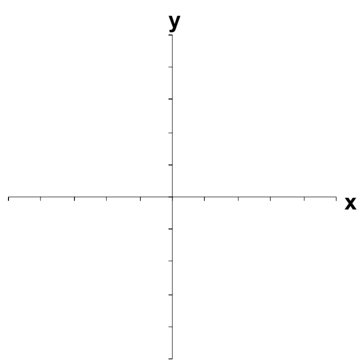
interpreted as the volume of _____________________________ ____.

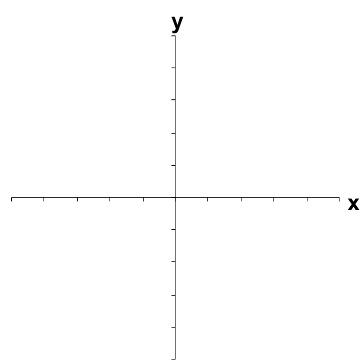
Additional notes

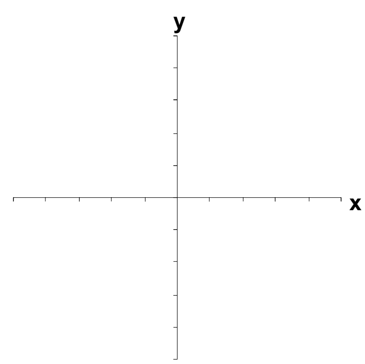












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Section 14.4 Center of Mass and Moments of Inertia

Objective: In this lesson you learned how to find the mass of a planar lamina, the center of mass of a planar lamina, and moments of inertia using double integrals.

I. Mass (Pages 1012–1013)

> ***What you should learn***
> How to find the mass of a planar lamina using a double integral

If ρ is a continuous density function on the lamina (of variable density) corresponding to a plane region R, then the mass m of the lamina is given by ____________________________ .

For a planar lamina, density is expressed as __________________ __________________________.

II. Moments and Center of Mass (Pages 1014–1015)

> ***What you should learn***
> How to find the center of mass of a planar lamina using double integrals

Let ρ be a continuous density function on the planar lamina R. The **moments of mass** with respect to the x- and y-axes are

$M_x =$ ____________________________ and

$M_y =$ ____________________________ . If m is the mass of the lamina, then the **center of mass** is ________________________ .

If R represents a simple plane region rather than a lamina, the point $(\overline{x}, \overline{y})$ is called the ______________________ of the region.

III. Moments of Inertia (Pages 1016–1017)

> ***What you should learn***
> How to find moments of inertia using double integrals

The moments M_x and M_y used in determining the center of mass of a lamina are sometimes called the ________________________ about the x- and y-axes. In each case, the moment is the product of ____. The **second moment,** or the **moment of inertia** of a lamina about a line, is a measure of ____ ______________________________. These second moments are denoted I_x and I_y, and in each case the moment is the product of ____.

$I_x =$ ______________________________ and

$I_y =$ ______________________________. The sum of the

moments I_x and I_y is called the ________________________ and is denoted by I_0.

The moment of inertia I of a revolving lamina can be used to measure its _________________________, which is given by _____________________, where ω is the angular speed, in radians per second, of the planar lamina as it revolves about a line.

The **radius of gyration** $\bar{\bar{r}}$ of a revolving mass m with moment of inertia I is defined to be ________________________.

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Section 14.5 Surface Area

Objective: In this lesson you learned how to use a double integral to find the area of a surface.

What you should learn
How to use a double integral to find the area of a surface

I. Surface Area (Pages 1020–1024)

If f and its first partial derivatives are continuous on the closed region R in the xy-plane, then the **area of the surface S** given by $z = f(x, y)$ over R is given by:

List several strategies for performing the often difficult integration involved in finding surface area.

Additional notes

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Section 14.6 Triple Integrals and Applications

Objective: In this lesson you learned how to use a triple integral to find the volume, center of mass, and moments of inertia of a solid region.

I. Triple Integrals (Pages 1027–1031)

What you should learn
How to use a triple integral to find the volume of a solid region

Consider a function *f* of three variables that is continuous over a bounded solid region *Q*. Then, encompass *Q* with a network of boxes and form the ____________________________ consisting of all boxes lying entirely within *Q*. The norm $||\Delta||$ of the partition is ____
_______________________________________.

If *f* is continuous over a bounded solid region *Q*, then the **triple integral of *f* over *Q*** is defined as

____, provided

the limit exists. The **volume** of the solid region *Q* is given by

______________________________.

Let *f* be continuous on a solid region *Q* defined by $a \le x \le b$, $h_1(x) \le y \le h_2(x)$, and $g_1(x, y) \le z \le g_2(x, y)$, where h_1, h_2, g_1, and g_2 are continuous functions. Then,

____.

To evaluate a triple iterated integral in the order *dz dy dx*, ______
____
____
____.

Describe the process for finding the limits of integration for a triple integral.

II. Center of Mass and Moments of Inertia
(Pages 1032–1034)

> ***What you should learn***
> How to find the center of mass and moments of inertia of a solid region

Consider a solid region Q whose density is given by the density function ρ. The **center of mass** of a solid region Q of mass m is given by $(\bar{x}, \bar{y}, \bar{z})$ where

$m =$ ______________________________

$M_{yz} =$ ______________________________

$M_{xz} =$ ______________________________

$M_{xy} =$ ______________________________

$\bar{x} =$ __________________

$\bar{y} =$ __________________

$\bar{z} =$ __________________

The quantities M_{yz}, M_{xz}, and M_{xy} are called the ________________ __________________ of the region Q about the yz-, xz-, and xy-planes, respectively. The first moments for solid regions are taken about a plane, whereas the second moments for solids are taken about a ______________________. The **second moments** (or **moments of inertia**) about the x-, y-, and z-axes are as follows.

Moment of inertia about the x-axis: $I_x =$ ____

Moment of inertia about the y-axis: $I_y =$ ____

Moment of inertia about the z-axis: $I_z =$ ____

For problems requiring the calculation of all three moments, considerable effort can be saved by applying the additive property of triple integrals and writing

____

where

$I_{xy} =$ ____

$I_{xz} =$ ____

$I_{yz} =$ ____

Additional notes

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Section 14.7 Triple Integrals in Cylindrical and Spherical Coordinates

Objective: In this lesson you learned how to write and evaluate triple integrals in cylindrical and spherical coordinates.

I. Triple Integrals in Spherical Coordinates
(Pages 1038–1040)

> ***What you should learn***
> How to write and evaluate a triple integral in cylindrical coordinates

The rectangular conversion equations for cylindrical coordinates are x = ____________________, y = _________________, and z = ______________________.

If f is a continuous function on the solid Q, the iterated form of the triple integral in cylindrical form is

____.

To visualize a particular order of integration, it helps to view the iterated integral in terms of ________________________________
____.

For instance, in the order $dr\,d\theta\,dz$, the first integration occurs
____
____
____.

II. Triple Integrals in Spherical Coordinates
(Pages 1041–1042)

> ***What you should learn***
> How to write and evaluate a triple integral in spherical coordinates

The rectangular conversion equations for spherical coordinates are x = _________________________, y = ______________________, and z = ____________________________.

The triple integral in spherical coordinates for a continuous function f defined on the solid region Q is given by

____

As with cylindrical coordinates, you can visualize a particular order of integration by ______________________________________ ____ ____.

Additional notes

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Section 14.8 Change of Variables: Jacobians

Objective: In this lesson you learned how to use a Jacobian to change variables in a double integral.

I. Jacobians (Pages 1045–1046)

What you should learn How to understand the concept of a Jacobian

If $x = g(u, v)$ and $y = h(u, v)$, then the **Jacobian** of x and y with respect to u and v, denoted by $\partial(x, y)/\partial(u, v)$, is

.

____

In general, a change of variables is given by a one-to-one transformation T from a region S in the uv-plane to a region R in the xy-plane, to be given by ______________________________ ___________________________, where g and h have continuous first partial derivatives in the region S. In most cases, you are hunting for a transformation in which ______________________ ____.

II. Change of Variables for Double Integrals (Pages 1047–1049)

What you should learn How to use a Jacobian to change variables in a double integral

Let R be a vertically or horizontally simple region in the xy-plane, and let S be a vertically or horizontally simple region in the uv-plane. Let T from S to R be given by $T(u, v) = (x, y) = (g(u, v), h(u, v))$, where g and h have continuous first partial derivatives. Assume that T is one-to-one except possibly on the boundary of S. If f is continuous on R, and $\partial(x, y)/\partial(u, v)$ is nonzero on S, then

____

Additional notes

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Chapter 15 Vector Analysis

Section 15.1 Vector Fields

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Objective: In this lesson you learned how to sketch a vector field, determine whether a vector field is conservative, find a potential function, find curl, and find divergence.

I. Vector Fields (Pages 1058–1061)

What you should learn
How to understand the concept of a vector field

A **vector field over a plane region *R*** is ____________________

____.

A **vector field over a solid region *Q* in space** is _____________

____.

A vector field $\mathbf{F}(x, y, z) = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}$ is **continuous** at a point if and only if __________________________

____.

List some common physical examples of vector fields and give a brief description of each.

Let $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ be a position vector. The vector field **F** is **an inverse square field** if

_________________________________, where *k* is a real number

and **u** = **r**/||**r**|| is a unit vector in the direction of **r**.

Because vector fields consist of infinitely many vectors, it is not possible to create a sketch of the entire field. Instead, when you sketch a vector field, your goal is to ______________________
____.

II. Conservative Vector Fields (Pages 1061–1063)

> ***What you should learn***
> How to determine whether a vector field is conservative

The vector field **F** is called **conservative** if ________________
____. The function f is called the ______________________________ for **F**.

Let M and N have continuous first partial derivatives on an open disk R. The vector field given by $\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$ is conservative if and only if ______________________________ .

III. Curl of a Vector Field (Pages 1064–1065)

> ***What you should learn***
> How to find the curl of a vector field

The **curl** of a vector field $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is

____ .

If curl $\mathbf{F} = \mathbf{0}$, then **F** is said to be _________________________.

The cross product notation use for curl comes from viewing the gradient ∇f as the result of the ____________________________
____________________________ acting on the function f.

The primary use of curl is in a test for conservative vector fields in space. The test states ___________________________________
____
____.

IV. Divergence of a Vector Field (Page 1066)

> ***What you should learn***
> How to find the divergence of a vector field

The curl of a vector field **F** is itself ________________________. Another important function defined on a vector field is **divergence,** which is _________________________________.

The **divergence** of **F**$(x, y) = M\mathbf{i} + N\mathbf{j}$ is

____.

The **divergence** of **F**$(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is

____.

If div **F** = 0, then **F** is said to be ___________________________.

Divergence can be viewed as ______________________________
____
____
____.

If **F**$(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is a vector field and M, N, and P have continuous second partial derivatives, then __________________.

Additional notes

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Section 15.2 Line Integrals

Objective: In this lesson you learned how to find a piecewise smooth parametrization, and write and evaluate a line integral.

I. Piecewise Smooth Curves (Page 1069)

What you should learn How to understand and use the concept of a piecewise smooth curve

A plane curve C given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $a \le t \le b$, is **smooth** if ____ ____. A space curve C given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $a \le t \le b$, is **smooth** if ____ ____. A curve C is **piecewise smooth** if ____ ____ ____.

II. Line Integrals (Pages 1070–1073)

What you should learn How to write and evaluate a line integral

If f is defined in a region containing a smooth curve C of finite length, then the **line integral of f along C** is given by

for a plane

____

or by

____

for space, provided this limit exists.

Let f be continuous in a region containing a smooth curve C. If C is given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, where $a \le t \le b$, then

____

If C is given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, where $a \le t \le b$, then

____

If $f(x, y, z) = 1$, the line integral gives ________________

__________________________________.

III. Line Integrals of Vector Fields (Pages 1074–1076)

> ***What you should learn***
> How to write and evaluate a line integral of a vector field

Let **F** be a continuous vector field defined on a smooth curve C given by $\mathbf{r}(t)$, $a \le t \le b$. The **line integral** of **F** on C is given by

____

IV. Line Integrals in Differential Form (Pages 1077–1078)

> ***What you should learn***
> How to write and evaluate a line integral in differential form

If **F** is a vector field of the form $\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$, and C is given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, then $\mathbf{F} \cdot d\mathbf{r}$ is often written in **differential form** as _____________________.

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Section 15.3 Conservative Vector Fields and Independence of Path

Objective: In this lesson you learned how to use the Fundamental Theorem of Line Integrals, independence of path, and conservation of energy.

I. Fundamental Theorem of Line Integrals (Pages 1083–1085)

What you should learn How to understand and use the Fundamental Theorem of Line Integrals

Let C be a piecewise smooth curve lying in an open region R and given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $a \le t \le b$. The **Fundamental Theorem of Line Integrals** states that if $\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$ is conservative in R, and M and N are continuous in R, then

____

where f is a potential function of **F.** That is, $\mathbf{F}(x, y) = \nabla f(x, y)$.

The Fundamental Theorem of Line Integrals states that ________

____

____

____.

II. Independence of Path (Pages 1086–1088)

What you should learn How to understand the concept of independence of path

Saying that the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is **independent of path** means that ____

____

____.

If **F** is continuous on an open connected region, then the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path if and only if __________

__________________________.

A curve C given by $\mathbf{r}(t)$ for $a \le t \le b$ is **closed** if

____________________.

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ have continuous first partial derivatives in an open connected region R, and let C be a piecewise smooth curve in R. The following conditions are equivalent.

1.

2.

3.

III. Conservation of Energy (Page 1089)

> ***What you should learn***
> How to understand the concept of conservation of energy

State the Law of Conservation of Energy.

The **kinetic energy** of a particle of mass m and speed v is

________________________.

The **potential energy** p of a particle at point (x, y, z) in a conservative vector field **F** is defined as ________________

____________________, where f is the potential function for **F.**

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Section 15.4 Green's Theorem

Objective: In this lesson you learned how to evaluate a line integral using Green's Theorem.

I. Green's Theorem (Pages 1093–1098)

What you should learn
How to use Green's Theorem to evaluate a line integral

A curve C given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, where $a \le t \le b$, is **simple** if ________________________________ —that is, $\mathbf{r}(c) \ne \mathbf{r}(d)$ for all c and d in the open interval (a, b). A plane region R is **simply connected** if ______________________ ____.

Let R be a simply connected region with a piecewise smooth boundary C, oriented counterclockwise (that is, C is traversed once so that the region R always lies to the left). Then **Green's Theorem** states that if M and N have continuous first partial derivatives in an open region containing R, then

____________________________________ .

Line Integral for Area

If R is a plane region bounded by a piecewise smooth simple closed curve C, oriented counterclockwise, then the area of R is given by ____________________________ .

II. Alternative Forms of Green's Theorem (Pages 1098–1099)

What you should learn
How to use alternative forms of Green's Theorem

With appropriate condition on **F,** C, and R, you can write Green's Theorem in the following vector form

For the second vector form of Green's Theorem, assume the same conditions for **F,** C, and R. Using the arc length parameter s for C, you have ______________________________. So, a unit tangent vector **T** to curve C is given by ____________________________________. The outward unit normal vector **N** can then be written as ______________________________. The second alternative form of Green's Theorem is given by

____________________________________.

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Section 15.5 Parametric Surfaces

Objective: In this lesson you learned how to sketch a parametric surface, find a set of parametric equations to represent a surface, find a normal vector, find a tangent plane, and find the area of a parametric surface.

I. Parametric Surfaces (Pages 1102–1103)

What you should learn
How to understand the definition of a parametric surface, and sketch the surface

Let x, y, and z be functions of u and v that are continuous on a domain D in the uv-plane. The set of points (x, y, z) given by $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$ is called a ______________ __________________. The equations $x = x(u, v)$, $y = y(u, v)$, and $z = z(u, v)$ are the ________________________________ for the surface.

If S is a parametric surface given by the vector-valued function **r,** then S is traced out by ______________________________ _____________________________________.

II. Finding Parametric Equations for Surfaces (Page 1104)

What you should learn
How to find a set of parametric equations to represent a surface

Writing a set of parametric equations for a given surface is generally more difficult than identifying the surface described by a given set of parametric equations. One type of surface for which this problem is straightforward, however is the surface given by $z = f(x,y)$. You can parameterize such a surface as _____________________________________.

III. Normal Vectors and Tangent Planes (Pages 1105–1106)

What you should learn
How to find a normal vector and a tangent plane to a parametric surface

Let S be a smooth parametric surface $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$ defined over an open region D in the uv-plane. Let (u_0, v_0) be a point in D. A normal vector at the point $(x_0, y_0, z_0) = (x(u_0, v_0), y(u_0, v_0), z(u_0, v_0))$ is given by

.

____

IV. Area of a Parametric Surface (Pages 1106–1108)

What you should learn
How to find the area of a parametric surface

Let S be a smooth parametric surface $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$ defined over an open region D in the uv-plane. If each point on the surface S corresponds to exactly one point in the domain D, then the **surface area** S is given by ______________________________________,

where $\mathbf{r}_u =$ ____________________________ and

$\mathbf{r}_v =$ ____________________________.

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Section 15.6 Surface Integrals

Objective: In this lesson you learned how to evaluate a surface integral, determine the orientation of a surface, and evaluate a flux integral.

I. Surface Integrals (Pages 1112−1115)

> ***What you should learn***
> How to evaluate a surface integral as a double integral

Let S be a surface with equation $z = g(x, y)$ and let R be its projection onto the xy-plane. If g, g_x, and g_y are continuous on R and f is continuous on S, then the **surface integral of *f* over *S*** is

____

II. Parametric Surfaces and Surface Integrals (Page 1116)

> ***What you should learn***
> How to evaluate a surface integral for a parametric surface

For a surface S given by the vector-valued function $\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$ defined over a region D in the uv-plane, you can show that the surface integral of $f(x, y, z)$ over S is given by

____

III. Orientation of a Surface (Page 1117)

> ***What you should learn***
> How to determine the orientation of a surface

Unit normal vectors are used to __________________________
__________________________. A surface is called **orientable** if
____
____
____.
If this is possible, S is called ______________________________.

IV. Flux Integrals (Pages 1118−1121)

> ***What you should learn***
> How to understand the concept of a flux integral

Suppose an oriented surface S is submerged in a fluid having a continuous velocity field **F**. Let ΔS be the area of a small patch of the surface S over which **F** is nearly constant. Then the amount of fluid crossing this region per unit time is approximated by____ __________________. Consequently, the volume of fluid crossing the surface S per unit time is called ________________ _______________.

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, where M, N, and P have continuous first partial derivatives on the surface S oriented by a unit normal vector $\mathbf{N}$. The **flux integral of F across *S*** is given by ____________________________.

Let S be an oriented surface given by $z = g(x, y)$ and let R be its projection onto the xy-plane. If the surface is oriented upward, $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS =$ ____. If the surface is oriented downward, $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS =$

____.

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Section 15.7 Divergence Theorem

Objective: In this lesson you learned how to use the Divergence Theorem and how to use it to calculate flux.

I. Divergence Theorem (Pages 1124–1128)

> ***What you should learn***
> How to understand and use the Divergence Theorem

The **Divergence Theorem** gives the relationship between ______ ____ __________________________.

In the Divergence Theorem, the surface S is **closed** in the sense that it ____ ________________________.

Let Q be a solid region bounded by a closed surface S oriented by a unit normal vector directed outward from Q. The **Divergence Theorem** states that if **F** is a vector field whose component functions have continuous first partial derivatives in Q, then ___________________________________.

II. Flux and the Divergence Theorem (Pages 1129–1130)

> ***What you should learn***
> How to use the Divergence Theorem to calculate flux

Consider the two sides of the equation $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS = \iiint_Q \operatorname{div} \mathbf{F}\, dV$. The flux integral on the left determines ____ ______________________________. This can be approximated by ____.

The triple integral on the right measures ____________________ ____ _______________________________ _____________________.

The point (x_0, y_0, z_0) in a vector field is classified as a **source** if ____________________; a **sink** if ____________________, or **incompressible** if __________________________.

In hydrodynamics, a *source* is a point at which ____. A *sink* is a point at which ____.

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Section 15.8 Stokes's Theorem

Objective: In this lesson you learned how to use Stokes's Theorem to evaluate a line integral or a surface integral and how to use curl to analyze the motion of a rotating liquid.

I. Stokes's Theorem (Pages 1132–1134)

> ***What you should learn***
> How to understand and use Stokes's Theorem

Stokes's Theorem gives the relationship between ____________
____
____.

The positive direction along C is ____________________ relative to the normal vector **N.** That is, if you imagine grasping the normal vector **N** with your right hand, with your thumb pointing in the direction of **N,** your fingers will point __________________
____.

Let S be an oriented surface with unit normal vector **N,** bounded by a piecewise smooth simple closed curve C with a positive orientation. **Stokes's Theorem** states that if **F** is a vector field whose component functions have continuous first partial derivatives on an open region containing S and $C,$ then

____.

II. Physical Interpretation of Curl (Pages 1135–1136)

> ***What you should learn***
> How to use curl to analyze the motion of a rotating liquid

curl $\mathbf{F}(x, y, z) \cdot \mathbf{N}$ = ______________________________________

The rotation of **F** is maximum when _____________________
____________________________________. Normally, this tendency to rotate will vary from point to point on the surface S, and Stokes's Theorem says that the collective measure of this rotational tendency taken over the entire surface S (surface integral) is equal to ____________________________________
____.

If curl $\mathbf{F} = \mathbf{0}$ throughout region Q, the rotation of $\mathbf{F}$ about each unit normal $\mathbf{N}$ is ____________________. That is, $\mathbf{F}$ is ___________________________.

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Chapter 16 Additional Topics in Differential Equations

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Section 16.1 Exact First-Order Equations

Objective: In this lesson you learned how to recognize and solve exact differential equations.

I. Exact Differential Equations (Pages 1144–1146)

What you should learn How to solve an exact differential equation

The equation $M(x, y)dx + N(x, y)dy = 0$ is an **exact differential equation** if ____

____

____.

The general solution of the equation is ______________________.

State the **Test for Exactness.**

A ____ is actually a special type of an exact equation.

Example 1: Test whether the differential equation $(5x - x^3 y)dx + \left(y - \frac{1}{4}x^4\right)dy = 0$ is exact.

A general solution $f(x, y) = C$ to an exact differential equation can be found by ____

____.

II. Integrating Factors (Pages 1147–1148)

> ***What you should learn***
> How to use an integrating factor to make a differential equation exact

If the differential equation $M(x, y)dx + N(x, y)dy = 0$ is not exact, it may be possible to make it exact by ________________
____
____.

Consider the differential equation $M(x, y)dx + N(x, y)dy = 0$. If $\dfrac{1}{N(x, y)}[M_y(x, y) - N_x(x, y)] = h(x)$ is a function of x alone, then ____________________________ is an integrating factor. If $\dfrac{1}{M(x, y)}[N_x(x, y) - M_y(x, y)] = k(y)$ is a function of y alone, then ____________________________ is an integrating factor.

Additional notes

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Section 16.2 Second-Order Homogeneous Linear Equations

Objective: In this lesson you learned how to solve second-order homogeneous linear differential equations and higher-order homogeneous linear differential equations.

I. Second-Order Linear Differential Equations (Pages 1151–1154)

What you should learn
How to solve a second-order linear differential equation

Let $g_1, g_2, \ldots g_n$ and f be functions of x with a common (interval) domain. An equation of the form

$$y^{(n)} + g_1(x)y^{(n-1)} + g_2(x)y^{(n-2)} + \cdots + g_{n-1}(x)y' + g_n(x)y = f(x)$$

is called a ____.

If $f(x) = 0$, the equation is _________________________;

otherwise, it is ____________________________.

The functions $y_1, y_2, \ldots, y_n$ are _______________________ if the only solution of the equation $C_1y_1 + C_2y_2 + \cdots + C_ny_n = 0$ is the trivial one, $C_1 = C_2 = \cdots = C_n = 0$. Otherwise, this set of functions is ____________________________.

If y_1 and y_2 are linearly independent solutions of the differential equation $y'' + ay' + by = 0$, then the general solution is

_______________________, where C_1 and C_2 are constants.

In other words, if you can find two linearly independent solutions, you can obtain the general solution by ____________

____.

The **characteristic equation** of the differential equation $y'' + ay' + by = 0$ is _________________________.

The solutions of $y'' + ay' + by = 0$ fall into one of the following there cases, depending on the solutions of the characteristic equation, $m^2 + am + b = 0$.

1.

2.

3.

II. Higher-Order Linear Differential Equations (Page 1155)

Describe how to solve higher-order homogeneous linear differential equations.

What you should learn
How to solve a higher-order linear differential equation

III. Application (Pages 1156–1157)

Describe Hooke's Law.

What you should learn
How to use a second-order linear differential equation to solve an applied problem

The equation that describes the undamped motion of a spring is

________________________________.

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Section 16.3 Second-Order Nonhomogeneous Linear Equations

Objective: In this lesson you learned how to solve second-order nonhomogeneous linear differential equations.

I. Nonhomogeneous Equations (Page 1159)

Let $y'' + ay' + by = F(x)$ be a second-order nonhomogeneous linear differential equation. If y_p is a particular solution of this equation and y_h is the general solution of the corresponding homogeneous equation, then ______________________ is the general solution of the nonhomogeneous equation.

> ***What you should learn***
> How to recognize the general solution of a second-order nonhomogeneous linear differential equation

II. Method of Undetermined Coefficients (Pages 1160–1162)

If $F(x)$ in $y'' + ay' + by = F(x)$ consists of sums or products of x^n, e^{mx}, $\cos \beta x$, or $\sin \beta x$, you can find a particular solution y_p by the method of ______________________________.

Describe how to use this method.

> ***What you should learn***
> How to use the method of undetermined coefficients to solve a second-order nonhomogeneous linear differential equation

III. Variation of Parameters (Pages 1163–1164)

Describe the conditions to which the method of variation of parameters is best suited.

> ***What you should learn***
> How to use the method of variation of parameters to solve a second-order nonhomogeneous linear differential equation

To use the method of variation of parameters to find the general solution of the equation $y'' + ay' + by = F(x)$, use the following steps.

1.

2.

3.

4.

Homework Assignment

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Section 16.4 Series Solutions of Differential Equations

Course Number
Instructor
Date

Objective: In this lesson you learned how to use power series to solve differential equations.

I. Power Series Solution of a Differential Equation (Page 1167–1168)

What you should learn How to use a power series to solve a differential equation

Recall that a power series represents a function f on ___________ ______________________, and you can successively differentiate the power series to obtain a series for f', f'', and so on.

Describe how to use power series in the solution of a differential equation.

II. Approximation by Taylor Series (Page 1169)

What you should learn How to use a Taylor series to find the series solution of a differential equation

What type of series can be used to solve differential equations with initial conditions?

Describe how to use this method.

Additional notes

Homework Assignment

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